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INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech IV Semester End Examinations (Regular/Supplementary) - July, 2021

Regulation: R18

COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTIONS

Time: 3 Hours

(EEE)

Max Marks: 70

Answer FIVE Questions choosing ONE question from each module
(NOTE: Provision is given to answer TWO questions from any ONE module)

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

MODULE – I

1. (a) If $f(z)$ is analytic in a region R and $f'(z) = 0$ everywhere in R . Show that $f(z)$ of constant absolute value is constant. [7M]
 (b) Construct the analytic function $f(z)$ for which the imaginary part is $v = e^{-x}(x \cos y + y \sin y)$. [7M]
2. (a) If $u = x^2 - y^2$, $v = \frac{-y}{x^2 + y^2}$, prove that u and v are harmonic functions but $u+iv$ is not an analytic function. [7M]
 (b) Find the bilinear transformation that maps the points $z = 1, i, -1$ onto $w = i, 0, -i$. respectively. [7M]

MODULE – II

3. (a) Evaluate $\int_0^{3+i} z^2 dz$ along the line joining the points $(0,0)$ and $(3,1)$. [7M]
 (b) Evaluate $\int_C \frac{e^{2z}}{z^2+1} dz$, where C is $|z| = \frac{1}{2}$. [7M]
4. (a) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y=x^2$. [7M]
 (b) Evaluate $\int_C \frac{z^2+1}{z^2-1} dz$, where C is $|z+1| = 1$, using Cauchy's integral formula. [7M]

MODULE – III

5. (a) Expand $f(z)=\cos z$ as a Taylor's series about $z = \frac{\pi}{3}$. [7M]
 (b) Evaluate $\int_C \frac{\cos \pi z^2 - \sin \pi z^2}{(z+1)(z+2)} dz$, where C is $|z| = 3$, using Cauchy's residue's theorem. [7M]
6. (a) Evaluate $f(z) = \frac{1}{(z+1)(z+3)}$ as Laurent's series valid for the region $1 < |z| < 3$. [7M]
 (b) Evaluate $\int_0^{2\pi} \frac{1}{13+5 \cos \theta} dz$, by contour integration. [7M]

MODULE – IV

7. (a) The density function of a continuous random variable X is given by $f(x) = Kx(2-x)$, $0 \leq x \leq 2$. Find its rth moment. [7M]
- (b) A random variable x has the probability function shown in Table 1. Find i) k ii) $P(x \leq 3)$ iii) $P(0 < x < 4)$ [7M]

Table 1

x	0	1	2	3	4
p(k)	k	3k	5k	7k	9k

8. (a) Find the moment generating function of the random variable X whose probability function is given by
- $$f(x) = \begin{cases} x & ; 0 < x \leq 1 \\ 2 - x & ; 1 \leq x < 2 \end{cases} \quad [7M]$$
- (b) The cumulative distribution function(cdf) of a continuous random variable X is $F(x) = 1 - (1+x)e^{-x}$, $x > 0$. Find the probability density function of X, mean and variance of X. [7M]

MODULE – V

9. (a) 6 dice are thrown 729 times. How many times do you expect atleast three dice to show 5 or 6? [7M]
- (b) A book of 500 pages contains 50 mistakes. Find the probability that there are atleast four mistakes per page. [7M]
10. (a) A manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective. What is the probability that a box will fail to meet the guaranteed quality ? [7M]
- (b) A normal distribution has mean 30 and standard deviation 5. Find the probabilities that
i) $26 \leq X \leq 40$ ii) $X \geq 45$ [7M]

