

Mathematical Transform Techniques

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Mathematical Transform Techniques

Module-I: Root Finding Techniques and Interpolation

Root finding techniques: Solving algebraic and transcendental equations by bisection method, method of false position, Newton-Raphson method; Interpolation: Finite differences, forward differences, backward differences and central differences; Symbolic relations; Newton's forward interpolation, Newton's backward interpolation; Gauss forward central difference formula, Gauss backward central difference formula; Interpolation of unequal intervals: Lagrange's interpolation.

Module-II: Curve Fitting and Numerical Solution of Ordinary Differential Equations

Fitting a straight line; Second degree curves ; Exponential curve, power curve by method of least squares; Taylor's series method ; Step by step methods : Euler's method, modified Euler's method and Runge-Kutta method for first order differential equations.

Module-III: Laplace Transforms

Definition of Laplace transform, linearity property, piecewise continuous function, existence of laplace transform, function of exponential order, first and second shifting theorems, change of scale property, laplace transforms of derivatives and integrals, multiplied by 't', divided by 't', laplace transform of periodic functions.

Inverse laplace transform: Definition of inverse laplace transform, linearity property, first and second shifting theorems, change of scale property, multiplied by 's', divided by 's', convolution theorem and applications.

Module-IV: Fourier Transforms

Fourier integral theorem, fourier sine and cosine integrals; fourier transforms; fourier sine and cosine transform, properties, inverse transforms, finite fourier transforms.

Module-V: Partial Differential Equations and Applications

Formation of partial differential equations by elimination of arbitrary constants and arbitrary functions, solutions of first order linear equation by lagrange method; Charpit's method; method of separation of variables; One dimensional heat and wave equations under initial and boundary conditions.

Mathematical Transform Techniques

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