

Linear Algebra and Calculus *for Engineers*

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 **SPECTRUM**
UNIVERSITY PRESS
An Innovative Series...

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Spectrum University Press, Hyderabad

A Part of StudentsHelpline Publishing House (P) Ltd. , Hyderabad

(An ISO 9001 : 2015 Certified Company)

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326/C, 1st Floor, Surneni Nilayam

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www.studentshelpline.org

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First Edition-2018

ISBN 978-93-83640-20-1

˘ **299/- Student Edition**

˘ **499/- Library Edition with HB**

Printed at StudentsHelpline Group, S R Nagar, Hyderabad-38

Published by Surneni Mohan Naidu for Spectrum University Press, Hyderabad - 38

Linear Algebra and Calculus for Engineers

Module-I: Theory of Matrices and Linear Transformations

Real Matrices: Symmetric, Skew-Symmetric and Orthogonal Matrices, Complex Matrices: Hermitian, Skew-Hermitian and Unitary Matrices, Elementary row and column transformations, Rank of matrix, Echelon form and Normal form, Inverse by Gauss-Jordan method, Cayley-Hamilton theorem: Statement, Verification, finding inverse and powers of matrix; Linear dependence and Independence of vectors, Eigen values and Eigen vectors of matrix and Properties (without proof); Diagonalization of matrix by linear transformation.

Module-II: Functions of Single and Several Variables

Mean value theorems: Rolle's Theorem, Lagrange's theorem, Cauchy's theorem- without proof, Functions of several variables: Partial differentiation, chain rule, total derivative, Euler's theorem, Functional dependence, Jacobian, Maxima and Minima of function of two variables without constraints and with constraints, Method of Lagrange multipliers.

Module-III: Higher Order Linear Differential Equations and their Applications

Linear Differential Equations of Second and Higher Order with Constant Co-efficients, Non-homogeneous Term of the Type $f(x) = e^{ax}$, $\sin ax$, $\cos ax$ and $f(x) = x^n$, $e^{ax}v(x)$, $x^n v(x)$; Method of variation of parameters; Applications to electrical circuits

Module-IV: Multiple Integrals

Double and triple integrals; Change of order of Integration. Transformation of coordinate system; Finding the area of a region using double integration and volume of a region using triple integration.

Module-V: Vector Calculus

Scalar and Vector Point Functions: Definitions of Gradient, Divergent and Curl with examples, Solenoidal and Irrotational Vector point functions: Scalar potential Function; Line integral, Surface Integral and Volume Integral, Vector integral theorems: Green's theorem in a plane, Stoke's theorem and Gauss divergence theorem without proofs.

Linear Algebra and Calculus for Engineers

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