LECTURE NOTES

ON

HEAT TRANSFER

III B TECH II SEM

JNTUH - R13

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UNIT - 1

HEAT TRANSFER

- WHAT IS HEAT TRANSFER?
- HOW IS HEAT TRANSFERRED?
- WHY IS IT IMPORTANT TO STUDY IT?
- HOW IS HEAT TRANSFER DIFFERENT FROM THERMODYNAMICS

WHAT IS HEAT TRANSFER

 HEAT TRANSFER IS ENERGY IN TRANSIT DUE TO A TEMPERATURE DIFFERENCE

MODES OF HEAT TRANSFER

- CONDUCTION
- CONVECTION
- RADIATION

CONDUCTION

• Conduction through a solid or a stationary fluid



CONVECTION

 CONVECTION FROM A SURFACE TO A MOVING FLUID

$$\boldsymbol{q}^{\prime\prime} = \boldsymbol{h} \big(\boldsymbol{T}_{\infty} - \boldsymbol{T}_{S} \big)$$



RADIATION

• NET RADIATION HEAT EXCHANGE BETWEEN TWO SURFACES



CONDUCTION – ATOMIC OR MOLECULAR ACTIVITY

- A GAS WITH TEMPERATURE GRADIENT
- NO BULK MOTION $T_1 > T_2$



FOURIER'S LAW OF CONDUCTION

 1-D Conduction $q'x' = -k \frac{dT}{dx}$ $\frac{dT}{dx} = \frac{T_2 - T_1}{L}$ $\boldsymbol{q}_{\boldsymbol{x}}^{\prime\prime} = -\boldsymbol{k} \frac{\boldsymbol{T}_2 - \boldsymbol{T}_1}{\boldsymbol{L}}$ $\boldsymbol{q}_{x}^{\prime\prime}=\kappa \frac{\boldsymbol{T}_{1}-\boldsymbol{T}_{2}}{\boldsymbol{I}}$



CONVECTION

Energy transfer due to

Random Molecular Motion (Diffusion)

Motion Of Fluid (Advection) Diffusion

+ Advection - Convection

Diffusion - Within The Velocity Boundary Layer Advection - Outside The Boundary Layer

> FLUID MECHANICS IS INDISPENSABLE FOR CONVECTION Boundary Layer Development



u(y)



BOILING AND CONDENSATION – involve phase change





Droplets

Heating

NEWTON'S LAW OF COOLING $q'' = h(T_{\infty} - T_{s})$

• *q*" – positive – heat is transferred from the surface

 $T_{S} > T_{\infty}$

- q" negative heat is transferred to the surface
 T_S < T_∞
- *h f* (surface geometry, fluid, nature of flow)

TYPICAL VALUES OF h

Process	h (W/m ² .K)
Free convection	
Gases	2-25
Liquids	50-1000
Forced Convection	
Gases	25-250
Liquids	50-20000
Boiling and condensation	2500-1,00,000

RADIATION

- Energy emitted by matter that is at a finite temperature
- Radiation solid, liquid, gas
- Energy of radiation transported E.M.Waves
- No medium is required (vacuum is perfect medium)

EMISSIVE POWER - Rate at which energy is released per unit area

STEFAN-BOLTZMAN LAW
$$E_{b} = \sigma T_{s}^{4}$$

 T_s – Absolute temperature of the surface (K) σ - Stefan-Boltzman constant (5.67 x 10⁻⁸ W/m².K⁴) *Eb* – Emissive Power (W/m²) BLACK SURFACE – Stefan Boltzman Law REAL SURFACE – Less Than Black Surface $E = \varepsilon \sigma T_s^4$

 ε - Emissivity $0 \le \varepsilon \le 1$



Surface of ε , α and T_s

- **G** IRRADIATION Rate at which radiation is incident on a unit area of the surface (receiving/sending)
- Gabs ABSORPTION A portion of the irradiation may be absorbed by the surface

 $\begin{aligned} \textbf{G}_{abs} &= \alpha \ \textbf{G} \\ \textbf{0} &\leq \alpha \leq \textbf{1} \end{aligned}$

RELATIONSHIP TO THERMODYNAMICS

THERMODYNAMICS

- equilibrium states of matter (no temp gradient)
- amount of energy required in the form of heat for a system to pass from one equilibrium state to another

HEAT TRANSFER

- thermodynamic non-equilibrium process
- rate at which heat transfer occurs

STEADY STATE CONDUCTION

Objectives

The aim of this lecture is to understand the Fourier's law of conduction (both physically and mathematically) and introduce various thermal properties like thermal conductivity and thermal diffusivity

INTRODUCTION

•Heat transfer has direction as well as magnitude.

•The rate of heat conduction in a specified direction is proportional to the temperature gradient.

•Heat conduction in a medium is three dimensional and time dependent

T = f(x, y, z, t)

•Heat conduction in a medium is said to be

-STEADY - temperature does not vary with time.

-UNSTEADY (transient) - temperature varies with time

- One dimensional

T=*f*(*x*) or *T*=*f*(*y*) or *T*=*f*(*z*)

- Two dimensional

T=f(x,y) or *T=f(y,z)* or *T=f(x,z)*

- Three dimensional

T = f(x,y,z)

CONDUCTION RATE EQUATION



A cylindrical rod of known material is insulated on its lateral surface.

Its end faces are maintained at different temperatures with $T_1 > T_2$

The heat transfer rate q_x depends upon,

- The temperature difference, T
- The rod length, x
- The cross sectional area, A

The heat transfer rate varies as,

$$q'' x \alpha A \frac{\Delta T}{\Delta x}$$
 (2.1)

the proportionality may be converted to an equality by introducing a coefficient that is a measure of the material behavior.

Therefore,
$$q''_x = \frac{\Delta T}{k \Delta x} A$$

where k is the thermal conductivity (W/m.K)

Applying limits,

$$\lim_{\Delta x \to 0} q''_{x} = \lim_{\Delta x \to 0} k A \frac{\Delta T}{\Delta x}$$
(2.2a)

Therefore,
$$q^{''} = -kA \frac{dT}{d}$$
 (2.3)

and the heat flux

$$q''_{x} = \frac{q_{x}}{A} = -k \frac{dT}{dx}$$
(2.4)

The minus sign is necessary because heat is always transferred in the direction of decreasing temperature.

Fourier's law, as written in Equation 2.4, follows that is normal to the cross sectional area A called an isothermal surface as illustrated in Figure 2.2.



Generalizing the conduction rate equation for three dimension gives,

$$\begin{array}{c} \mathbf{r} \\ \mathbf{q} = -k \nabla T = -k | i \end{array} \left(\begin{array}{c} \widehat{\partial T} & \widehat{\partial T} & \widehat{\partial T} \\ +j & \widehat{\partial T} + k & \widehat{\partial T} \end{array} \right)$$

$$\left(\begin{array}{c} \partial \mathbf{x} \\ 25 \end{array} \right) \partial \mathbf{y} \quad \partial \mathbf{z} \end{array} \right)$$

$$(2.4)$$

• In Cartesian coordinates, the general expression for q" is

$$q = iq_{x} + jq_{y} + kq_{z}$$
 (2.5)

• and from Equation 2.4a, it follows that,

$$q''_{x} = -k \frac{\partial T}{\partial x}; q''_{y} = -k \frac{\partial T}{\partial y}; q''_{z} = -k \frac{\partial T}{\partial z}$$
(2.6)

NOTE: Here we have assumed that,

- The medium in which conduction occurs is isotropic.
- The thermal conductivity is independent of the coordinate direction in an isotropic medium.

HEAT TRANSFER

UNIT - **2**

Objectives

- One dimensional steady conduction in plane wall, composite wall and cylinder is introduced. The approach is to reduce the heat diffusion equation for the case chosen.
- Using the appropriate boundary conditions, the heat diffusion equation is solved for temperature distribution and heat transfer rate is computed
- Analogy between thermal and electrical systems is drawn in order to aid the solving of conduction problems on the basis of electrical circuits

ONE-DIMENSIONAL STEADY STATE CONDUCTION

- We treat situations for which heat is transferred by diffusion under one dimensional, steady state conditions.
- In a one-dimensional system, temperature gradients exist along a single coordinate direction, and heat transfer occurs exclusively in that direction.
- The system is characterized by steady state conditions if the temperature at each point is independent of time.

We begin our consideration of one-dimensional, steady state conduction by discussing heat transfer with no internal generation.

THE PLANE WALL

For one dimensional conduction in a plane wall, temperature is a function of the *x* coordinate only and heat is transferred exclusively in this direction. In Figure 2.5, a plane wall separates two fluids of different temperatures.

Heat transfer occurs,

- by convection from the hot fluid at $T_{\infty,1}$ to one surface of the wall at $T_{S,1}$
- by conduction through the wall, and
- by convection from the other surface of the wall at $T_{s,2}$ to the cold fluid at $T_{\infty,2}$.

Let us first determine the temperature distribution, from which we can then obtain the conduction heat transfer rate.



FIGURE 2.5 HEAT TRANSFER THROUGH A PLANE WALL

Temperature Distribution

• The temperature distribution in the wall can be determined by solving the heat equation using proper boundary conditions. For steady state conditions with no energy source within the wall, the appropriate form of the heat equation is

$$\frac{d}{dx} \begin{pmatrix} \partial T \\ \partial x \end{pmatrix} = 0$$
(2.22)

Integrating the above equation twice to obtain the general solution

$$T(x) = C_1 x + C_2$$
 (2.23)

Applying boundary conditions of the first kind at x=0 and x=L to obtain the constants of integration, C_1 and, C_2 .

$$T(0) = T_{S,1} \text{ and } T(L) = T_{S,2}$$

Applying the condition at x = 0 to the general solution, it follows that

Similarly, at x=L,
which gives,

$$T_{s,1} = C_2$$

 $I_{2} = C_1 L + C_2 = C_1 L + T_1$
 $S_{s,2} = C_1 L + C_2 = C_1 L + T_1$
 $T_{s,2} - T_{s,1}$
 $L = C_1$

substituting into the general solution, the temperature distribution is then

$$T(X) = (T_{s,2} - T_{s,1}) \frac{X}{L} + T_{s,1}$$
 (2.24)

From this result, it is evident that, for one dimensional, steady state conduction in a plane wall with no heat generation and constant thermal conductivity, the temperature varies linearly with x.

Using Fourier's law, to determine the conduction heat transfer rate. That is,

$$q_{x=-kA}\frac{dT}{dxL} = \frac{kA}{dxL} \left(T_{s,1} - T_{s,2} \right)$$
(2.25)

Note that A is the area of the wall normal to the direction of heat transfer and for the plane wall, it is a constant independent of x.

The heat flux is then,

$$q''_{x} = \frac{q}{A} = \frac{k}{A} (T_{s,1} - T_{s,2})$$
 (2.26)

Equations 2.25 and 2.26 indicate that both the heat rate q_x and heat flux are constants, independent of x.

Thermal Resistance

- There exists an analogy between the diffusion of heat and electrical charge.
- Thermal resistance may be associated with the conduction of heat in the same fashion as an electrical resistance is associated with the conduction of electricity.
- Defining resistance as the ratio of a driving potential to the corresponding transfer rate
- it follows from Equation 2.25 that the thermal resistance for conduction is

(2.27)

$$R_{t,\text{COND}} = \frac{\begin{pmatrix} T_{s,1} & T_{s,2} \\ q_x \end{pmatrix}}{q_x} = \frac{L}{kA}$$
Similarly, for electrical conduction, Ohm's law provides an electrical resistance of the form

$$\boldsymbol{R}_{e} = \frac{\begin{pmatrix} \boldsymbol{E} & \boldsymbol{E} \\ \boldsymbol{s}, 1 & -\boldsymbol{s}, 2 \end{pmatrix}}{I} = \frac{\boldsymbol{L}}{\sigma A}$$
(2.28)

Therefore,

- the rate of heat transfer through a plane wall corresponds to the electric current
- the thermal resistance corresponds to electrical resistance and
- the temperature difference corresponds to voltage difference across the plane wall. (Figure. 2.6)



Figure 2.6 Analogy Between Thermal And Electrical Resistance Concepts

Consider convection heat transfer from a solid surface of area A (Figure 2.7) and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_{∞} , with a convection heat transfer coefficient h. Newton's law of cooling for convection heat transfer rate

The thermal resistance for convection is

$$q = hA\left(T_{s} - T_{\infty}\right) \tag{2.29}$$

$$R_{t,\text{CONV}} = \frac{\left(T_{s} - T_{\infty}\right)}{q} = \frac{1}{hA}$$
(2.30)



Figure 2.7 Schematic For Convection Resistance at a Surface

Consider steady one-dimensional heat flow through a plane wall of thermal conductivity k and thickness L that is exposed to convection on both sides to fluids at temperatures and with heat transfer coefficients h_1 and h_2 , respectively, as shown in Figure. 2.8.



Figure 2.8 Thermal Resistance Network For Heat Transfer Through A Plane Wall
Under steady state conditions, we have

$$q_{x} = h_{1} A \left(T_{\infty,1} - T_{s,1} \right) = \left(T_{s,1} - T_{s,2} \right) \frac{kA}{L} = h_{2} A \left(T_{s,2} - T_{\infty,2} \right)$$
(2.31)
$$q_{x} = \frac{\left(T_{\infty,1} - T_{s,1} \right)}{\frac{1}{h_{1} A}} = \frac{\left(T_{s,1} - T_{s,2} \right)}{\frac{L}{kA}} = \frac{\left(T_{s,2} - T_{\infty,2} \right)}{\frac{1}{h_{2} A}}$$
(2.32)

In terms of the overall temperature difference, , and the total thermal resistance, *R*_{tot} the heat transfer rate may also be expressed as

$$q_{x} = \frac{\begin{pmatrix} T & T \\ \infty & ,1 - \infty & ,2 \end{pmatrix}}{\kappa_{tot}}$$
(2.33)

In general, we can write

 $q = \frac{\left(\frac{T_{i} - T_{j}}{\sum}\right)}{\sum \text{ All resistances between the driving } T_{i} - T_{i}}$

Equivalent thermal circuits concepts is used for more complex systems, such as composite walls (i.e. any number of series and parallel thermal resistances due

to layers of different materials). Consider the series composite wall of Figure. 2.9. The one dimensional heat transfer rate for this system may be expressed as

$$q_{\mathbf{X}} = \frac{\begin{pmatrix} \mathbf{T} & \mathbf{T} \\ \infty & 1 & -\infty & 4 \end{pmatrix}}{\sum \mathbf{R}_{t}}$$
(2.34)

T Twhere $\infty, 1 - \infty, 4$ is the overall temperature difference and the summation includes all thermal resistances. Hence,



Figure 2.9 Equivalent Thermal Circuit For A Series Composite Wall

Also, the heat transfer rate can be related to the temperature difference and resistance associated with each element. For example,

$${}^{q}{}_{x} = \frac{T_{1} - T_{1}}{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}} = \frac{T_{1} - T_{2}}{\begin{pmatrix} L \\ A \\ K \\ A \end{pmatrix}} = \frac{T_{1} - T_{2}}{\begin{pmatrix} L \\ B \\ K \\ B \end{pmatrix}} = \dots$$
(2.36)

With the composite systems, it is usually convenient to work with an overall heat transfer coefficient, *U*, which is defined by the expression analogous to Newton's law of cooling. Accordingly,

$$q_X = UA \Delta T \tag{2.37}$$

where T is the overall temperature difference.

From Equations 2.34 and 2.37 we see that $UA = 1/R_{tot}$. Hence, for the composite wall of Figure.2.9,

$$U = \frac{1}{k_{tot}} = \frac{1}{\left[\left(\begin{array}{c} 1\\ 1\\ 1\end{array}\right) + \left(\begin{array}{c} 1\\ 1\\ 1\right) + \left(\begin{array}{c} 1\\ 1\\ 1\end{array}\right) + \left(\begin{array}{c} 1\\ 1\\ 1\right) + \left(\begin{array}{c} 1\\ 1\\ 1\end{array}\right) + \left(\begin{array}{c} 1\\ 1\\ 1\right) + \left(\begin{array}{c} 1\\ 1\\ 1\end{array}\right) + \left(\begin{array}{c} 1\\ 1\\ 1\right) + \left(\begin{array}{c} 1\\ 1\right) + \left(\begin{array}{$$

In general, we write



Contact Thermal Resistance:

- In heat conduction analysis through composite walls, we have assumed "perfect contact" at the interface of two layers, and thus no temperature drop at the interface.
- This would be the case when the surfaces are perfectly smooth and they produce a perfect contact at each point.
- In reality, however, even flat surfaces that appear smooth to the eye turn out to be rather rough when examined under a microscope, as shown in Figure 2.11, with numerous peaks and valleys.
- That is, a surface is microscopically rough no matter how smooth it appears to be.



• Ideal Thermal Contact



• Actual Thermal Contact

Figure 2.11 Temperature Distribution Along Two Solid Plates Pressed Against Each Other

- When two such surfaces are pressed against each other, the peaks will form good material contact but the valleys will form voids filled with air.
- As a result, an interface will contain numerous air gaps of varying sizes that act as insulation because of the low thermal conductivity of air.

Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called thermal contact*R*, *c* resistance, given by

$$R''_{t,c} = \frac{T_1 - T_2}{q'_x}$$
(2.40)

- For solids whose thermal conductivities exceed that of the interfacial fluid, the contact resistance may be reduced by increasing the area of the contact spots.
- Such an increase may be effected by increasing the joint pressure and/or by reducing the roughness of the mating surfaces.
- The contact resistance may also be reduced by selecting an interfacial fluid of large thermal conductivity.
- In this respect, no fluid (an evacuated interface) eliminates conduction across the gap, thereby increasing the contact resistance.

THE CYLINDER

Consider a hollow cylinder, whose inner and outer surfaces are exposed to fluids at different temperatures (Figure. 2.12). For steady state conditions with no heat generation, the appropriate form of the heat equation,

$$\frac{1}{r}\frac{d}{dr}\left(\frac{dT}{dr}\right) = 0$$
(2.41)

where, for a moment k is treated as a variable. The physical significance of this result becomes evident if we also consider the appropriate form of Fourier's law. The rate at which energy is conducted across the cylindrical surface in the solid may be expressed as

$$q_r = -kA \qquad \frac{dT}{dr} \qquad 2 \underset{=-k(\pi rL)}{2} \frac{dT}{dr} \qquad (2.42)$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer.

NOTE: Since, Equation 2.41 prescribes that the quantity $\frac{kr dT}{dr}$ is independent of *r*,

it follows from Equation 2.42 that the conduction heat transfer rate q_r (not the heat flux q_r) is a constant in the radial direction.



Figure 2.12 Hollow Cylinder With Convective Surface Conditions

Assuming the value of k to be constant, Equation 2.41 may be integrated twice to obtain the general solution

$$T(r) = C_1 \ln r + C_2$$
 (2.43)

Applying the boundary conditions to the general solution, i.e. $T(r_1) = T_{s,1}$ and $T(r_2) = T_{s,2}$ we obtain,

$$T_{s,1} = C_{1} \ln r_{1} + C_{2}$$
$$T_{s,2} = C_{1} \ln r_{2} + C_{2}$$

Solving for C₁ and C₂ and substituting into the general solution, we then obtain

$$T(r) = \frac{T_{1} - T_{2}}{(r_{1}/r_{1})} \ln \left(\frac{r}{r_{2}}\right)^{+T_{s,2}}$$
(2.44)

NOTE: that the temperature distribution associated with radial conduction through a cylindrical wall is logarithmic, not linear, as it is for the plane wall. The logarithmic distribution is shown in Figure. 2.12.

If the temperature distribution, Equation 2.44, is now used with Fourier's law, Equation 2.42, we obtain the following expression for the heat transfer rate:

$$q_{r} = \frac{2 \pi \left(\frac{r_{s,1}}{r_{s,1}} \right)}{\left(\frac{r_{2}}{r_{1}} \right)}$$
(2.45)

From this result it is evident that, for radial conduction in a cylindrical wall, the thermal resistance is of the form

$$R_{t,cond} = \frac{\frac{\binom{r_2}{\binom{l_1}{\binom{r_1}{2\pi Lk}}}}{2\pi Lk}}$$
(2.46)

This resistance is shown in Figure. 2.12. Note that since the value of q_r is independent of r, the foregoing result could have been obtained by using the alternative method, that is, by integrating Equation 2.42.

Consider now the composite system of Figure 2.13. Recalling how we treated the composite plane wall and neglecting contact resistances between the interface, the heat transfer rate may be expressed as

$$q_{r} = \frac{T_{\infty,1} - T_{\infty,2}}{\begin{pmatrix} 1\\ 2\pi r_{1} L h_{1} \end{pmatrix} + \begin{pmatrix} r_{2}\\ -\frac{1}{2\pi k A L} \\ -\frac{1}{2\pi k A L} \\ -\frac{1}{2\pi k A L} \\ -\frac{1}{2\pi k B L} + \begin{pmatrix} r_{3}\\ -\frac{1}{2\pi k B L} \\ -\frac{1}{2\pi$$

The above result may also be expressed in terms of an overall heat transfer coefficient. That is,

$$q_{r} = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}} = UA\left(T_{\infty,1} - T_{\infty,4}\right)$$
(2.48)

If U is defined in terms of the inside area, $A_1=2\pi r_1L$ Equations 2.47 and 2.48 may be equated to yield

$$U = \frac{1}{1 + \frac{r_1}{1 + \frac{r_2}{n_1}} + \frac{r_1}{r_1}} \begin{pmatrix} r_3 \\ r_1 \\ r_1 \\ k_B \\ r_2 \\ k_C \\ r_3 \\ r_4 \\ r_4$$

Note:

- UA is constant, while U is not
- In radial system q" is constant, while q is not



Figure 2.13 Temperature Distribution For A Composite Cylindrical Wall

Objectives

- One dimensional steady conduction in sphere is introduced
- The approach is to reduce the heat diffusion equation for the case chosen.
- Using the appropriate boundary conditions, the heat diffusion equation is solved for temperature distribution.
- Concept of critical radius of insulation is presented.

THE SPHERE

Consider a hollow sphere, whose inner and outer surfaces are exposed to fluids at different temperatures (Fig. 2.14).



Fig. 2.14 Conduction in a spherical shell

For steady state conditions with no heat generation, the appropriate form of the heat equation,

$$\frac{1}{r} \frac{d}{dr} \left(\frac{2}{r} \frac{dT}{dr} \right) = 0$$
 (2.50)

where, for a moment k is treated as a variable. The physical significance of this result becomes evident if we also consider the appropriate form of Fourier's law. The rate at which energy is conducted across the cylindrical surface in the solid may be expressed as

$$q_r = -kA \quad \frac{dT}{dr} \qquad \frac{4}{4} \sum_{r=-k(\pi r)}^{2} \frac{dT}{dr} \qquad (2.51)$$

where $A = 4\pi r^2$ is the area normal to the direction of heat transfer. Since, Equation 2.50 states that the quantity $kr^2 dT$ is independent of r, it follows from Equation 2.51 that the conduction heat transfer rate q_r (not the heat flux q_r ") is a constant in the radial direction.

Assuming the value of k to be constant, Equation 2.50 may be integrated twice to obtain the general solution

$$T(r) = \frac{C_1}{r} + C_2$$
 (2.52)

Applying, the following boundary conditions

 $T(r_1) = T_{s,1}$ and $T(r_2) = T_{s,2}$ we then obtain

$$T_{s,1} = \frac{C_1}{r_1} + C_2$$
$$T_{s,2} = \frac{C_1}{r_2} + C_2$$

Solving for C₁ and C₂ and substituting into the general solution, we then obtain

$$T(r) = T_{s,1} + \frac{T_{s,1} - T_{s,2}}{\left| \left(\frac{1}{r_{2}} - \frac{1}{r_{1}} \right) \right| \left| \frac{1}{r_{1}} - \frac{1}{r_{1}} \right|}$$
(2.53)

Note that the temperature distribution associated with radial conduction through a spherical wall is not linear, as it is for the plane wall under the same conditions.

If the temperature distribution, Equation 2.53, is now used with Fourier's law, Equation 2.51, we obtain the following expression for the heat transfer rate:

$$q_{r} = \frac{4 \frac{k}{\pi} \left(\frac{T}{s, 1 - s, 2} \right)}{\left| \left(\frac{1}{2} - \frac{1}{2} \right) \right|}$$
(2.54)

From this result it is evident that, for radial conduction in a spherical wall, the thermal resistance is of the form

$$\boldsymbol{R}_{t,cond} = \frac{\left| \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \right|}{\frac{\left(r_1 r_2 \right)}{4\pi k}}$$
(2.55)

Note that since the value of q_r is independent of r, the above result could have been obtained by using the alternative method, that is, by integrating Equation 2.51.

Spherical composites may be treated in much the same way as composite walls and cylinders, where approximate forms of the total resistance and overall heat transfer coefficient may be determined

$$\frac{dT}{dx} = \frac{q'_x}{k} = \frac{200 \times 10^3}{100} = 2000 \text{ K/M}$$

Case (b): In this case temperature decreases in the positive direction of *x*, and hence temperature gradient would be negative, and therefore Fourier equation becomes,

$$q''_{x} = -k \frac{dT}{dx}$$

$$\int_{0}^{L} q''_{x} dx = -k \int_{0}^{T_{2}} dT$$

$$q''_{x} = k \frac{T - T}{L} = 100 \frac{(600 - 400)}{0.1} = 200 \text{ kW/M}^{2}$$

$$\frac{dT}{dx} = -\frac{q''_{x}}{k} = -\frac{200 \times 10^{3}}{100} = -2000 \text{ K/M}$$

 $dx \quad k = 100$ Case (c): In this case temperature increases in the positive direction of *x*, and

Integrating,

hence temperature gradient would be positive, and therefore Fourier equation becomes, dT

$$q_X = k$$
 —

Integrating,

$$\int_{0}^{L} q''_{X} dx = \int_{1}^{1} \int_{1}^{2} dT$$

$$q''_{X} = k \frac{\left(\frac{T - T}{2}\right)}{L} = 100 \frac{\left(\frac{600 - 400}{0.1}\right)}{0.1} = 200 \text{ kW/M}^{2}$$

$$\frac{dT}{dx} = \frac{q''_{X}}{k} = \frac{200 \times 10^{3}}{100} = 2000 \text{ K/M}$$

Comments:

- 1. In case (a), the direction of heat transfer is in the negative direction of x.
- 2. In case (b), the direction of heat transfer is in the positive direction of x
- 3. In case (c), the direction of heat transfer is in the negative direction of x.

TRANSIENT HEAT CONDUCTION

STRUCTURE

Introduction Lumped System Analysis Criteria Of The Lumped System Analysis Transient Heat Conduction In Large Plane Walls, Long Cylinders, And Spheres Transient Heat Conduction In Semi-infinite Solids Transient Heat Conduction In Multidimensional Systems TIME DEPENDENT CONDUCTION - Temperature history inside a conducting body that is immersed suddenly in a bath of fluid at a different temperature. Ex: Quenching of special alloys

The temperature of such a body varies with time as well as position.

T(x,y,z,t)

(x,y,z) - Variation in the x,y and z directions

t - Variation with time

In this chapter, we consider the variation of temperature with *time* as well as *position* in one and multi-dimensional systems.

LUMPED SYSTEM ANALYSIS

COPPER BALL WITH UNIFORM TEMPERATURE



Temperature of the copper ball changes with time, but it does not change with position at any given time.

Temperature of the ball remains uniform at all times

POTATA TAKEN FROM BOILING WATER

Large potato put in a vessel with boiling water.



After few minutes, if you take out the potato, temperature distribution within the potato is not even close to being uniform. Thus, lumped system analysis is not applicable in this case.

Hot metal forging that is initially at a uniform temperature T_i and is quenched by immersing it in a liquid of lower temperature $T_{\infty} < T_i$



During a differential time interval *dt*, the temperature of the body rises by a differential amount *dT*. An energy balance of the solid for the time interval *dt* can be expressed as

$-hA(T-T_{\infty})dt = \rho VC_p dT$

 $\begin{array}{c|c} (1) \\ \tau = & \hline hA \\ \hline s \end{array} \end{array} \quad |(\rho V C_p) = R_t C_t \quad \begin{array}{c} R_t \cdot \text{Resistance to convection heat transfer} \\ C_t - \text{Lumped thermal capacitance of the solid} \end{array}$

Transient Temperature Response of Lumped Capacitance Solids



The rate of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling

$$Q = h A_{S} \left[T(t) - T_{\infty} \right]$$



Lumped system analysis is *exact* when *Bi* = 0

Generally, accepted norm for assuming lumped system analysis

$Bi \leq 0.1$

Spherical Copper
Ball
$$k = 401$$
 W/m K
 $D = 12$ cm
 $\pi \frac{D^3}{6}$
 $L_c = \frac{V}{A} = \frac{6}{\pi D^2} = \frac{D}{6} = 0.02 m$
 $Bi = \frac{h L_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$

Small bodies with higher thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis

Heat conduction in a specified direction *n* per unit surface area is expressed as

$$q = -k \frac{\partial T}{\partial n}$$

Larger the thermal conductivity \Rightarrow the smaller the temperature gradient


When the convection coefficient *h* is high and *k* is low, large temperature differences occur between the inner and outer regions of a large solid

Physical significance of the Fourier number

 $\tau = \underline{\alpha} \underline{t} = \underline{k} L^{2} (1/L) \underline{\Delta T} = \underline{The \ rate \ at \ whichheat \ is \ conducted \ across \ L \ of \ a \ body \ of \ volume \ L^{3}} L^{2} \rho \ C_{P} / t^{3} \Delta T L^{3} The rate at whichheat \ is stored \ in a \ body \ of \ volume \ L}$



What constitutes an infinitely large plate or an infinitely long cylinder?

A plate whose thickness is small relative to the other dimensions can be modeled as an infinitely large plate, except very near the outer edges.

But the edge effects on large bodies are usually negligible, and thus a large plane wall such as the wall of a house can be modeled as an infinitely large wall for heat transfer purposes. Similarly, a long cylinder whose diameter is small relative to its length can be analyzed as an infinitely long cylinder.

TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS

- A semi-infinite solid is an idealised body that has a single plane surface and extends to infinity in all directions. This idealised body is used to indicate that the temperature change in the part of the body in which we are interested (the region close to the surface) is due to the thermal conditions on a single surface.
- Ex: Earth temperature variation near its surface

Thick wall – temperature variation near one of its surfaces

• For short periods of time, most bodies can be modeled as

semi-infinite solids since heat does not have sufficient time to penetrate deep into the body and the thickness of the body does not enter into the heat transfer analysis.



Schematic of the semi-infinite medium

HEAT TRANSFER

UNIT - **3**

CONVECTIVE HEAT TRANSFER



Convection heat transfer involves 20°C • fluid motion

heat conduction

The fluid motion enhances the heat transfer, since it brings hotter and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in fluid. Therefore, the rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rate of heat transfer.

c) Conduction

Convection heat transfer strongly depends on

- fluid properties dynamic viscosity , thermal conductivity k, density and specific heat
- fluid velocity V
- Geometry and the roughness of the solid surface
- Type of fluid flow (such as being laminar or turbulent).

NEWTON'S LAW OF COOLING

$$\dot{Q}_{conv} = hA_s \left(T_s - T_\infty\right)$$

h = Convection heat transfer coefficient

- A_s = Heat transfer surface area
- *T*_{*s*} = Temperature of the surface

 T_{∞} = Temperature of the fluid sufficiently far from the surface LOCAL HEAT FLUX q''

conv

$$q_{conv''} = h_l \left(T_s - T_\infty \right)$$

*h*_l is the local convection coefficient

TOTAL HEAT TRANSFER RATE \dot{Q}_{conv}

$$\mathcal{Q}_{conv} = \int q_{conv} dA_{s} \qquad dA_{s} \qquad \mathcal{Q}_{conv} = \left(T_{s} - T_{\infty}\right) \int_{A_{s}} h_{l} dA_{s}$$

$$h = \frac{1}{A} \int_{s} h_{l} dA_{s}$$

$$h = \frac{1}{A} \int_{s} h_{l} dA_{s}$$

Local and total convection transfer (a) Surface of arbitrary shape. (b) Flat plate.



A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.



A similar phenomenon occurs for the temperature. When two bodies at different temperatures are brought into contact, heat transfer occurs until both bodies assume the same temperature at the point of contact.

Therefore, a fluid and a solid surface will have the same temperature at the point of contact. This is known as NO-TEMPERATURE-JUMP CONDITION.

An implication of the no-slip and the no-temperature jump conditions is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction*, since the fluid layer is motionless,

$$q_{conv} = q_{cond} = -k fluid \left. \frac{\partial T}{dy} \right|_{y=0}$$

T represents the temperature distribution in the fluid $(\partial T/\partial y)_{y=0}$ is the *temperature gradient* at the surface.

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$$Nu = {^{hL}c}k$$

NUSSELT NUMBER

k is the thermal conductivity of the fluid L_c is the characteristic length

Heat transfer through a fluid layer of thickness *L* and temperature difference



Heat transfer through the fluid layer will be by *convection* when the fluid involves some motion and by *conduction* when the fluid layer is motionless.

$$q_{conv} = h\Delta T \qquad q_{cond} = k\frac{\Delta I}{L}$$

$$\frac{q_{conv}}{q_{conv}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = Nu$$

$$\frac{h\Delta T}{k} = Nu$$

Nusselt number - enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. Larger the Nusselt number, the more effective the convection.

Nu = 1 for a fluid layer - heat transfer across the layer by pure conduction

Internal and external flows

EXTERNAL FLOW - The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe

INTERNAL FLOW - flow in a pipe or duct, if the fluid is completely bounded by solid surfaces



Laminar versus Turbulent Flow

Some flows are smooth and orderly while others are rather chaotic. The highly ordered fluid motion characterized by smooth streamlines is called laminar. The flow of high-viscosity fluids such as oils at low velocities is typically laminar.

The highly disordered fluid motion that typically occurs at high velocities characterized by velocities fluctuations is called turbulent. The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping



Os or e Rey olds i 88's, dis o ered that the flo regi e depe ds ai ly o the ratio of the *inertia forces to viscous forces* in the fluid.



The Reynolds number can be viewed as the ratio of the inertia forces to viscous forces acting on a fluid volume element.

$$Re = \frac{Inertia \ forces}{Viscous} = \frac{VL_c}{\nu} = \rho \ \frac{VL_c}{\mu}$$

One, Two and Three Dimensional Flows

V(x, y, z) in cartesian or $V(r, \theta, z)$ in cylindrical coordinates



One-dimensional flow in a circular pipe

Turbulent flow



Time (t)

The eddying motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow).

$$\overline{ ' '} = \mu \int \frac{\partial \overline{u}}{\partial \overline{u}}$$

$$\tau_t = -\rho u v$$

Random eddy motion of groups of particles resembles the random motion of molecules in a gas-colliding with each other after traveling a certain distance and exchanging momentum and heat in process.

Therefore, momentum and heat transport by eddies in turbulent boundary layers is analogous to the molecular momentum and heat diffusion.

$$\tau_{t} = -\rho \, u \, v \qquad t \quad \frac{\partial \overline{u}}{\partial y} \qquad \dot{q}_{t} = \rho \, C_{p} \overline{v' \, T'} = -k_{t} \frac{\partial T}{\partial y}$$

 μ_t - Turbulent viscosity - momentum transport by turbulent eddies k_t - Turbulent thermal conductivity - thermal energy transport by turbulent eddies.

Total shear stress and total heat flux can be expressed conveniently as

$$\tau = \mu + \mu \quad \frac{\partial \overline{u}}{\partial y} = \rho \, v + \varepsilon \quad \frac{\partial \overline{u}}{\partial y} \qquad \varepsilon = \mu \quad \rho$$

$$t \text{ otal } \begin{pmatrix} t \end{pmatrix} \quad \begin{pmatrix} m \end{pmatrix} \qquad M \quad t$$

$$q_{total} = \begin{pmatrix} -k + k \ t \end{pmatrix} \frac{\partial \overline{T}}{\partial t} = -\rho C_p \left(\alpha + \varepsilon H\right) \frac{\partial \overline{T}}{\partial t} \qquad \varepsilon H = kt \quad \rho C_p$$

 ∂y

.

•Eddy motion and thus eddy diffusivities are much larger than their molecular counterparts in the core region of a turbulent boundary layer.

- •The eddy motion loses its intensity close to the wall, and diminishes at the wall because of the no-slip condition.
- •Therefore, the velocity and temperature profiles are nearly uniform in the core region of a turbulent boundary layer, but very steep in the thin layer adjacent to the wall, resulting in large velocity and temperature gradients at the wall surface.
- •Therefore, the velocity and temperature profiles are nearly uniform in the core region of a turbulent boundary layer, but very steep in the thin layer adjacent to the wall, resulting in large velocity and temperature gradients at the wall surface.

VELOCITY BOUNDARY LAYER



Development of a boundary layer on a surface is due to the no-slip condition

Surface Shear Stress

$$\begin{aligned} \tau &= \mu \frac{\partial u}{\partial y} \\ s & \overline{\partial y} \\ y=0 \end{aligned}$$

Skin friction coefficient

$$\tau_s = C_f \frac{\rho V^2}{2}$$

Friction force over the entire surface

 $F_f = C_f A_s \frac{\rho V^2}{2}$

THERMAL BOUNDARY LAYER



Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface)

The *thickness* of the thermal boundary layer, at any location along the surface is define as *the distance from the surface at which the temperature difference* $T - T_s$ equals 0.99($T_{\infty} - T_s$).

For the special case of $T_s = 0$, we have T = 0.99 at the outer edge of the thermal boundary layer, which is analogous to u = 0.99 for the velocity boundary layer.

- Shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it.
- In flow over a heated (or cooled) surface, both velocity and thermal boundary layers will develop simultaneously.
- Noting that the fluid velocity will have a strong influence on the temperature profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convection heat transfer.

PRANDTL NUMBER

The relative thickness of the velocity and the thermal boundary layers is described by the *dimensionless* parameter Prandtl number, defined as

$$Pr = \frac{Molecular \ diffusivity \ of \ momentum}{Molecular \ diffusivity \ of \ heat} = \frac{v}{\alpha} = \frac{\mu C_p}{k}$$

$$\underline{\delta} \approx Pr^n$$

n is positive exponent

 δ_t

 $Pr \cong 1$ for gases \Rightarrow both momentum and heat dissipate through the fluid at about the same rate

Heat diffuses very quickly in liquid metals (*Pr* < 1)

Heat diffuses very slowly in oils (Pr > 1) relative to momentum Therefore, thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

$$\delta = \delta t \quad \text{for } Pr = 1$$

$$\delta > \delta t \quad \text{for } Pr > 1 \qquad Pr = \frac{Molecular \ diffusivity \ of \ momentum}{Molecular \ diffusivity \ of \ heat} = \frac{v}{\alpha} = \frac{\mu C_p}{k}$$

$$\delta < \delta t \quad \text{for } Pr < 1$$

NATURAL CONVECTION



The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a natural convection current Heat transfer that is enhanced as a result of this natural convection current is called natural convection heat transfer

Natural convection occurs because of the presence of a. Density difference b. Gravity

There is no gravity in space. Therefore, there is no natural convection heat transfer in a spacecraft, even if the spacecraft is filled with atmospheric air.

In heat transfer studies, the primary variable is temperature. Therefore, we need to express net buoyancy force in terms of temperature differences.

Density difference is to be expressed in terms of temperature difference, which requires a knowledge of a property that represents the variation of the density of a fluid with temperature at constant pressure.

The property which provides this information is volume expansion coefficient, $\boldsymbol{\beta}$



Coefficient of volume expansion is a measure of change in volume of the substance with temperature at constant pressure

$$\beta = - \begin{pmatrix} dV \\ - \end{pmatrix} = - \begin{pmatrix} d\rho \\ - \end{pmatrix}$$

V\dT)P \dT)P
Equation of motion and Grashoff number

Consider a vertical hot body immersed in a quiescent cold fluid



Assumptions

- Natural convection Flow is laminar
- Flow is two dimensional
- Velocity Flow is steady
- profile Fluid is Newtonian and properties
 are constant
 - one exception: the density difference between the fluid inside and outside the boundary layer that gives rise to the buoyancy force and sustains flow (This is known as Boussinesq approximation)



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial v} \\ \frac{\partial v}{\partial v} \\ \frac{\partial v}{\partial$$

$$\begin{pmatrix}
\frac{\partial \mathbf{v}}{\partial \mathbf{x}} & \frac{\partial \mathbf{v}}{\partial \mathbf{y}} & \frac{\partial \mathbf{v}}{\partial \mathbf{y}} & \frac{\partial \mathbf{P}}{\partial \mathbf{y}} & \frac{\partial^{2} \mathbf{v}}{\partial \mathbf{x}} & \frac{\partial^{2} \mathbf{v}}{\partial \mathbf{y}} \\
\frac{\partial \mathbf{z}}{\partial \mathbf{x}} & \frac{\partial \mathbf{y}}{\partial \mathbf{y}} & \frac{\partial \mathbf{y}}{\partial \mathbf{z}} & \frac{\partial \mathbf{p}}{\partial \mathbf{z}} & \frac{\partial \mathbf{p}}{\partial \mathbf{y}} \\
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\frac{\partial \mathbf{z}}{\partial \mathbf{x}} & \frac{\partial \mathbf{z}}{\partial \mathbf{y}} & \frac{\partial \mathbf{z}}{\partial \mathbf{z}} & \frac{\partial \mathbf{z}}{\partial \mathbf{z}} \\
\frac{\partial \mathbf{P}_{\infty}}{\partial \mathbf{x}} & \frac{\partial \mathbf{v}}{\partial \mathbf{y}} & \frac{\partial \mathbf{z}}{\partial \mathbf{z}} \\
\frac{\partial \mathbf{P}_{\infty}}{\partial \mathbf{x}} & \frac{\partial \mathbf{v}}{\partial \mathbf{z}} & \frac{\partial \mathbf{v}}{\partial \mathbf{z}} \\
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\frac{\partial \mathbf{z}}{\partial \mathbf$$

∂y)

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{y}} = \boldsymbol{0}$$

$$\begin{pmatrix} \partial \boldsymbol{v} & \partial \boldsymbol{v} \\ + \boldsymbol{v} & - \boldsymbol{v} \\ - \boldsymbol{v} \\ + \boldsymbol{v} & - \boldsymbol{v} \\ - \boldsymbol{v}$$

 $(\rho_{\infty} - \rho) \mathbf{g}$ Body force term – flow is driven by the density field generated by temperature field

$$\beta_{\beta} = \frac{1}{V} \left(\frac{dV}{dT} \right)_{P} = \frac{1}{\rho} \left(\frac{d\rho}{dT} \right)_{P} = \frac{1}{\rho} \frac{\rho_{\infty} - \rho}{\rho T_{\infty} - T}$$

$$\rho_{\infty} - \rho = \rho \beta (T - T_{\infty})$$

$$\begin{pmatrix} \partial \mathbf{V} & \partial \mathbf{V} \\ \mathbf{v} & \mathbf{v} & \mathbf{v} \end{pmatrix}^{2} = \mu \frac{\partial^{2} \mathbf{V}}{2 + \rho g \beta (T - T_{\infty})} \\ \begin{pmatrix} \partial \mathbf{X} & \partial \mathbf{y} \end{pmatrix}^{2} & \partial \mathbf{X} \\ \mathbf{u} \frac{\partial \mathbf{V}}{\partial \mathbf{X}} + \mathbf{v} \frac{\partial \mathbf{V}}{\partial \mathbf{y}} = \nu \frac{\partial^{2} \mathbf{V}}{\partial \mathbf{X}^{2}} + \mathbf{g} \beta (T - T_{\infty}) \\ \end{pmatrix}^{2} \end{pmatrix}$$

Governing equations and boundary conditions

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = v\frac{\partial^2 v}{\partial x^2} + g\beta \left(T - T\right)_{\infty}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$u = v = 0 : T = T_s \text{ at } x = 0$$

$$v = 0$$
; $T = T_{\infty}at x \rightarrow \infty$

Similarity approach



Re_⊥

- The role played by the Reynolds number in forced convection is played by the Grashoff number in natural convection
- Grashoff number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection
- For vertical plates, critical Grashoff number is around 10⁹



Re²

Forced convection dominates

Natural convection dominates

Both natural convection and forced are important **NATURAL CONVECTION OVER SURFACES**

$$Nu = \frac{hL}{c} = C (Gr_L Pr)^n = CRa_L^n k$$

$$Ra_{L} = Gr Pr = \frac{g\beta(T_{s} - T_{\infty})L^{3}c}{U^{2}}Pr$$

Properties of fluid are calculated at mean film temperature

$$Tf = (T_{s} + T_{\infty})/2$$

Criteria for forced and natural convection



$$0.1 < \frac{Gr}{Re^2} > 10$$

Natural convection can inhibit or enhance heat transfer depending on the directions of buoyancy induced flow and forced convection motion



Assisting flow

Opposing flow

Transverse flow

$$Nu_{combined} = \left(\begin{array}{cc} Nu & Nu_{natural} \\ Nu & Nu_{natural} \end{array} \right)^{1/2}$$

+ is for assisted flow
-is for opposing flow
n = 3 for vertical surfaces
For other geometries, n = 3 - 4

HEAT TRANSFER

UNIT - 4

TWO PHASE FLOW AND HEAT TRANSFER

Phase: macroscopic state of matter which is homogeneous in chemical composition and physical structure Ex: Gas, Liquid and solid

Gas-Liquid – steam and water air and water Liquid-Solid- plasma and platelets (Blood) Liquid-Liquid – Oil and water

Liquid-Vapor transformation – BOILING T_L > T_{sat} at a given pressure

Vapor-Liquid transformation – CONDENSATION T_v < T_{sat} at a given pressure

Boiling and Condensation (Convection)

- Latent Heat of Vaporization
- Surface Tension of the liquid-vapor interface
- Properties of the fluid in each phase

Typical values of the convection heat transfer coefficient

Process	h W/m ² .K
Free convection Gases Liquids	2-25 50-1000
Forced convection Gases Liquids	25-250 50-20,000
Convection with phase change Boiling and Condensation	2500-100,000

Boiling: the process of addition of heat to a liquid such a way that generation of vapor occurs.

Evaporation Air



Evaporation

occurs at the liquid-vapor interface

Pv < Psat of the liquid at a given temp

evaporation involves no bubble formation or bubble motion



Boiling occurs at a solid-liquid interface occurs at a solidliquid interface



Boiling occurs when a liquid is brought into contact with a surface at a temperature above the saturation temperature of the liquid

 $Q_{boiling} = h (T_w - T_{sat}) W/m^2$

Free and Forced convection depends on

• density, specific heat, viscosity and thermal conductivity of the fluid

Boiling Heat Transfer depends on

- density, specific heat, viscosity and thermal conductivity of liquid
- Latent heat of vaporization
- Surface tension at the liquid-vapor interface

Bubbles exist because of the surface tension at the liquid vapor interface due to the attraction force on molecules at the interface toward the liquid phase.

Surface tension $\downarrow \uparrow$ Temperature

Surface tension = 0 at critical temperature

No bubbles at supercritical pressures and temperatures

BOILING

- Pool Boiling
- Flow Boiling (Forced Convection Boiling



POOL BOILING:

Motion of the fluid is due to natural convection currents

Motion of the bubbles under the influence of buoyancy.

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Pool Boiling



Flow boiling

Fluid is forced to move in a heated pipe or surface by external means such as pump

Flow boiling is always accompanied by other convection effects.

Pool and Flow Boiling is classified as

- Subcooled Boiling
- Saturated Boiling



Subcooled Boiling





Saturated Boiling

sat Saturated water 100 C

Heating

BOILING REGIMES AND BOILING CURVE

- Nukiyama 1934
- **Boiling Regimes**
- Natural Convection Boiling
- Nucleate Boiling
- Transition Boiling
- Film Boiling

Typical Boiling Curve for water at 1 atm pressure



 $\Delta T = T_s - T_{sat} \ ^{\circ}C$

Natural convection Boiling

Governed by natural convection currents



Natural convection boiling

Heat transfer from the heating surface to the fluid is by natural convection

Nucleate Boiling



Heating Nucleate boiling The stirring and agitation caused by the entrainment of the liquid to the heater surface is primarily responsible for the increased heat transfer coefficient and heat flux in the region of nucleate boiling.

High heat transfer rates are achieved in nucleate boiling

Transition Boiling (Unstable film boiling)



Heat flux decreases because of larger fraction of heater surface is covered by a vapor film which acts as a insulation because of the low thermal conductivity of the vapor relative to the liquid

Film Boiling



The presence of a vapor film between the heater surface and the liquid is responsible for the low heat transfer rates

Heat transfer rate increases with increasing excess temperature as a result of heat transfer from the heated surface to the liquid through the vapor film by radiation, which becomes significant at higher temperatures. Various boiling regimes during boiling of methanol on a horizontal 1-cm-diameter steam-heated copper tube

- a. Nucleate boiling
- **b.** Transition boiling
- c. Film boiling



1. Natural convection

Onset of boiling



2. Individual bubble regime

3. Regime of slugs and bubble



Bubble nucleation – Boiling Inception

The process of bubble formation is called nucleation

Trapped pockets of gas Liquid **Rough wall**

Enlarged view of a boiling surface

The cracks and crevices do not, of themselves, constitute nucleation sites for the bubbles: they must also contain pockets of gas, probably air trapped when the vessel was filled with the liquid. It is from these pockets of trapped air that the vapor bubbles begin to grow during nucleate boiling
HEAT TRANSFER IN NUCLEATE BOILING

$$Nu = f(Re, Pr)$$

$$\frac{hL}{Mu = k_f}; Re = \mu_f; Pr = k_f$$

Velocity is taken as the liquid velocity in towards the surface which is to supply the vapour which is being produced, so

$$u \sim \frac{\mathbf{q}''}{\mathbf{h}_{fg} \rho_{f}}$$

Length scale is taken to be

$$L \sim \left[\frac{\sigma}{\left[\frac{\sigma}{\rho^{f} - \rho^{g}} \right]^{2}} \right]$$

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During boiling, disturbances of all wavelengths are present, there will be some disturbances at small wavelength and long wavelength that will amplify and cause the interface to be unstable.

Condition for the interface instability of a motionless liquid overlaying a motionless vapour region

$$\alpha > \alpha_{c} \qquad \begin{bmatrix} (\rho f - \rho g)g \\ = \end{bmatrix} \begin{bmatrix} \sigma \end{bmatrix}$$

This condition is called Rayleigh-Taylor Instability

CRITICAL WAVENUMBER

$$L = 2\pi \int_{\frac{|g(\rho)-\rho_g}{\rho_g}} \sigma \int_{\frac{1}{2}}^{\frac{1}{2}}$$

p = 1 bar; σ = 0.058988 N/m ρ_f = 958.63; ρ_g = 0.59034 kg/m³

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L_c = 16 mm

$$Nu = \frac{hL}{k_{f}} = \frac{h}{k_{f}} \left[\frac{\sigma}{g(\rho_{f} - \rho_{g})} \right]_{j,Re}^{\frac{1}{2}} = \frac{\rho_{f}uL}{\mu_{f}} = \frac{\rho_{f}}{\mu_{f}} \frac{q''}{n_{fg}\rho_{f}} \left[\frac{\sigma}{g(\rho_{f} - \rho_{g})} \right]_{j,Re}^{\frac{1}{2}} = \frac{\mu_{f}C_{\rho_{f}}}{\kappa_{f}}$$

$$Nu = \frac{1}{C_{sf}} Re 1 - n Pr - m$$

$$\frac{C_{\rho f} \left(T_{W} - T_{sat}\right)}{n_{fg}} = C_{st} \left[\frac{q''}{\mu_{f}} \frac{\sigma}{n_{fg}} \sqrt{\frac{\sigma}{g(\rho_{f} - \rho_{g})}} \right]_{j}^{\frac{1}{2}} \left[\frac{c_{\rho f}}{\kappa_{f}} \frac{\mu_{f}}{\mu_{f}} \right]_{j}^{1 + m}$$

n = 0.33 and 1+m = 1 for water and 1.7 for other fluids

Cst is the surface-fluid constant. It depends on both the surface and the fluid. Typical values are between 0.0025 and 0.015.

For a given ΔT_{sat} , the heat flux is proportional to $(C_{st})^{-3}$. Since C_{st} can vary by a factor of 10, the heat flux can vary by a factor of 1000

APPLICABLE ONLY FOR CLEAN SURFACES

FILM BOILING



([fg])

Taking the example for steam-water at 1 bar, for which $I_{sat} = 100^{\circ} \text{ C}; T_W = 700^{\circ} \text{ C}; \Delta T_{sat} = 600^{\circ} \text{ C}; hfg = 2256000 \text{ J/kg}$ $kg = 0.046 \text{ W/m.K}; \mu g = 21 \times 10^{-6} \text{ Pa.s}; \rho g = 0.32 \text{ kg/m}^3;$ $\rho f = 1000 \text{ kg/m}^3; d = 0.02 \text{ m}$ physical properties are evaluated at 400° C. Then from above eqn, $h = 142 \text{ W/m}^2.\text{K}$ and so $q'' = h \Delta T_{sat} = 85000 \text{ W/m}^2$



Critical heat flux for water at 1 bar is around 1.25 MW/m² So, although the tube is very hot, it is carrying only a fraction of the critical heat flux. So we around point A on the boiling curve (See Fig.12). The point C, which has the same heat flux as at point B, can be found. The temperature at C will be so high that radiative heat transfer is very important $\frac{23}{23}$

$$h_{total} = h_{film \ boiling} + a h_{radiative}; \quad a = \frac{3}{4}$$

$$h_{radiative} = \sigma \varepsilon \begin{pmatrix} T^4 - T^4 \\ \frac{w \quad sat}{2} \end{pmatrix} \quad \varepsilon = \text{surface emissivity} \\ -8 \quad 24 \quad (T_w - T_{sat}) \quad \sigma = 5.67 \times 10 \quad W/m \ K$$

COMPLETION OF THE BOILING CURVE



The transition boiling region is the main region not yet considered.

Actually comparatively little is known, and one assumption made is that the transition boiling line (B to

C) is the straight line connecting B and C when plotted (as above) on a log-log graph.

Tsat (log scale)

Upto A	Natural convection heat transfer
Α	Onset of natural convection boiling
A to B	Nucleate boiling heat transfer
В	Critical heat flux
B to C	Transition boiling
С	Minimum film boiling temperature
C onwards	Film boiling

,,	

RADIATION: PROCESSES AND PROPERTIES

- Thermal radiation requires no matter
- Applications: Industrial heating, cooling and drying processes, energy conversion methods – fossil fuel combustion and solar radiation

OBJECTIVES

- MEANS BY WHICH THERMAL RADIATION IS GENERATED
- SPECIFIC NATURE OF RADIATION
- •MANNER IN WHICH RADIATION INTERACTS WITH MATTER



Unlike conduction and convection, heat transfer by radiation can occur between two bodies, even when they are separated by a medium colder than both.

Solar radiation reaches the earth after passing through cold air layers at high altitudes.

FUNDAMENTAL CONCEPTS

- $T_s > T_{sur}$
- No conduction or convection still solid will cool
- Solid gets cooled emission of thermal radiation form the surface of the solid surroundings Radiation cooling of a Sur heated solid Vacuum **Radiation from** Surface radiation surroundings emission Т S

Radiation – propogation of electromagnetic waves

J.C.Maxwell – accelerated charges or changing electric currents give rise to electric and magnetic fields. These moving fields are called ELECTROMAGNETIC WAVES OR ELECTROMAGNETIC RADIATION

Electro-magnetic Radiation – energy emitted by matter as a result of the changes in the electronic configurations of the atoms or molecules

Characteristics of E.M.Radiation

- •Frequency v(Hz 1/sec)
- •Wavelength λ (m)

In Vacuum $c_0 = 2.998 \times 10^8 \text{ m/s}$

n – Index of refraction

c – speed of propogation of wave in that medium

n

 $\lambda = \underline{c}$



Material	n
Air and most gases	1.0
Glass	1.5
Water	1.33

OBSERVATIONS

λ & c – Depend On The Medium Through Which Wave Travels

Independent of the medium Depends only on the source

Electromagnetic Radiation: Propogation of a discrete packets of energy called photons or quanta

Each photon of frequency $\boldsymbol{\nu}$ is considered to have an energy of

$$e = hv = \frac{hc}{\lambda}$$

 $h = 6.625 \times 10^{-34} J.s - Planck's Constant$

Energy of the photon – inversely proportional to its wavelength

Shorter wavelength radiation possess larger photon energies

No wonder, we try to avoid very short wavelength radiation such as gamma rays and X-rays since they are highly destructive



Visible Light Region of the Electromagnetic Spectrum



Wavelength range of different colours

Color	Wavelength Band (m)
Violet	0.40 - 0.44
Blue	0.44 - 0.49
Green	0.49 - 0.54
Yellow	0.54 - 0.60
Orange	0.60 - 0.67
Red	0.63 - 0.76

Solar radiation:

Electromagnetic radiation emitted by sun Wavelength band – 0.3 - 3 μ m Half range is in the visible range Other half range is in the ultraviolet and infrared range IN HEAT TRANFER, WE ARE INTERESTED IN ENERGY EMITTED BY BODIES DUE TO THEIR TEMPERATURE ONLY – THERMAL RADIATION



ALL BODIES EMIT RADIATION – RADIATION AT ABSOLUTE ZERO TEMP – ZERO

THERMAL RADIATION – Rate at which energy is emitted by matter as a result of its finite temperature



MECHANISM OF EMISSION – energy released as a result of oscillations of many electrons that constitute matter

- Radiation is a volumetric phenomena
- Radiation is considered as surface phenomena
- Radiation interior molecules absorbed by adjoining molecules
- Radiation that is emitted from a solid or a liquid originates from molecules that within a distance of 1 μm from the exposed surface

Radiation emission



Solid or liquid

Emission Process

- a. Volumetric Phenomenon
- b. Surface Phenomenon

DESCRIPTION OF THE THERMAL RADIATION

Spectral Distribution

- Emitted Radiation continuous, non-uniform distribution of mono-chromatic (single wavelength) components
- Spectral distribution depends on
 - Nature of the emitting surface
 - Temperature of the emitting surface



Spectral distribution

Directional distribution

RADIATION INTENSITY

Radiation emitted by a surface propagates in all directions

Radiation incident on the surface may come from different directions

Response of the surface to the radiation depends on the direction

Directional effects – concept – RADIATION INTENSITY

SOLID ANGLE *d*ω

• Emission of radiation from a differential area dA_1 into a solid angle $d\omega$ subtended by dA_n at a point on dA_1



 ϕ ϕ $d\omega = \frac{dAn}{d\omega} steridians (sr)$

y

r 2



Quantifying the slice of a pizza

Quantifying the slice of a water

of plane angle α

melon of solid angle $\text{d}\omega$

Differential area dA_n is normal to (θ, ϕ) direction as in figure dA_n is normal to the direction of viewing since dA_n is viewed from the center of the sphere






Solid angle of a sphere

$$A_{n} = \int dA_{n} = \int_{\phi=0}^{2\pi} \int_{\sigma}^{\pi} \int_{r}^{2} \sin\theta d\theta d\phi = 2\pi r^{2} \int_{\theta=0}^{\pi} \sin\theta d\theta = 2\pi r^{2} \cdot -\cos\theta |_{\theta}^{\pi} = \theta = 4\pi r^{2}$$
sphere
$$A_{n} = 4\pi r^{2}$$

For a sphere with unit radius, solid angle is 4π

Differential solid angle *dw* subtended by a differential surface area *dA* when viewed from a point at a distance *r* from *dA* is expressed as

$$d\omega = \frac{dA}{r^2} = \frac{dA \cos \alpha}{r^2}$$

where α is the angle between the normal to the surface and the direction of viewing, and thus $dA_n = dA\cos\alpha$ is the normal (or projected) area to the direction of viewing

Small surfaces viewed from relatively a large distances can approximately treated as differential areas in solid angle calculations. For example, the solid angle subtended by 6 cm² plane surface when viewed from a point at a distance of 90 cm along the normal of the surface

$$\omega = \frac{A_n}{r^2} = \frac{6}{90^2} = 7.41 \times 10^{-4} \text{ sr}$$

If the surface is tilted so that the normal of the surface makes an angle of 60° with the line connecting the point of viewing to the centre of the surface, the projected

area would be $dA_n = dA\cos\alpha = 6 \cos 60^\circ = 3 \text{ cm}^2$ and the solid angle in this case would be half of the value just determined.

INTENSITY OF EMITTED RADIATION

- Consider the emission of radiation by a differential area element dA_1 of a surface
- Radiation is emitted in all directions into the hemispherical surface and the radiation streaming through the surface area dA_n is proportional to the solid angle dω subtended by area dA_n
- Radiation is also proportional to the radiating area dA₁ as seen by an observer on dA_n, which varies from a maximum of dA₁ when dA_n is at the top directly above dA₁ (θ = 0°) to a minimum of zero when dA_n is at the bottom (θ = 0°)
 The effective area of dA₁ for emission in the direction of θ is projection of dA₁ on a plane normal to θ, which is dA₁cosθ



Radiation intensity for emitted radiation $I_e(\theta, \phi)$ is defined as the rate at which the radiation energy dq is emitted in the (θ, ϕ) direction per unit area normal to this direction and per unit solid angle about this direction

$$I_e(\theta,\phi) = \frac{dq}{dA_1 \cos \theta \cdot d\omega} = \frac{dq}{dA_1 \cos \theta \cdot \sin \theta \, d\theta \, d\phi} \Big(W/m^2 \, sr \Big)$$

Radiation flux for emitted radiation is the EMISSIVE POWER (E) – rate at which radiation energy is emitted per unit area of the emitting surface which is expressed in the differential form

$$dE = \frac{dq}{dA_1} = I_e \ (\theta, \phi) \cos \ \theta \sin \theta \ d\theta \ d\phi$$

Hemisphere above the surface intercepts all the radiation rays emitted by the surface, the emissive power from the surface into the hemisphere surrounding it is given by $2\pi \pi/2$

$$E = \int dE = \iint Ie (\theta, \phi) \cos \theta \sin \theta \, d\theta \, d\phi \Big(W \, m^2 \Big)$$

hemisphere $\phi = 0 \ \theta = 0$

The intensity of radiation emitted by a surface, in general, varies with direction (especially with zenith angle θ . But, many surfaces in practice can be approximated as DIFFUSE. For a diffusely emitting surface, the intensity of emitted radiation is independent of direction and thus I_e = constant

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos \theta \sin n\theta \, d\theta \, d\phi = 2\pi \int_{\theta=0}^{\pi/2} \cos \theta \sin n\theta \, d\theta =$$
$$= \frac{2\pi}{2} \int_{\theta=0}^{\pi/2} \sin 2\theta \, d\theta = \frac{-\cos 2\theta}{2} \Big|_{0}^{\pi/2} \pi$$
$$= -\frac{\pi}{2} (\cos \pi - \cos 0) = -\frac{\pi}{2} (-1-1) = \pi$$

$$E = \int dE = \int \int_{e}^{2\pi\pi/2} I_{e} (\theta,\phi) \cos \theta \sin \theta \, d\theta \, d\phi (W \, m_{2}) \Rightarrow E = \pi I_{e}$$

hem isp here $\phi = 0 \ \theta = 0$

$$G = \frac{1}{|1000|} (\frac{W}{m})_{(5-0)\mu m+|1000|} (\frac{W}{m})_{(20-5)\mu m+|} \frac{1}{|1000|} (\frac{W}{m})_{(25-20)\mu m+0|} \frac{1}{2} (\frac{W}{m^2})_{(25-20)\mu m+0|} \frac{1}{2} (\frac{W}{m^2})_{(25-20)$$

Comments: Generally, radiation sources do not provides such a regular spectral distribution for the irradiation. However, the procedure of computing the total irradiation from knowledge of the spectral distribution remains the same, although evaluation of the integral is likely to involve more detail.

HEAT TRANSFER

UNIT - **5**

€-NTU APPROACH :

Known: inlet fluid temperatures, fluid mass flow rates, type and size of the heat exchanger

Predict outlet temperature of hot and cold stream in a specified HE Task is to determine

- Heat transfer performance of a specified heat exchanger
- If a heat exchanger available in storage will do the job

To solve this type of problem by LMTD approach would be tedious because of numerous iterations required.

Kays and London – Effectiveness NTU approach to avoid iterations (1955)

EFFECTIVENESS

 $\varepsilon = \frac{\dot{Q}}{Q_{max}} = \frac{Actual heat transfer rate}{Q_{max}}$ Maximum possible heat transfer rate

Actual heat transfer rate



These two limiting conditions will not be reached simultaneously unless C_C=C_H When $C_C \neq C_{H_c}$ the fluid with the smaller heat capacity would experience maximum temperature and the heat transfer would come to a halt



$$C p = 4.18 kJ / kg .deg .C$$

 $Ch = mhC ph = 2 \times 4.18 = 8.36 kW / deg C$
 $C_{c} = m_{c}C_{nc} = 8 \times 4.18 = 33.44 kW / deg C$

$$W_{max,1} = mhC \, ph \, (Th, i - Tc, i) = 8.36(70 - 10) = 501.6 kW$$

01.6kW = mcC pc
$$(T_{c}, o - T_{c}, i)$$
 = 33.44 $(T_{c}, o - 10)$ \Rightarrow Tc, $o = 25C$

Considering Qmax,2 = 2000.4kW – compute Th,i

0

2000.4kW = mhC ph
$$(T_{h,o} - T_{h,i}) = 8.36 (70 - T_{h,i}) \Rightarrow T_{h,i} = -170 C$$

Hence, the cold water will go on transferring heat to hot water until cold water temperature reaches 25°C, by this time the hot water would have reached already 10°C,



sfer between hot water and cold water t h е n t h e r е i S n 0 h е а t t r а n

Derivation of ϵ – NTU for Parallel flow heat exchanger



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l





Number of transfer units (NTU)

$$NTU = \frac{UA_s}{C_{min}} = \frac{UA_s}{\bigcup_{(mc_{pc})}}$$

- Non-dimensional thermal size of the heat exchanger
- Heat transfer unit heat capacity of fluid
- For specified values of C_{min} and U
 - NTU measure of heat transfer surface area
 - Larger NTU larger the heat exchanger

$$C = \frac{C}{m \text{ in}}$$

тах

$$\varepsilon = f(c, NTU)$$

Effectiveness relation for heat exchanger: NTU = UA_S/C_{MIn} $\mathbf{C} = \mathbf{C}_{\min} / \mathbf{C}_{\max} \quad \left(\begin{array}{c} c \\ c \\ c \end{array} \right)_{\min} / \left(\begin{array}{c} m c \\ c \end{array} \right)_{min} / \left(\begin{array}{c} m c \\ c \end{array} \right)_{max} \quad \text{Kays and London}$ and Heat exchanger type Effectiveness relation 1 **Double pipe:** $\varepsilon = \frac{1 - \exp\left[-\frac{1}{1 + C}\right]}{\left[(1 + C)_{(1 + C)}\right]}$ **Parallel** –flow $\varepsilon = \frac{1 - \exp[-NTU \ 1 - C]}{1 - C \exp[-NTU \ 1 - C]}$ **Counter flow** $\mathcal{E} = \mathbf{1} - \mathbf{C} \operatorname{CAP}_{1} \quad \dots \quad \sum_{\text{treel}} - NTU\sqrt{1 + C^{2}} \right|$ $\mathcal{E} = \mathbf{2} \left\{ 1 + C + 1 + \sqrt{C^{2}} + \frac{\left[- NTU\sqrt{1 + C^{2}} \right]}{\left[- NTU\sqrt{1 + C^{2}} \right]} \right\}$ $\varepsilon = 1 - \exp\left\{ \frac{NTU^{0.22}}{C} \left[\left[\exp\left(-C NTU^{0.78} \right) - 1 \right] \right] \right\}$ $\varepsilon = \mathbf{1} \left(1 - \exp\left\{ 1 - C\left[1 - \exp\left(-NTU \right) \right] \right\} \right)$ 2 Shell and tube : One shell pass 2, 4,...tube passes **3 Cross flow (single pass)** Both fluid unmixed C_{max} mixed, **Cmin unmixed C**min mixed $\varepsilon = \left(1 - \exp\left[-\frac{1}{C} \left[1 - \exp\left[-C NTU \right] \right] \right) \right)$ **C**max unmixed 4 All heat $\varepsilon = 1 - exp(-NTU)$ exchangers with C=0



Parallel Flow



Counter Flow



One-shell pass and 2,4,6....tube passes



Two-shell passes and 4,8,12....tube passes



Number of transfer units $NTU = A_s U/C_{min}$

Cross- flow with both fluids unmixed



Cross- flow with one fluid mixed and other unmixed



For a given value of NTU and $c = C_{MIN}/C_{max}$, the counterflow HE has the highest effectiveness, followed closely by the cross flow HE with both

fluids unmixed. Lowest effectiveness is encountered in parallel flow HE

• capacity ratio varies between 0 to 1.

 $c = 0 \Rightarrow c = C_{MIN}/C_{max} \Rightarrow 0 C_{max} \Rightarrow \infty, - CONDENSER AND$ BOILER c = 1 \Rightarrow c = C_{MIN}/C_{max} \Rightarrow 1; ε is lowest





• ε ranges from 0 to 1. It increases rapidly with NTU for small values (upto NTU = 1.5) but rather slowly for larger values.

•High ϵ is desirable from heat transfer point of view but undesirable from economic point of view. Hence, NTU larger than 3 is not justified.

- For a given value of NTU and c = C_{MIN}/C_{max}, the counterflow HE has the highest effectiveness, followed closely by the cross flow HE with both fluids unmixed. Lowest effectiveness is encountered in parallel flow HE
- ε is independent of capacity ratio c for NTU values of less than 0.3
- capacity ratio varies between 0 to 1.

 $c = 0 \Rightarrow c = C_{MIN}/C_{max} \Rightarrow 0 C_{max} \Rightarrow \infty$, - CONDENSER AND

BOILER c = 1 \Rightarrow c = C_{MIN}/C_{max} \Rightarrow 1; ε is lowest

Effectiveness relation for heat exchanger: NTU = UA_S/C_{MIN} $\mathbf{C}_{min}^{\mathbf{C}} = \mathbf{C}_{min}^{\mathbf{C}} \left(\begin{array}{c} \mathbf{C}_{P} \\ \mathbf{C}_{P} \end{array} \right)_{min}^{\mathbf{C}} \left(\begin{array}{c} \mathbf{C}_{P} \\ \mathbf{C}_{P} \end{array} \right)_{max}^{\mathbf{Kays}} \text{ and London}$ and Heat exchanger type **Effectiveness relation** 1 **Double pipe:** $NTU = -\frac{\ln \left[1 - \varepsilon \left(1 + C\right)\right]}{1 + C}$ Parallel –flow $NTU = \frac{1}{C-1} \begin{pmatrix} \varepsilon - 1 \\ \frac{\ln \varepsilon}{\varepsilon - 1} \end{pmatrix}$ **Counter flow** 2 Shell and tube : $\frac{1}{\sqrt{1+C^2}} \ln \left(\frac{2\varepsilon - 1 - C - \sqrt{1+C^2}}{2\varepsilon - 1 - C + \sqrt{1+C^2}} \right)$ **One shell pass** NTU = -2, 4,...tube passes 3 Cross flow (single pass) $NTU = - \ln \begin{bmatrix} \ln 1 - C\varepsilon \\ 1 + \frac{C}{C} \end{bmatrix}$ Both fluid unmixed C_{max} mixed, $NTU = -\frac{\begin{bmatrix} L & \mathbf{v} & \mathbf{v} \\ \mathbf{v} & (\mathbf{1} - \varepsilon \mathbf{C}) + \end{bmatrix}}{\mathbf{C}}$ **C**min unmixed **C**min mixed C_{max} unmixed $NTU = -\ln(1-\varepsilon)$ 4 All heat exchangers with C=0

DESIGN AND CHOICE OF HEAT EXCHANGER

- HEAT TRANSFER RATE
- PUMPING POWER PRESSURE DROP
- COST
- SIZE AND WEIGHT
- TYPE
- MATERIALS