



INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal-500043, Hyderabad

B.Tech III SEMESTER END EXAMINATIONS (REGULAR / SUPPLEMENTARY) - FEBRUARY 2023

Regulation:UG20

PROBABILITY THEORY AND STOCHASTIC PROCESSES

Time: 3 Hours (ELECTRONICS AND COMMUNICATION ENGINEERING) Max Marks: 70

Answer ALL questions in Module I and II

Answer ONE out of two questions in Modules III, IV and V

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

MODULE – I

1. (a) Explain joint probability and conditional probability. State and prove Baye's theorem.
[BL: Understand| CO: 1|Marks: 7]
- (b) If the probability that a communication system has high selectivity is 0.54 and the probability that it will have high fidelity is 0.81 and the probability that it will have both is 0.18, find the probability that
 - i) A system with high fidelity will also have high selectivity
 - ii) A system with high selectivity will also have fidelity [BL: Apply| CO: 1|Marks: 7]

MODULE – II

2. (a) List the properties of distribution function. Explain and determine central limit theorem
[BL: Understand| CO: 2|Marks: 7]
- (b) The joint p.d.f. of the two dimensional random variable is
$$\begin{cases} \frac{4xy}{9}, & 1 < x < 2, 1 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$
 - i) Find the marginal density functions of X and Y.
 - ii) Find the conditional density function of Y given X=x. [BL: Apply| CO: 2|Marks: 7]

MODULE – III

3. (a) Summarize joint moment generating function of random variables and derive its properties
[BL: Understand| CO: 3|Marks: 7]
- (b) Two random variables X and Y have the joint characteristic function $\phi_{X,Y}(\omega_1, \omega_2) = e^{-2\omega_1^2 - 8\omega_2^2}$. Show that X and Y are both zero mean random variables and also that they are uncorrelated.
[BL: Apply| CO: 3|Marks: 7]
4. (a) Describe jointly gaussian random variables, two random variables case and N random variable case Gaussian random variables.
[BL: Understand| CO: 4|Marks: 7]
- (b) Let two random variables U and V be linear transformations of X and Y given by $U = X - Y$; $V = X + Y$. If is a joint density function of X and Y, then find the joint density function of U and V.
[BL: Apply| CO: 4|Marks: 7]

MODULE – IV

5. (a) Write short note on the following
- Stationary random process
 - Wide sense stationary random process
 - Strict sense stationary
- [BL: Understand| CO: 5|Marks: 7]
- (b) Consider the random process $X(t) = A \cos \omega t + B \sin \omega t$ where A and B are random variables with $E(A)=0=E(B)$ and $E(AB)=0$. Prove that X(t) is mean ergodic. [BL: Apply| CO: 5|Marks: 7]
6. (a) Outline various properties of cross correlation function. Briefly explain about Gaussian random process. [BL: Understand| CO: 5|Marks: 7]
- (b) Consider two random processes $X(t) = A \cos \omega t + B \sin \omega t$ and $Y(t) = B \cos \omega t - A \sin \omega t$ where A and B are uncorrelated, zero mean random variables with same variance and ' ω ' is a constant. Show that X(t) and Y(t) are jointly stationary? [BL: Apply| CO: 5|Marks: 7]

MODULE – V

7. (a) Discuss power density spectrum of a random process and mention its properties [BL: Understand| CO: 6|Marks: 7]
- (b) An ergodic random process is known to have an auto correlation function of the form
- $$R_{XX}(\tau) = \begin{cases} 1-|\tau|, & |\tau| \leq 1 \\ 0, & |\tau| > 1 \end{cases}, \text{ . Find its spectral density.} \quad [\text{BL: Apply| CO: 6|Marks: 7}]$$
8. (a) Elucidate the concept of cross power density spectrum and write the relation between cross correlation and cross power spectrum density. [BL: Understand| CO: 6|Marks: 7]
- (b) Find the cross correlation function corresponding to the cross power spectrum
- $$S_{XY}(\omega) = \begin{cases} a + jb\omega, & |\omega| < 1 \\ 0, & \textit{elsewhere} \end{cases}$$
- [BL: Apply| CO: 6|Marks: 7]

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