INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal, Hyderabad -500 043

## MECHANICAL ENGINEERING

## TUTORIAL QUESTION BANK

| Course Name | $:$ | PROBABILITY \& STATISTICS |
| :--- | :---: | :--- |
| Course Code | $:$ | A30008 |
| Class | $:$ | II B. Tech I Semester |
| Branch | $:$ | MECH |
| Year | $:$ | $2016-2017$ |
| Course Coordinator | $:$ | Mrs. L. Indira, Associate Professor |
| Course Faculty | $:$ | Mrs. L. Indira, Associate Professor |

## OBJECTIVES

To meet the challenge of ensuring excellence in engineering education, the issue of quality needs to be addressed, debated and taken forward in a systematic manner. Accreditation is the principal means of quality assurance in higher education. The major emphasis of accreditation process is to measure the outcomes of the program that is being accredited.

In line with this, Faculty of Institute of Aeronautical Engineering, Hyderabad has taken a lead in incorporating philosophy of outcome based education in the process of problem solving and career development. So, all students of the institute should understand the depth and approach of course to be taught through this question bank, which will enhance learner's learning process.

| S No | QUESTION | Bloms taxonomy <br> level | Course <br> Outcomes |
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| SINGLE RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS |  |  |  |,



| 10 | Write the properties of continuous random variable |  |  |  |  |  |  |  | Understand | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UNIT - IIMULTIPLE RANDOM VARIABLES, CORRELATION \&REGRESSION |  |  |  |  |  |  |  |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |  |  |  |  |  |  |  |
| 1 | State the properties of joint distribution function of two random variable |  |  |  |  |  |  |  | Analyze | 5 |
| 2 | The equations of two regression lines obtained in a correlation analysis are $3 x+12 y=19,3 y+9 x=46$. Find means of $x$ and $y$ |  |  |  |  |  |  |  | Evaluate | 5 |
| 3 | Given $\mathrm{n}=10, \sigma_{x}=5.4, \sigma_{y}=6.2$ and sum of the product of deviation from the mean of X and Y is 66 find the correlation coefficient |  |  |  |  |  |  |  | Evaluate | 5 |
| 4 | From the following data calculate (i) correlation c coefficient (ii) standard deviation of Y$\text { bxy }=0.85, \text { byx }=0.89, \sigma_{x}=3$ |  |  |  |  |  |  |  | Evaluate | 6 |
| 5 | If $r_{12}=0.77, r_{13}=0.72, r_{23}=0.52$ Find the multiple correlation coefficient. |  |  |  |  |  |  |  | Evaluate | 5 |
| 6 | Determine the probability of getting at least 60 heads when 100 coins are tossed. |  |  |  |  |  |  |  | Understand \& Evaluate | 6 |
| 7 | Explain about random vector concepts |  |  |  |  |  |  |  | Analyze | 6 |
| 8 | If a random variable $\mathrm{W}=\mathrm{X}+\mathrm{Y}$ where X and Y are two independent random variables what is the density function of W |  |  |  |  |  |  |  | Analyze | 6 |
| 9 | Explain types of correlations |  |  |  |  |  |  |  | Remember | 7 |
| 10 | Write the properties of rank correlation coefficient |  |  |  |  |  |  |  | Analyze | 7 |
| 11 | Write the properties of regression lines |  |  |  |  |  |  |  | Analyze | 7 |
| 12 | Write the difference between correlation and regression |  |  |  |  |  |  |  | Remember | 7 |
| 13 | The rank correlation coefficient between the marks in two subjects is 0.8 .the sum of the squares of the difference between the ranks is 33.find the number of students |  |  |  |  |  |  |  | Evaluate | 7 |
| 14 | Find the angle between the regression lines if S.D of Y is twice the S.D of X and $\mathrm{r}=0.25$ |  |  |  |  |  |  |  | Evaluate | 7 |
| 15 | Derive the angle between the two regression lines |  |  |  |  |  |  |  | Evaluate | 7 |
| Part - B (Long Answer Questions) |  |  |  |  |  |  |  |  |  |  |
| 1 | Consider the joint probability density function $\mathrm{f}(\mathrm{x}, \mathrm{y})=\mathrm{xy}, 0<\mathrm{x}<1$, $0<y<2$. Find marginal density function |  |  |  |  |  |  |  | Evaluate | 6 |
| 2 | Two independent variable X and Y have means 5 and 10 and variances 4 and 9 respectively. Find the coefficient of correlation between $U$ and $V$ where $U=3 x+4 y, V=3 x-y$ |  |  |  |  |  |  |  | Understand \& Evaluate | 7 |
| 3 | The probability density function of a random variable x is $f(x)=\frac{1}{2} \exp \left[-\frac{x}{2}\right], \quad x>0$. Find the probability of $1<\mathrm{x}<2$. |  |  |  |  |  |  |  | Evaluate | 6 |
| 4 | Let X and Y random variables have the joint density function $f(x, y)=2,0<x<y<1$ then find marginal density function |  |  |  |  |  |  |  | Evaluate | 6 |
| 5 | Find the rank correlation coefficient for the following ranks of 16 students$\begin{aligned} & (1,1),(2,10),(3,3),(4,4),(5,5),(6,7),(7,2),(8,6),(9,8),(10,11),(11,15),(12 \\ & , 9),(13,14),(14,12),(15,16),(16,13) \end{aligned}$ |  |  |  |  |  |  |  |  |  |
| 6 | Calculate the coefficient of correlation between age of cars and annual maintain cost and comment: |  |  |  |  |  |  |  |  |  |
| 7 | For 20 army personal the regression of weight of kidneys (Y) on weight of heart $(\mathrm{X})$ is $\mathrm{Y}=3.99 \mathrm{X}+6.394$ and the regression of weight of heart on weight of kidneys is $\mathrm{X}=1.212 \mathrm{Y}+2.461$. Find the correlation coefficient between the two variable and also their |  |  |  |  |  |  |  |  |  |


|  | means |  |  |
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| 8 | From 10 observations on price X and supply Y the following data was obtained <br> Find coefficient of correlation, line of regression of Yon X and X on Y |  |  |
| 9 | If the variance of X is 9.The two regression equations are 8 X $10 \mathrm{Y}+66=0$ and $40 \mathrm{X}-18 \mathrm{Y}-214=0$. Find correlation coefficient between X and Y and standard deviation of Y . |  |  |
| Part - C (Problem Solving and Critical Thinking) |  |  |  |
| 1 | Derive the angle between the two regression lines | Evaluate | 7 |
| 2 | If $\theta$ is the angle between two regression lines then show that $\sin \theta \leq 1$. $\mathrm{r}^{2}$ | Apply | 7 |
| 3 | What is the marginal distributions of X and Y . | Analyze | 6 |
| 4 | Write the normal equations of straight line | Analyze | 7 |
| 5 | Find mean value of the variables X and Y and coefficient of correlation from the following regression equations $2 \mathrm{Y}-\mathrm{X}-50=0,3 \mathrm{Y}-$ 2X-10=0 | Evaluate | 7 |
| 6 | Define regression and give its uses | Remember | 7 |
| 7 | What are normal equations for regression lines? | Analyze | 7 |
| 8 | When the Regression coefficient is independent | Analyze | 7 |
| 9 | Find correlation coefficient if bxy $=085 \mathrm{y}$, byx $=089 \mathrm{x}{ }^{\sigma_{x}=3}$ | Evaluate | 7 |
| 10 | When the coefficient of correlation is maximum | Analyze | 7 |
| UNIT-IIISAMPLING DISTRIBUTIONS AND TESTING OF HYPOTHESIS |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |
| 1 | Explain different Types and Classification of sampling | Analyze | 8 |
| 2 | Write about Point Estimation, Interval Estimation | understand | 9 |
| 3 | What is the maximum error one can expect to make with probability 0.9 when using mean of a random sample of size $n=64$ to estimate the means of a population with $\sigma^{2}=256$ | understand | 9 |
| 4 | A random sample of 500 apples was taken from a large consignment and 60 were found to be bad, find the standard error. | Evaluate | 9 |
| 5 | Three masses are measured as $62.34,20,48,35.97 \mathrm{kgs}$ with S.D $0.54,0.21,0.46 \mathrm{kgs}$. Find the mean and S.D of the sum of masses. | Evaluate | 9 |
| 6 | What is the value of correction factor if $\mathrm{n}=5$ and $\mathrm{N}=200$. | Apply | 9 |
| 7 | Find the value of finite population correction factor for $\mathrm{n}=10$ and $\mathrm{N}=100$. | Evaluate | 9 |
| 8 | Write a short note on Hypothesis, Null and Alternative with suitable examples | understand | 9 |
| 9 | Write a short Note on Type I \& Type II error in sampling theory | understand | 9 |
| 10 | Prove that Sample Variance is not an Unbiased Estimation of Population Variance | understand | 9 |
| 11 | Write Properties of t-distribution | Analyze | 10 |
| 12 | Explain about Chi-Square | Analyze | 10 |
| 13 | Write a short note on Distinguish between t, F, Chi square test | understand | 10 |
| 14 | Explain about Bayesian estimation | Analyze | 9 |
| 15 | Compare Large Samples and Small sample tests | Create | 10 |
| Part - B (Long Answer Questions) |  |  |  |
| 1 | The mean of a random sample is an unbiased estimate of the mean of the population $3,6,9,15,27$. (i) List of all possible samples of size 3 that can be taken without replacement from the finite population. (ii) Calculate the mean of the each of the samples listed in (iii) And | Apply | 8 |



| 3 | Calculate traffic intensity if inter arrival time is 125 minutes and inter service time is 10 minutes. | Evaluate | 11 |
| :---: | :---: | :---: | :---: |
| 4 | If average number of arrivals is 4 per hour and average number of services is 6 per hour. What is the probability that a new arrival need not wait for the service. | Understand | 11 |
| 5 | If $\lambda=8$ and $\mu=12$ per hour. Calculate the average time spent by a customer in the system | Apply | 11 |
| 6 | What is the probability that there are more than or equal to 10 customers in the system. | Understand | 11 |
| 7 | Explain pure birth process | Analyze | 11 |
| 8 | Explain pure death process | Analyze | 11 |
| 9 | Derive expected number of customers | Evaluate | 11 |
| 10 | Derive average waiting time in queue | Evaluate | 12 |
| 11 | If $\lambda=6$ and $\mu=18$ per hour. Calculate the service time. | Evaluate | 12 |
| 12 | Define transient state and steady sate | Remember | 12 |
| 13 | Explain M/M/1 model | Analyze | 12 |
| 14 | Explain M/M/1 with infinite population | Analyze | 12 |
| 15 | Derive probability of having n customers $\mathrm{P}(\mathrm{n})$ in a queue $\mathrm{M} / \mathrm{M} / 1$, having Poisson arrival | Evaluate | 12 |
| Part - B (Long Answer Questions) |  |  |  |
| 1 | Consider a box office ticket window being managed by a single server. Customer arrive to purchase ticket according to Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 910 sec. Determine the following: a)Fraction of the time the server is busy b)The average number of customers queuing for service | Apply | 11 |
| 2 | Patients arrive at a clinic in a Poisson manner at an average rate of 6 per hour. The doctor on average can attend to 8 patients per hour. Assuming that the service time distribution is exponential, find Average number of patients waiting in the queue, Average time spent by a patient in the clinic | Evaluate | 12 |
| 3 | A bank plans to open a single server drive in banking facilities at a particular centre. It is estimated that 20 customers will arrive each hour on an average. If on an average, it required 2 minutes to process a customers transaction, determine 1.The proportion of time that the system will be idle 2. On the average how long a customer will have to wait before reaching the server? 3. Traffic intensity of Bank? 4.The fraction of customers who will have to wait | Analyze | 12 |
| 4 | A car park contains five cars .The arrival of cars in Poisson with a mean rate of 10 per/hour. The length of time each car spends in the car park has negative exponential distribution with mean of two hours. how many cars are in the car park on average and what is the probability of newly arriving costumer finding the car park full and having to park his car else where | Evaluate | 12 |
| 5 | Consider a self service store with one cashier. Assume Poisson arrivals and exponential service time. Suppose that 9 customers arrive on the average of every 5 minutes and the cashier can serve 19 in 5 minutes. Find (i) the average number of customers queuing for service. (ii)the probability of having more than 10 customers in the system. (iii) the probability that the customer has to queue for more than 2 minutes | Evaluate | 12 |
| 6 | A self service canteen employs one cashier at its counter. 8 customers arrive per every 10 minutes on an average. The cashier | Evaluate | 12 |


|  | can serve on average one per minute. Assuming that the arrivals are Poisson and the service time distribution is exponential, determine: (i)the average number of customers in the system; (ii) the average queue length; <br> (iii) average time a customer spends in the system; (iv) average waiting time of each customer |  |  |
| :---: | :---: | :---: | :---: |
| 7 | Customers arrive at a one window drive in bank according to a Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes The car space in front of the window including that for the serviced can accommodate a maximum of 3 cars. Other cars can wait outside the space. i) What is the probability that an arriving customer can drive directly to the space in front of the window? Ii) What is the probability that an arriving customer will have to wait outside the indicated space? Iii) How long is an arriving customer expected to wait before starting service | Apply | 12 |
| 8 | A fast food restaurant has one drive window. Cars arrive according to a Poisson process. Cars arrive at the rate of 2 per 5 minutes. The service time per customer is 1.5 minutes. Determine i) The Expected number of customers waiting to be served. ii) The probability that the waiting line exceeds 10 iii ) Average waiting time until a customer reaches the window to place an order. iv) The probability that the facility is idle | Apply | @ 12 |
| 9 | At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the various number of trains in the system. Find also the average waiting time of a new train coming into the yard | Apply | 12 |
| 10 | Consider a single server queuing system with Poisson input and exponential service time. Suppose the mean rate is 3 calling units per hour with the expected service time as 0.25 hours and the maximum permissible number of calling units in the system is two. Obtain the steady state probability distribution of the number of calling units in the system and then calculate the expected number in the system | Apply | 12 |
| Part - C (Problem Solving and Critical Thinking) |  |  |  |
| 1 | What is probability of arrivals during the service time of any given customer? | Analyze | 11 |
| 2 | What is FIFO means? | Remember | 11 |
| 3 | Define Jack eying. | Understand | 11 |
| 4 | Define reneging. | Understand | 11 |
| 5 | Define $\mathrm{m} / \mathrm{m} / 1: \mathrm{FIFO}$ | Understand | 11 |
| 6 | Model of queuing system. | Analyze | 11 |
| 7 | Define balking. | Understand | 10 |
| 8 | What is the pattern according to which customers are served? | Analyze | 11 |
| 9 | What is variance of queue length? | Analyze | 11 |
| 10 | How to calculate the idle time of the server according to queue theory | Evaluate | 10 |
| UNIT-VSTOCHASTIC PROCESSES |  |  |  |
| Part - A (Short Answer Questions) |  |  |  |
| 1 | Define stochastic process | Remember | 13 |
| 2 | Define a regular Markov chain | Remember | 13 |


| 3 | Find whether the matrix $\left[\begin{array}{ccc}0.75 & 0.25 & 0 \\ 0 & 0.5 & 0.5 \\ 0.6 & 0.4 & 0\end{array}\right]$ is a regular transition matrix or not. | Evaluate | 13 |
| :---: | :---: | :---: | :---: |
| 4 | Find periodic and aperiodic states in each of following transition probability matrices. <br> (i) $\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$ (ii) $\left[\begin{array}{cc}\frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2}\end{array}\right]$ | Evaluate | 13 |
| 5 | Define reducible and non-reducible states. | Remember | 13 |
| 6 | Consider the Markov chain with transition probability matrix $\left[\begin{array}{cccc}0 & 0 & 1 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1\end{array}\right]$ i <br> is this matrix irreducible? | Analyze | 13 |
| 7 | Explain different types of stochastic process | Analyze | 13 |
| 8 | Give examples of stochastic process | Create | 13 |
| 9 | Find the expected duration of the game for double stakes | Evaluate | 13 |
| 10 | Define Markov's chain | Understand | 13 |
| 11 | Explain Markov's property | Understand | 13 |
| 12 | Explain transition probabilities | Understand | 13 |
| 13 | Explain stationary distribution | Understand | 13 |
| 14 | Explain limiting distribution | Understand | 13 |
| 15 | Explain irreducible and reducible | Understand | 13 |
| Part - B (Long Answer Questions) |  |  |  |
| 1 | The transition probability matrix is given by $P=\left[\begin{array}{lll}0.1 & 0.4 & 0.5 \\ 0.2 & 0.2 & 0.6 \\ 0.7 & 0.2 & 0.1\end{array}\right]$ and $P_{0}=\left[\begin{array}{lll}0.4 & 0.4 & 0.2\end{array}\right]$ (a) Find the distribution after three transitions. (b) Find the limiting probabilities. | Evaluate | 13 |
| 2 | If the transition probability matrix of market shares of three brands $A, B$, and $C$ is $\left[\begin{array}{ccc}0.4 & 0.3 & 0.3 \\ 0.8 & 0.1 & 0.1 \\ 0.35 & 0.25 & 0.4\end{array}\right]$ and the initial market shares are $50 \%, 25 \%$ and $25 \%$, Find (a) The market shares in second and third periods (b) The limiting probabilities. | Evaluate | 13 |
| 3 | Define the stochastic matrixes which of the following stochastic matrices are regular. (a) $\left[\begin{array}{ccc}1 / 2 & 1 / 4 & 1 / 4 \\ 0 & 1 & 0 \\ 1 / 2 & 0 & 1 / 2\end{array}\right]$ <br> (b) $\left[\begin{array}{ccc}2 & 1 / 2 & 0 \\ 1 / 2 & 1 / 2 & 0 \\ 1 / 4 & 1 / 4 & 1 / 2\end{array}\right]$ | Remember \& Evaluate | 13 |
| 4 | Three boys A, B, C are throwing a ball to each other. A always throws the ball to B ; B always throws the ball to C ; but C is just as likely to throw the ball to $B$ as to A. Show that the process is Markovian. Find the transition matrix and classify the states. Do all the states are ergodic | Understand \& Apply | 13 |
| 5 | A gambler has Rs.2. He bets Rs. 1 at a time and wins Rs. 1 with probability 0.5 . He stops Playing if he loses Rs. 2 or wins Rs.4.i)What is the Transition probability matrix of the related markov chain? (b) What is the probability that he has lost his money at the end of 5 | Understand \& Apply | 14 |


|  | plays |  |  |
| :---: | :---: | :---: | :---: |
| 6 | Check whether the following markov chain is regular and $\text { ergodic? }\left[\begin{array}{cccc} \mathbf{1} & \mathbf{1} / \mathbf{2} & \mathbf{1} / \mathbf{2} & \mathbf{0} \\ \mathbf{1} / \mathbf{2} & 0 & 0 & \mathbf{1} / 2 \\ \mathbf{1} \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{1} / \mathbf{2} \\ \mathbf{0} & \mathbf{1} / \mathbf{2} & \mathbf{1} / \mathbf{2} & \mathbf{1} / \mathbf{2} \end{array}\right]$ | Apply | 13 |
| 7 | The transition probability matrix of a marker chain is given by $\left[\begin{array}{ccc}0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8\end{array}\right]$ irreducibleor not? | Evaluate | 13 |
| 8 | . Which of the following matrices are Stochastic <br> i) $\left[\begin{array}{cc}1 / 2 & 0 \\ 0 & 1\end{array}\right]$ <br> ii) $\left[\begin{array}{ll}\mathbf{0} & 1 \\ 1 & 0\end{array}\right]$ <br> iii) $\left[\begin{array}{ccc}1 / 2 & 1 / 4 & 1 / 4 \\ 1 & 1 & 0 \\ 1 / 2 & 1 / 2 & 0\end{array}\right]$ | Apply | 13 |
| 9 | Which of the following Matrices are Regular <br> i) $\left[\begin{array}{cc}1 / 2 & 1 / 2 \\ 0 & 1\end{array}\right]$ <br> ii) $\left[\begin{array}{ll}\mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0}\end{array}\right]$ <br> iii) $\left[\begin{array}{ccc}1 / 2 & 1 / 4 & 1 / 4 \\ 0 & 1 & 0 \\ 1 / 2 & 1 / 2 & 0\end{array}\right]$ | Apply | 13 |
| 10 | a) Is the Matrix $\left[\begin{array}{cccc}\mathbf{0 . 4} & 0.6 & 0 & 0 \\ 0.3 & 0.7 & \mathbf{0} & \mathbf{0} \\ 0.2 & 0.4 & \mathbf{0 . 1} & \mathbf{0 . 3} \\ 0 & \mathbf{0} & \mathbf{0} & \mathbf{1}\end{array}\right]$ irreducible? (b) Is the Matrix $\mathrm{p}=\left[\begin{array}{ccc}\mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} 2 & \mathbf{1} / \mathbf{6} & \mathbf{1} / 3 \\ \mathbf{1} / 3 & 2 / 3 & \mathbf{0}\end{array}\right]$ Stochastic? | Evaluate | 13 |
| Part - C (Problem Solving and Critical Thinking) |  |  |  |
| 1 | What do you call the random variable in stochastic process? | Analyze | 13 |
| 2 | When the state is said to be Ergodic. | Analyze | 13 |
| 3 | What is null persistent state? | Understand | 13 |
| 4 | What is Markov process? | Understand | 13 |
| 5 | Give an example of discrete parameter Markov chain. | Create | 13 |
| 6 | When a matrix is said to be regular. | Understand | 13 |
| 7 | What is the use of Markov process? | Understand | 13 |
| 8 | When the state is said to be commute with each other. | Understand | 13 |
| 9 | Let $p=\frac{1}{2}, q=\frac{1}{2}, z=500, a=1000$ then find the expected duration of the game | Evaluate | 14 |
| 10 | If the stakes are doubled while the initial capital remain unchanged the probability ruin decreases for the player whose probability of success is $\mathrm{P}<1 / 2$ and increases for the adversary | Apply | 14 |

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