INSTITUTE OF AERONAUTICAL ENGINEERING

(AUTONOMOUS)

Dundigal, Hyderabad - 500 043

HEAT TRANSFER

III B. TECH II SEMESTER

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UNIT I – INTRODUTION TO HEAT TRANSFER

Thermodynamics & Heat Transfer

Study of
Heat and Work transfer
(quantitatively)

→ Thermodynamics

Study of
"How heat flows"

→ Heat Transfer

every activity involves
heat transfer

Conduction

The transfer of energy in a solid or fluid via molecular contact without bulk motion

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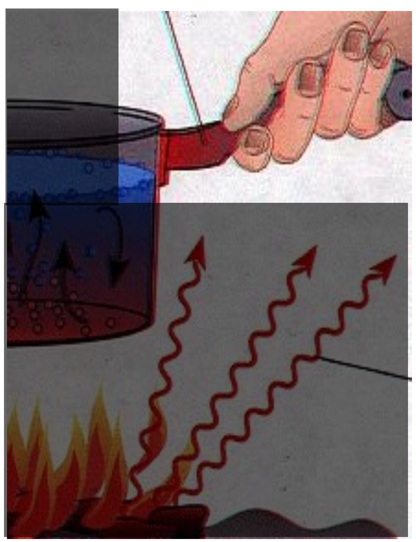
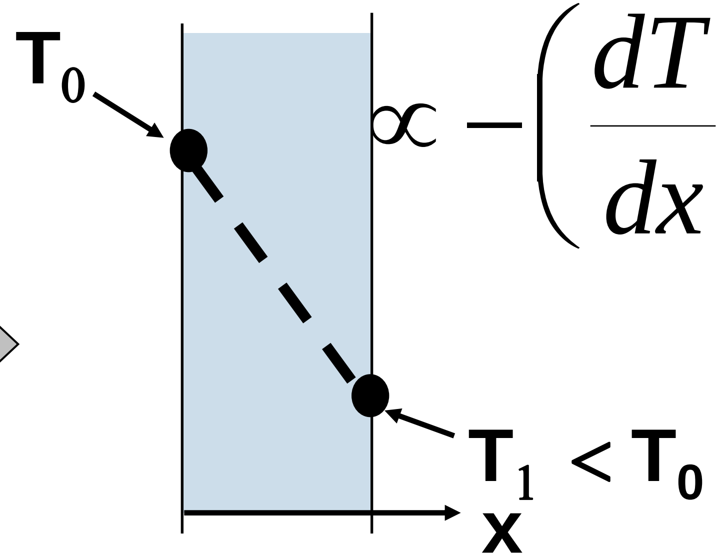
MODE

Solids > Lattice vibrations

Fluids > Molecular collisions



PHYSICAL PHENOMENON



$$\propto -\left(\frac{dT}{dx}\right)$$

MATHEMATICAL EQUATION

Fourier Law of Heat Conduction

Conduction (contd.)

$$q'' \propto -\left(\frac{dT}{dx}\right) \Rightarrow q_x'' = \left(-k\frac{\Delta T}{\Delta x}\right)$$

- The heat flux, **q** is directly proportional to temperature gradient
- The proportionality constant, **k**, is defined as the thermal conductivity, a thermo physical property.

Thermal Conductivity, k

Conduction (contd.)

Silver = 410 $Wm^{-1}K^{-1}$

METALS

	k/k_{silver}
Silver	1
Gold	0.7
Copper	0.93
Aluminum	0.86
Brass (70% Cu:30% Ni)	0.33
Platinum, Lead	0.25
Mild steel (0.1% Cu), Cast iron	0.12
Bismuth	0.07
Mercury	0.04

NON-METALS

	k/k_{silver}
Air	0.19
Water	0.0014
Granite, Sandstone	0.011
Average rock	0.012
Limestone	0.007
Ice	0.015
Glass (crown)	0.0058
Concrete (1:2:4)	0.0042
Brick	0.0038
Snow (fresh or average)	0.005
Soil (sandy, dry)	0.002
Soil (8% moist)	0.0033
Wood	0.0045

Convection

Convection occurs in liquids and gases.
Energy is carried with fluid motion when convection occurs.

$$= hA(T_{} - T_a)$$

PHYSICAL PHENOMENON

MATHEMATICAL EQUATION

Convection (contd.)

Newton's Law of Cooling

$$\dot{Q} = hA(T_w - T_a)$$

- The quantity **h** is called the convective heat transfer coefficient (W/m2-K).
- It is dependent on the type of fluid flowing past the wall and the velocity distribution.
- Thus, **h** is not a thermo physical property.

Convection Process	$h(W/m^2\text{-}K)$
Free convection	
Gases	2–25
Liquids	
50–1000	
Forced convection	
Gases	25–250
Liquids	
50–20,000	
Convection phase change	2,500–200,000

Convective Processes

Convection (contd.)

Single phase fluids (gases and liquids)

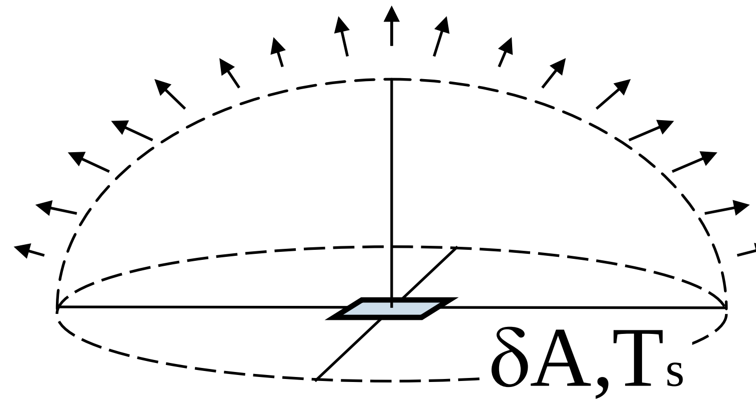
- Forced convection
- Free convection, or natural convection
- Mixed convection (forced plus free)

Convection with phase change

- Boiling
- Condensation

Radiation

Energy transfer in the form of electromagnetic waves



$$\dot{E} = \varepsilon \sigma T_s^{\,4}$$

PHYSICAL PHENOMENON

MATHEMATICAL EQUATION

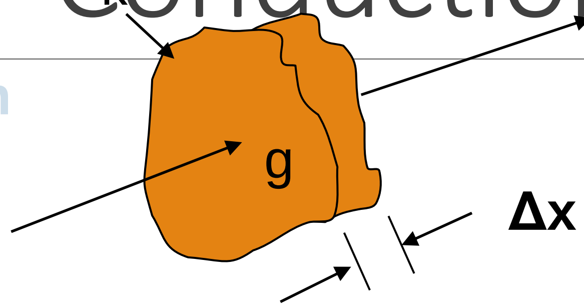
Radiation (contd.)

Stefan-Boltzman Law

$$\dot{E}_b \propto T_s^4$$

The emissive power of a black body over all wave lengths is proportional to fourth power of temperature

One Dimensional Heat Conduction

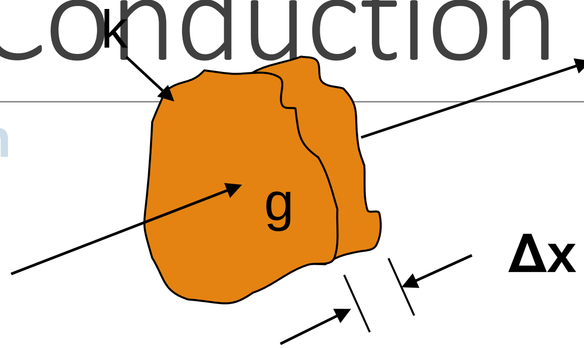


$$\begin{pmatrix}\text{Net rate of heat gain}\\ \text{by conduction}\end{pmatrix} + \begin{pmatrix}\text{Rate of energy}\\ \text{generation}\end{pmatrix} = \begin{pmatrix}\text{Rate of increase}\\ \text{of internal energy}\end{pmatrix}$$

$$[Aq]_x - [Aq]_{x+\Delta x} \quad + \quad A\,\Delta x\, g \quad = \quad A\,\Delta x\;\rho\; c_p\,\frac{\partial T(x,t)}{\partial t}$$

$$-\frac{1}{A}\frac{[Aq]_{x+\Delta x} - [Aq]_x}{\Delta x} + g = \rho c_p \frac{\partial T(x,t)}{\partial t}$$

One Dimensional Heat Conduction (contd.)



$$-\frac{1}{A}\frac{[Aq]_{x+\Delta x}-[Aq]_x}{\Delta x}+g=\rho c_p\frac{\partial T(x,t)}{\partial t}$$

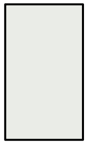
As Δx → 0, the first term on the LHS, by definition, becomes the derivative of [Aq] with respect to x

$$-\frac{1}{A}\frac{\partial}{\partial x}(Aq)+g=\rho c_p\frac{\partial T(x,t)}{\partial t}$$

$$\frac{1}{A}\frac{\partial}{\partial x}\left(Ak\frac{\partial T}{\partial x}\right)+g=\rho c_p\frac{\partial T(x,t)}{\partial t}$$

One Dimensional Heat Conduction (contd.)

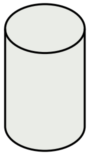
Rectangular Coordinates



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+g=\rho c_p\frac{\partial T(x,t)}{\partial t}$$

n = 0

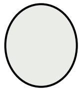
Cylindrical Coordinates



$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+g=\rho c_p\frac{\partial T(r,t)}{\partial t}$$

n = 1

Spherical Coordinates



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+g=\rho c_p\frac{\partial T(r,t)}{\partial t}$$

n = 2

A Compact Equation

$$\boxed{\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^nk\frac{\partial T}{\partial r}\right)+g=\rho c_p\frac{\partial T(r,t)}{\partial t}}$$

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

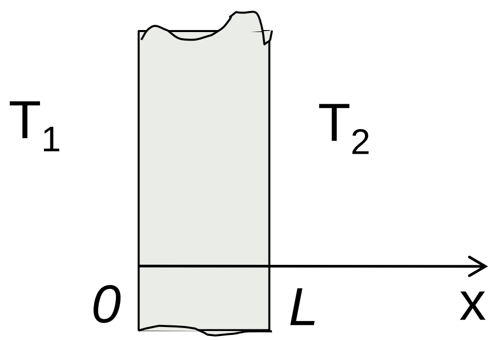
Convection BC (Third kind)

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

Convection BC (Third kind)



$$T(x,t)\,|_{x=0} = T(0,t) = T_1$$

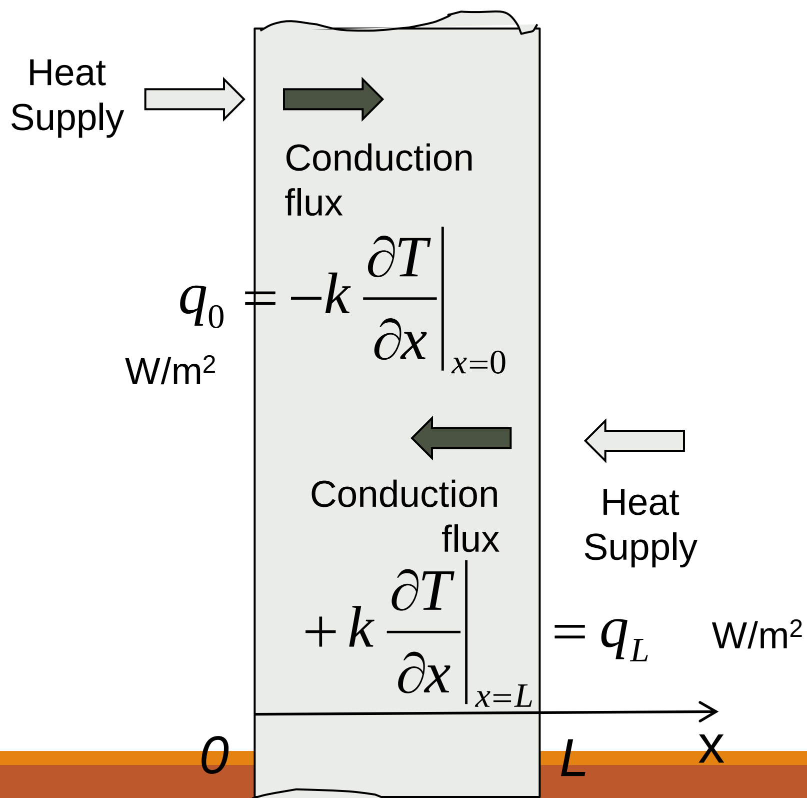
$$T(x,t)\,|_{x=L} = T(L,t) = T_2$$

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

Convection BC (Third kind)



Plate

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=0} = q_0$$

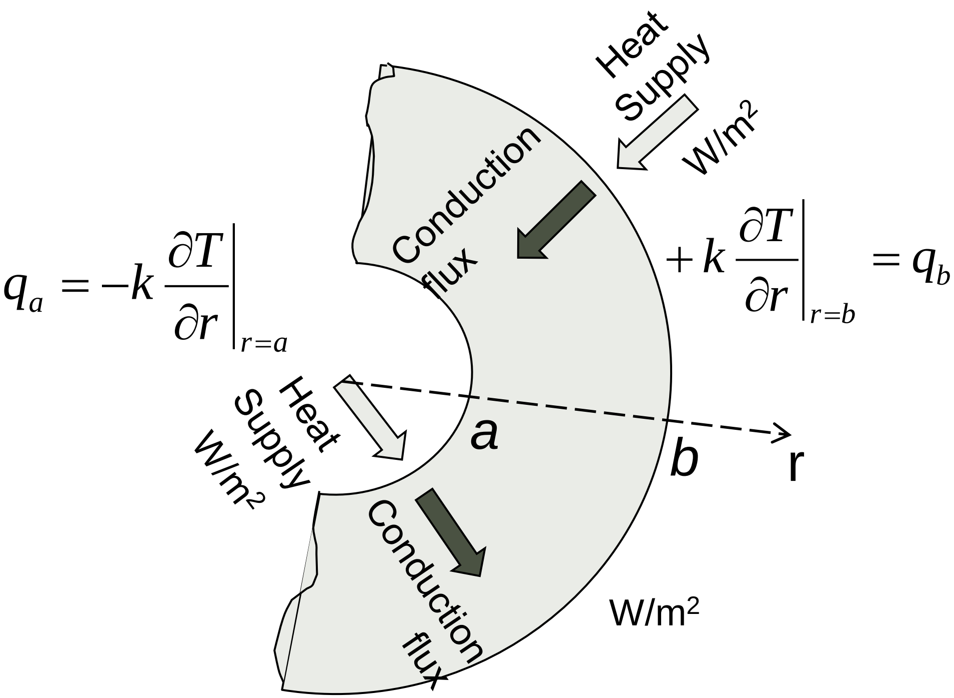
$$+k \left.\frac{\partial T}{\partial x}\right|_{x=L} = q_L$$

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

Convection BC (Third kind)



Hollow Cylinder or hollow sphere

$$-k \frac{\partial T}{\partial r}\Big|_{r=a} = q_a$$

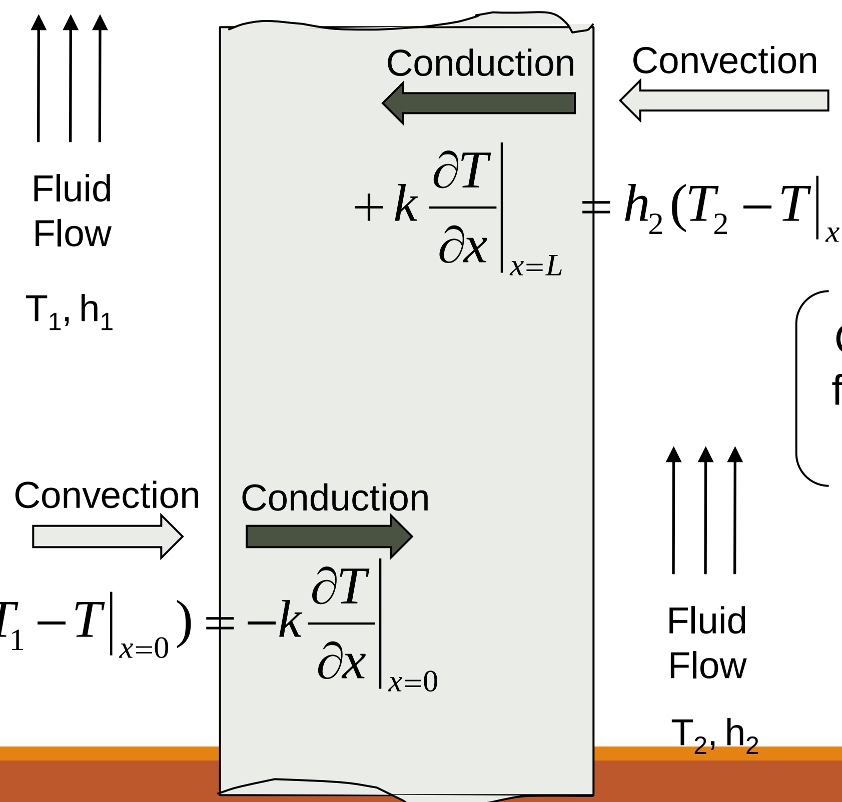
$$+k \frac{\partial T}{\partial r}\Big|_{r=b} = q_b$$

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

Convection BC (Third kind)



$$+k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h_2(T_2 - T|_{x=L})$$

$$h_1(T_1 - T|_{x=0}) = -k\left.\frac{\partial T}{\partial x}\right|_{x=0}$$

Plate

(Convection heat flux from the fluid at T_1 to the surface at x = 0) = (Conduction heat flux from the surface at x= 0 into the plate)

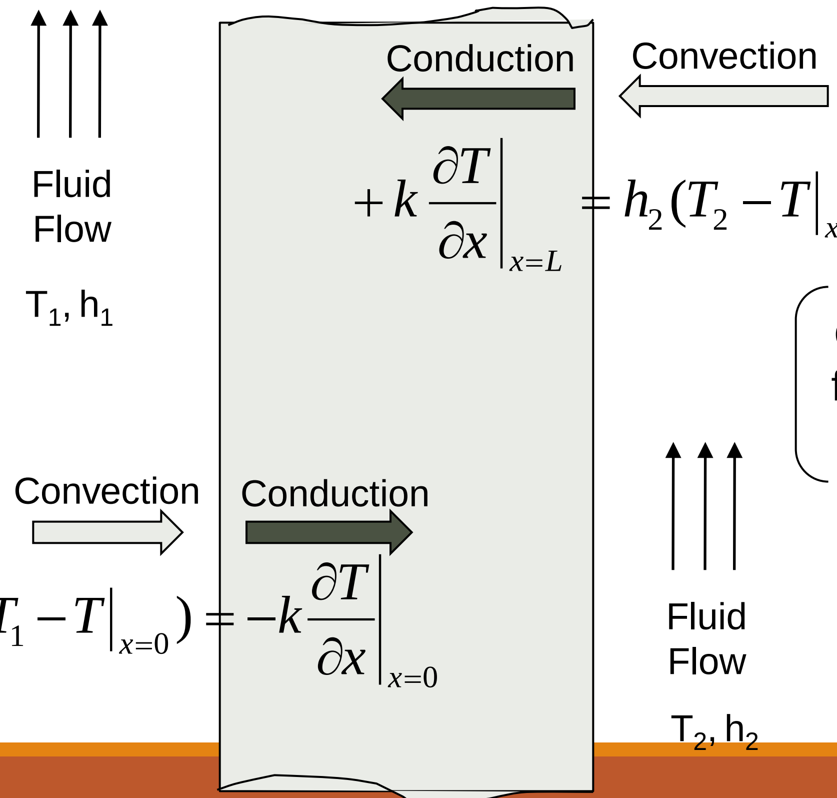
$$h_1(T_1 - T|_{x=0}) = -k\left.\frac{\partial T}{\partial x}\right|_{x=0}$$

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

Convection BC (Third kind)



$$+k\frac{\partial T}{\partial x}\bigg|_{x=L} = h_2(T_2 - T|_{x=L})$$

$$h_1(T_1 - T|_{x=0}) = -k\frac{\partial T}{\partial x}\bigg|_{x=0}$$

Plate

(Convection heat flux from the fluid at T_2 to the surface at x = L) = (Conduction heat flux from the surface at x = L into the plate)

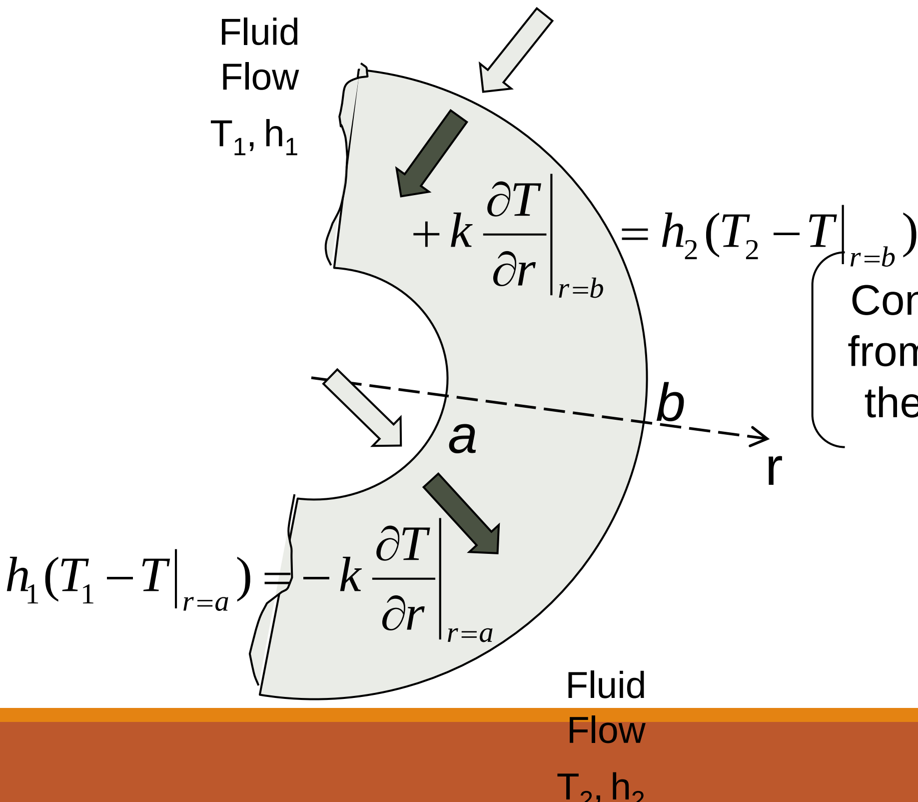
$$h_2(T_2 - T|_{x=L}) = +k\frac{\partial T}{\partial x}\bigg|_{x=L}$$

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

Convection BC (Third kind)



Hollow Cylinder or hollow sphere

(Convection heat flux from the fluid at T_1 to the surface at r = a) = (Conduction heat flux from the surface at r= a into the plate)

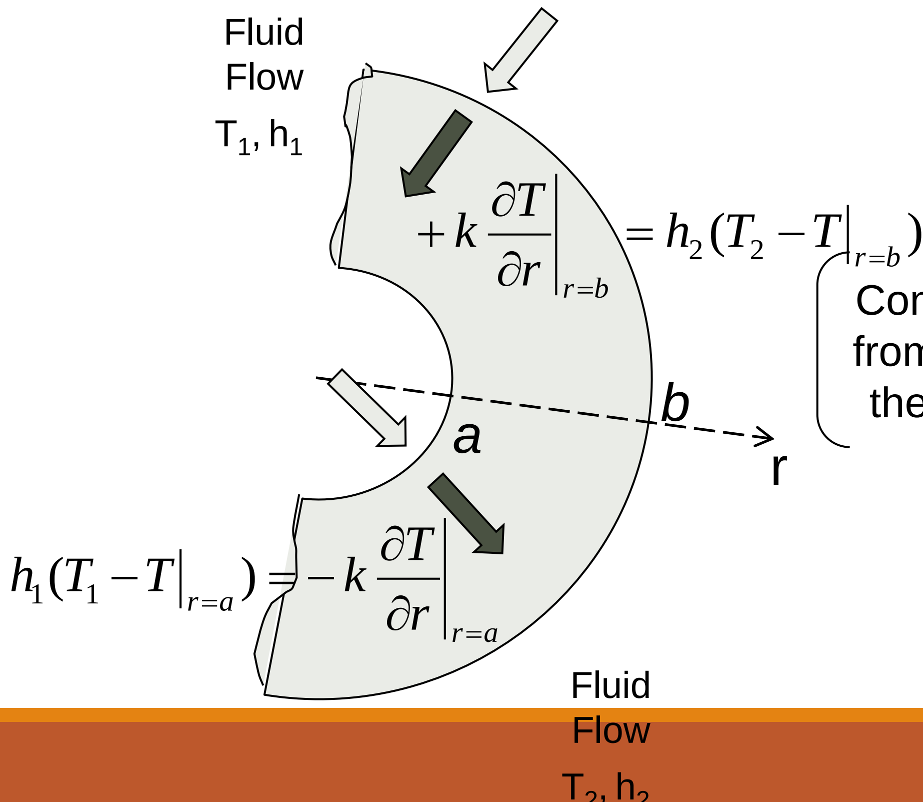
$$h_1 (T_1 - T|_{r=a}) = -k \left.\frac{\partial T}{\partial r}\right|_{r=a}$$

Boundary Conditions

Prescribed **Temperature** BC (First kind)

Prescribed **Heat Flux** BC (Second kind)

Convection BC (Third kind)



Hollow Cylinder or hollow sphere

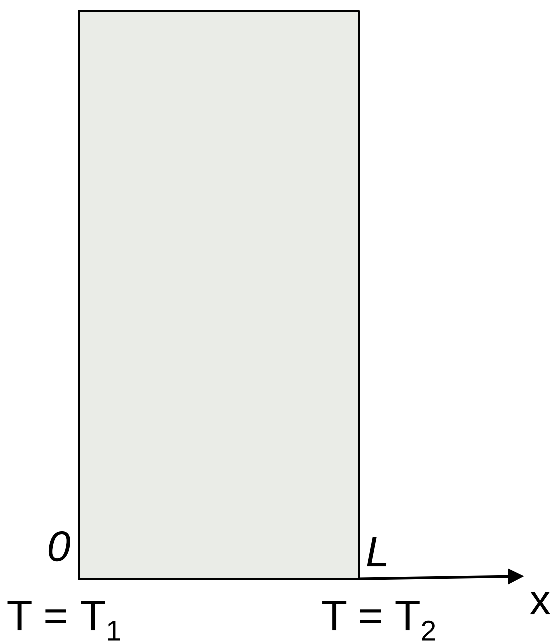
(Convection heat flux from the fluid at T_2 to the surface at r = b) = (Conduction heat flux from the surface at r= b into the plate)

$$h_2(T_2 - T|_{r=b}) = +k\frac{\partial T}{\partial r}\Big|_{r=b}$$

UNIT II - CONDUCTION

Steady State One Dimensional Heat Conduction

Rectangular Coordinates



$$\frac{\partial^2 T}{\partial x^2} = 0$$

Governing Equation

$$T(x) = c_1 x + c_2$$

$$T(x) = \left(\frac{T_2 - T_1}{L}\right) x + T_1$$

$$Q_x = \frac{K.A.(T_1 - T_2)}{L}$$

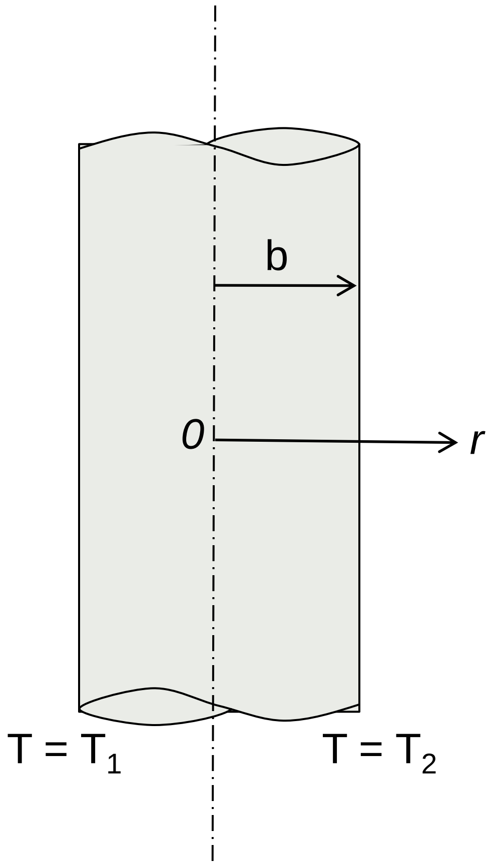
$$R = \frac{L}{K.A}$$

Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Solid Cylinder)

Governing Equation

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dT(r)}{dr}\right]+\frac{g_0}{k}=0$$



$$\left[\frac{dT(r)}{dr}=0\right]_{at\ \ r=0} \quad \left[T(r)=T_2\right]_{at\ \ r=b}$$

$$T(r)=-\frac{g_0}{2k}r+c_1\ln r+c_2$$

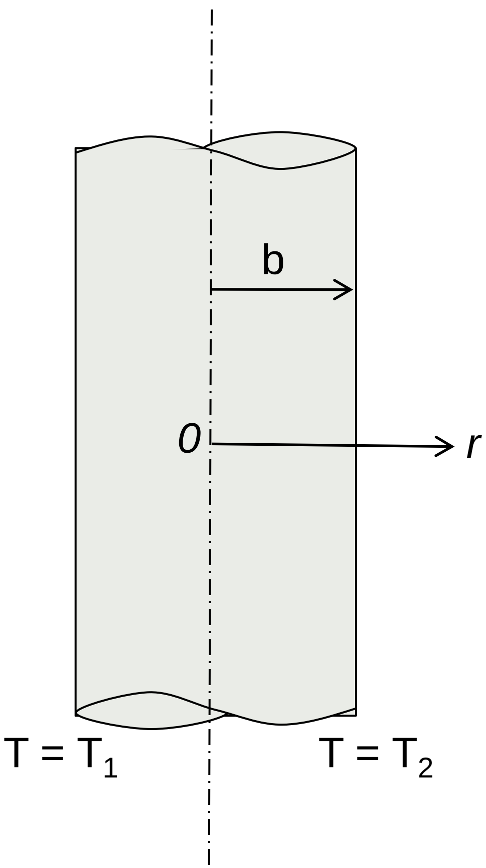
Solving, $$T(r)=-\frac{g_0}{4k}\left[1-\left(\frac{r}{b}\right)^2\right]+T_2$$

$$q(r)=-k\frac{dT(r)}{dr}=\frac{g_0 r}{2}$$

Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Solid Cylinder)

Solved Example



For r=1cm

$g_0 = 2 \times 10^8$ W/m^3

k = 20 W/(m.°C)

T2 = 100 °C

What will be the

1. Centre temperature T(0)
2. Heat flux at the boundary surface (r=1cm)

Equations to use (derive)

$$T(r) = -\frac{g_0}{4k}\left[1-\left(\frac{r}{b}\right)^2\right]+T_2$$

$$q(r) = \frac{g_0 r}{2}$$

Solution

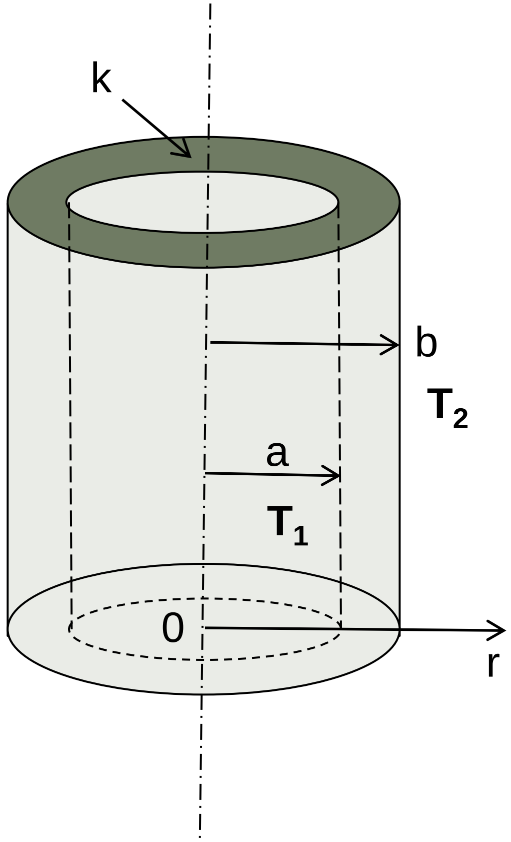
T(0) = 350 °C

q(r) = 10^6 W/m^2

Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Hollow Cylinder)

Determination of Temperature Distribution



Mathematical formulation of this problem is

$$\frac{d}{dr}\left[r \frac{dT(r)}{dr} \right] = 0 \qquad \textit{in } a < r < b$$

$$T(r) = c_1 \ln r + c_2$$

Solving, $$c_1 = \frac{T_2 - T_1}{\ln(b/a)}$$

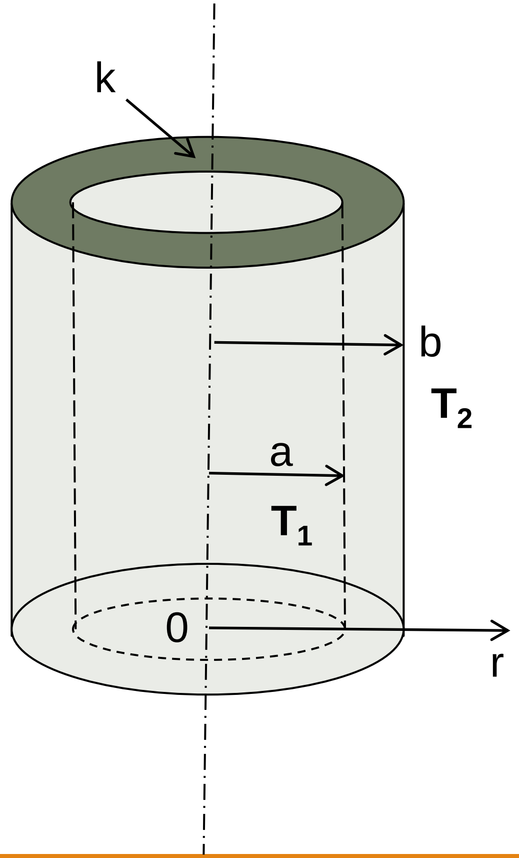
$$c_2 = T_1 - (T_2 - T_1)\frac{\ln(a)}{\ln(b/a)}$$

$$\frac{T(r) - T_1}{T_2 - T_1} = \frac{\ln(r/a)}{\ln(b/a)}$$

Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Hollow Cylinder)

Expression for radial heat flow Q over a length H



The heat flow is determined from,

$$Q = q(r).area = -k\frac{dT(r)}{dr}2\Pi rH$$

$$= -k2\Pi Hc_1$$

Since, $dT(r)/dr = (1/r)c_1$

$$Q = \frac{2\Pi kH}{\ln(b/a)}(T_1 - T_2)$$

Rearranging,

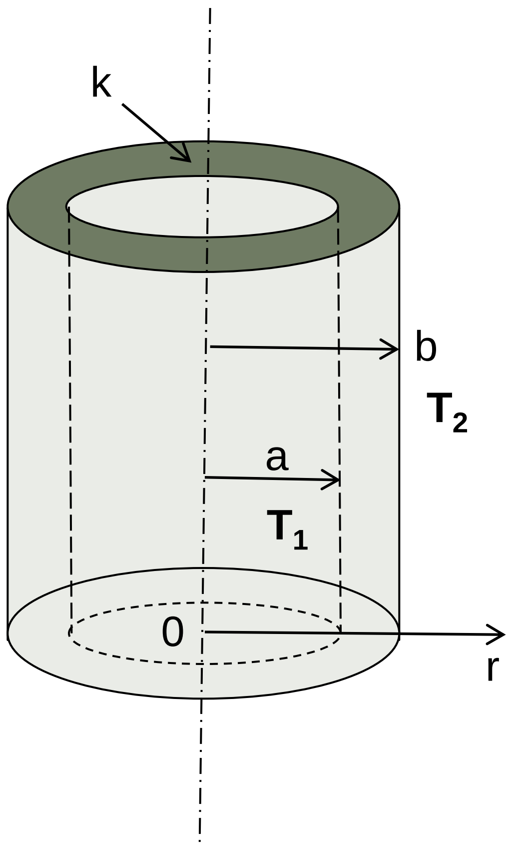
$$Q = \frac{T_1 - T_2}{R}$$

where, $R = \dfrac{\ln(b/a)}{2\Pi kH}$

Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Hollow Cylinder)

Expression for thermal resistance for length H



$$R = \frac{\ln(b/a)}{2\Pi kH}$$

Above equation can be rearranged as,

$$R = \frac{\ln(b/a)}{2\Pi kH} = \frac{(b-a)\ln[2\Pi bH/(2\Pi aH)]}{(b-a)2\Pi Hk}$$

$$R = \frac{t}{kA_m} \quad \textit{where,} \quad A_m = \frac{A_1 - A_0}{\ln(A_1 - A_0)}$$

here, $A_0 = 2\pi aH$ *=area of inner surface of cylinder*

$A_1 = 2\pi bH$ *=area of outer surface of cylinder*

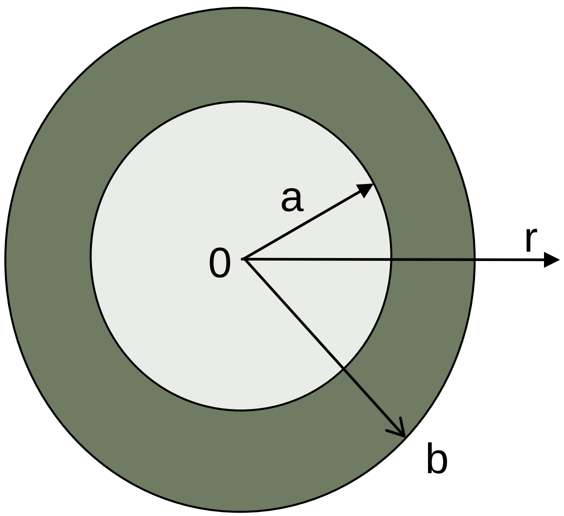
A_m *= logarithmic mean area*

$t = b - a$ *= thickness of cylinder*

Steady State One Dimensional Heat Conduction

Spherical Coordinates (Hollow Sphere)

Expression for temperature distribution



The mathematical formulation is given by,

$$\frac{d}{dr}\left(r^2 \frac{dT(r)}{dr}\right) = 0 \quad in \;\; a < r < b$$

$$T(r) = -\frac{c_1}{r} + c_2$$

$$where, \;\; c_1 = -\frac{ab}{b-a}(T_1 - T_2)$$

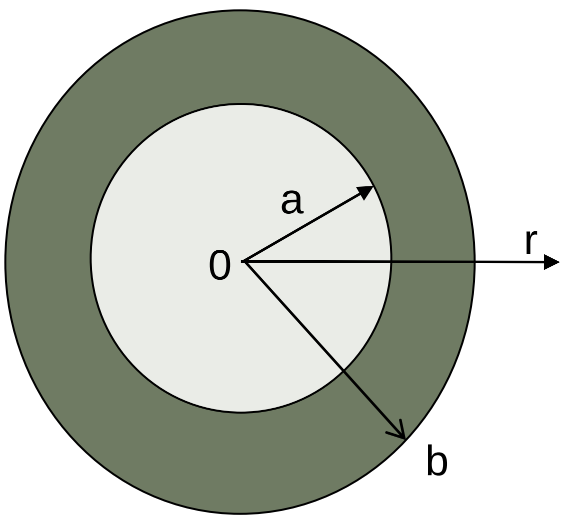
$$c_2 = \frac{bT_2 - aT_1}{b-a}$$

$$T(r) = \frac{a}{r}.\frac{b-r}{b-a}.T_1 + \frac{b}{r}.\frac{r-a}{b-a}.T_2$$

Steady State One Dimensional Heat Conduction

Spherical Coordinates (Hollow Sphere)

Expression for heat flow rate Q and thermal resistance R



Heat flow rate is determined using the equation,

$$Q = (4\Pi r^2)\left[-k\frac{dT(r)}{dr}\right]$$

$$= (4\Pi r^2)\left(-k\frac{c_1}{r^2}\right) = -4\Pi k c_1$$

using, $c_1 = -\frac{ab}{b-a}(T_1 - T_2)$

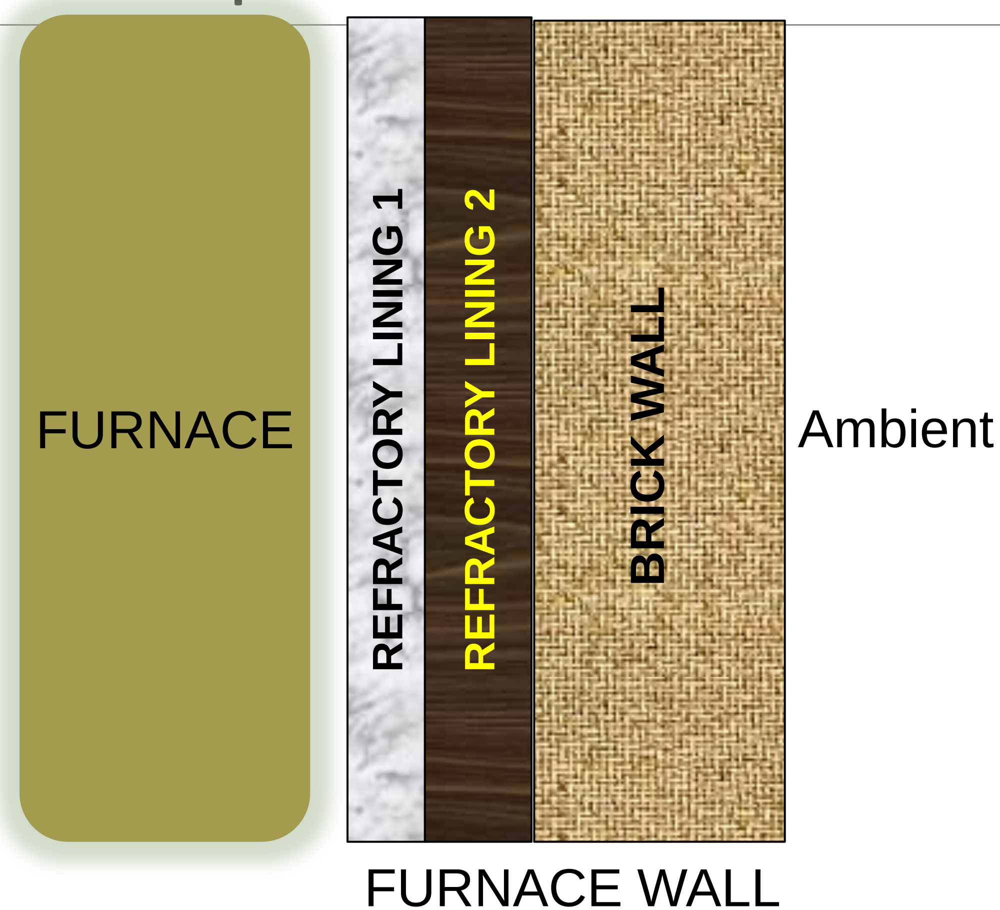
from last slide

$$Q = 4\Pi k\frac{ab}{b-a}(T_1 - T_2) = \frac{T_1 - T_2}{R}$$

where, $R = \frac{b-a}{4\Pi kab}$

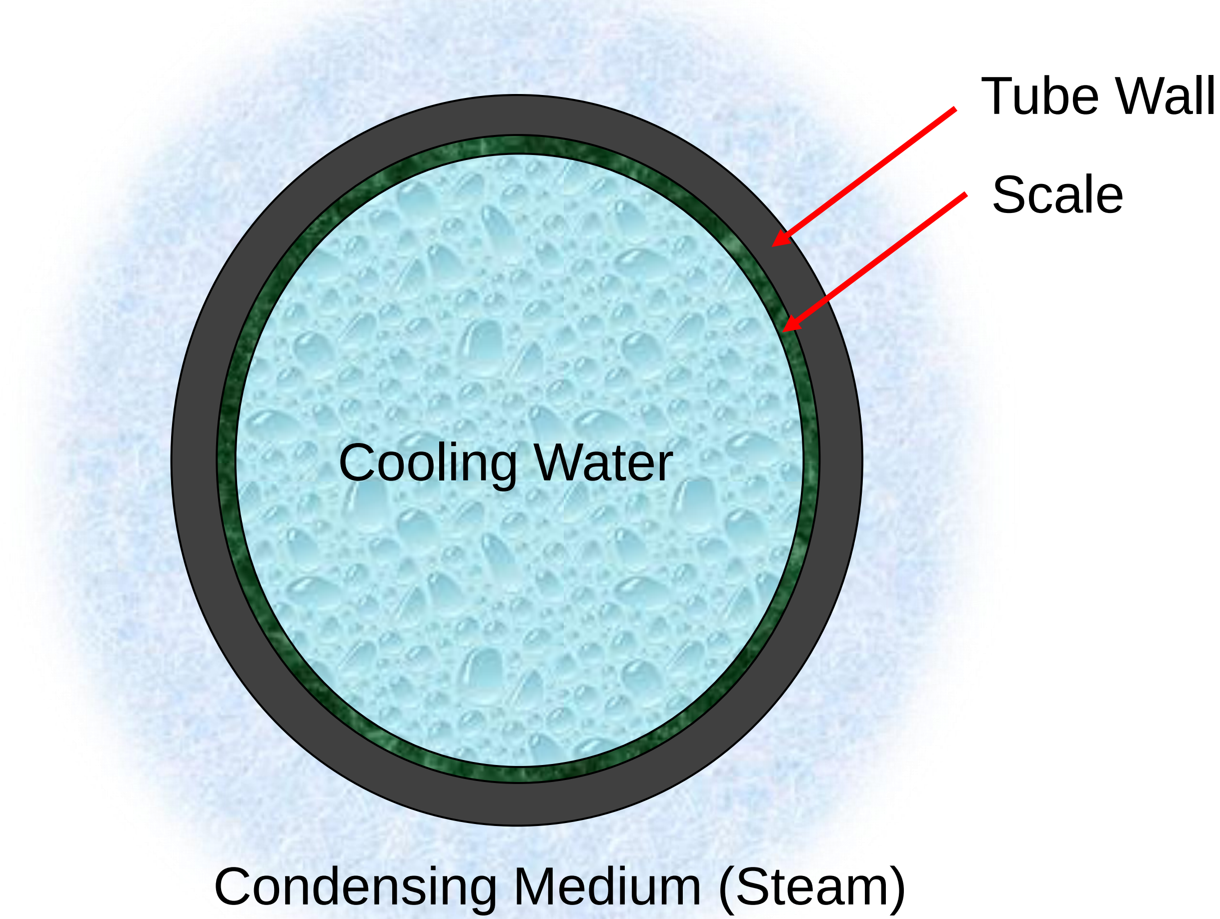
Composite Medium

Example (Furnace Wall)



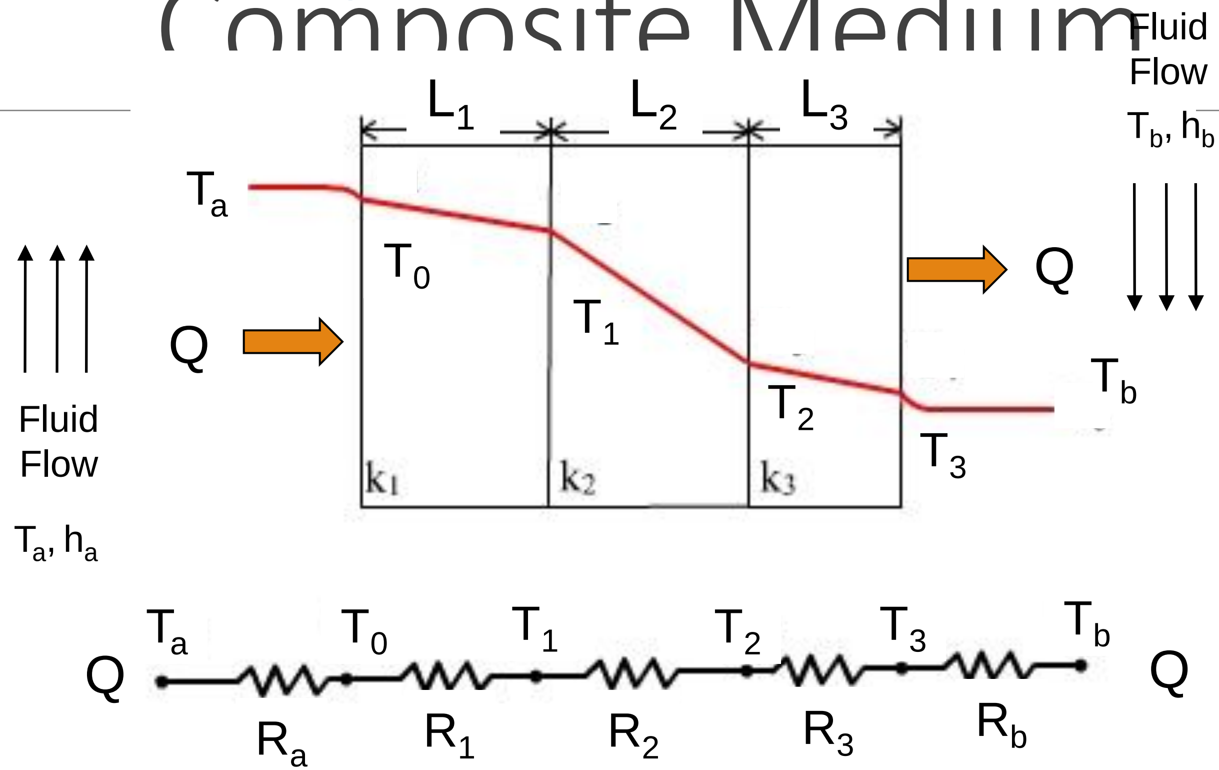
Example (Condenser Water Tube)

Composite Medium



Composite Slab (resistance in series)

Composite Medium



$$Q = \frac{T_a - T_0}{R_a} = \frac{T_0 - T_1}{R_1} = \frac{T_1 - T_2}{R_2} = \frac{T_2 - T_3}{R_3} = \frac{T_3 - T_b}{R_b}$$

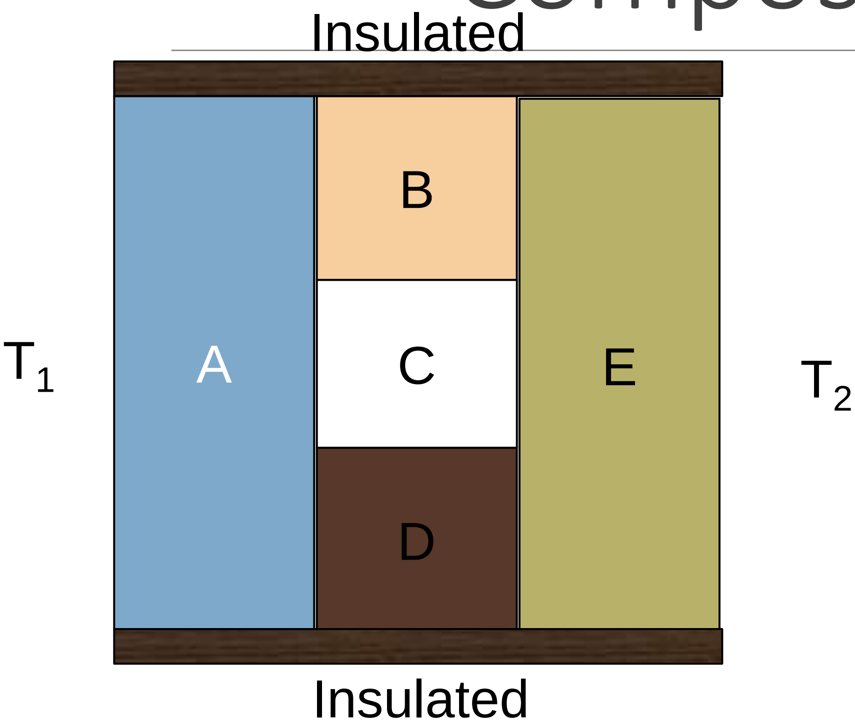
$$R_a = \frac{1}{Ah_a}; R_1 = \frac{L_1}{Ak_1}; R_2 = \frac{L_2}{Ak_2}; R_3 = \frac{L_3}{Ak_3}; R_b = \frac{1}{Ah_b}$$

$$Q = \frac{T_a - T_b}{R} \quad W$$

$$R = R_a + R_1 + R_2 + R_3 + R_b$$

Composite Slab (resistance in parallel)

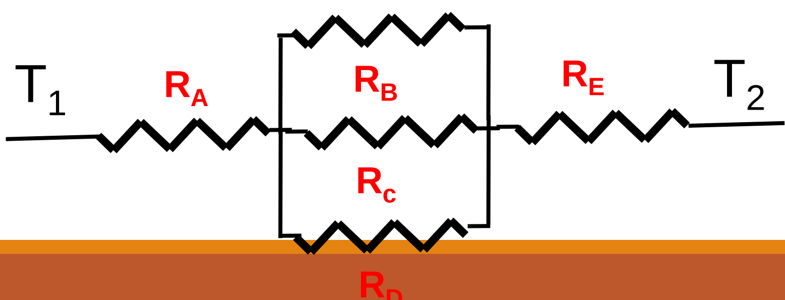
Composite Medium



$$Q = \frac{T_1 - T_2}{R} \quad W$$

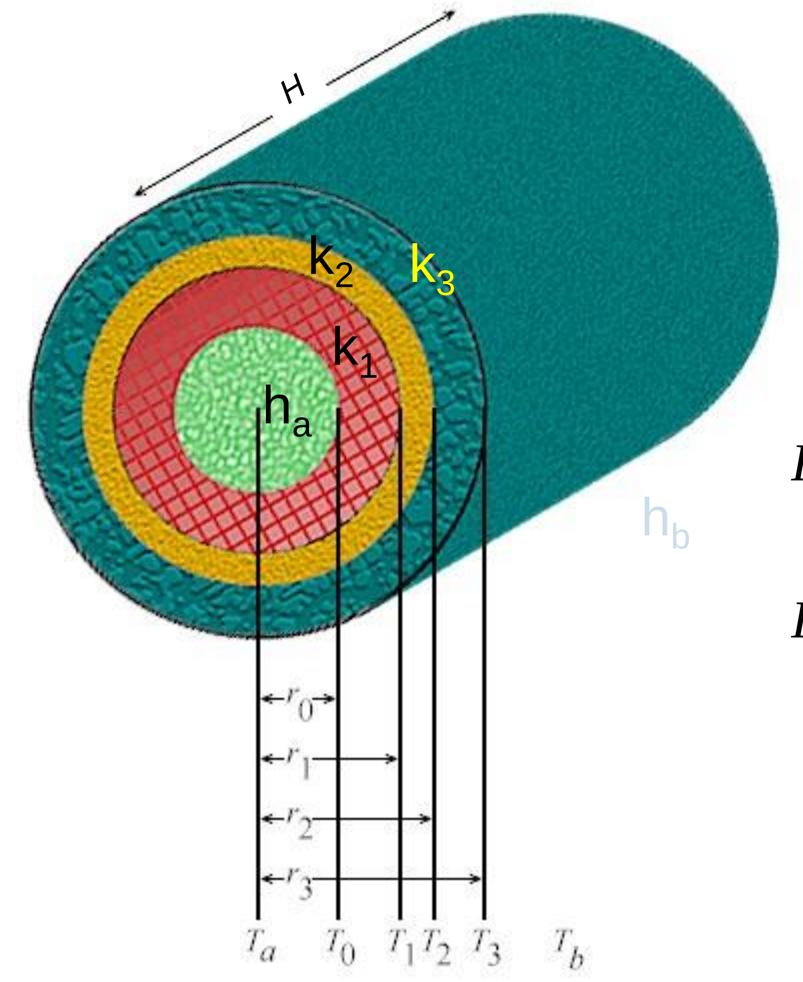
$$R = R_A + R_{eq.p} + R_E$$

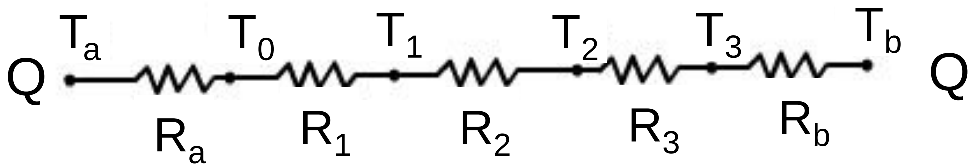
$$\frac{1}{R_{eq.p}} = \frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_D}$$



Composite Cylinder

Composite Medium





$$Q = \frac{T_a - T_0}{R_a} = \frac{T_0 - T_1}{R_1} = \frac{T_1 - T_2}{R_2} = \frac{T_2 - T_3}{R_3} = \frac{T_3 - T_b}{R_b}$$

$$R_a = \frac{1}{2\Pi r_0 H h_a}; R_1 = \frac{1}{2\Pi H k_1} \ln \frac{r_1}{r_0}; R_2 = \frac{1}{2\Pi H k_2} \ln \frac{r_2}{r_1}$$

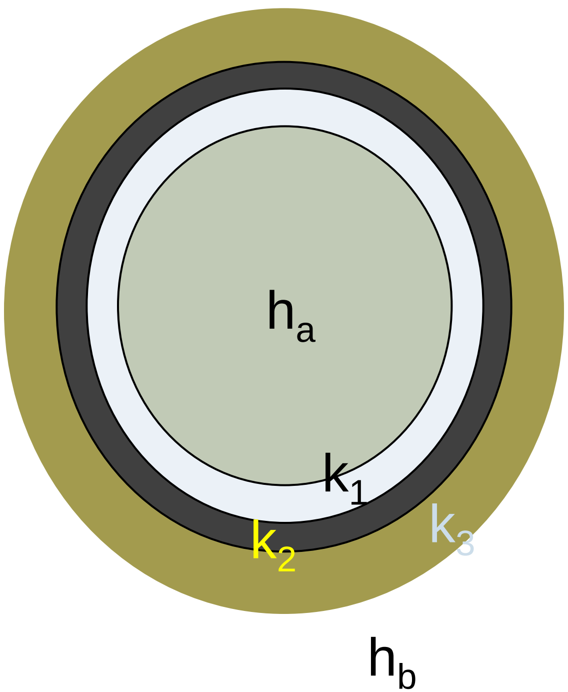
$$R_3 = \frac{1}{2\Pi H k_3} \ln \frac{r_3}{r_2}; R_b = \frac{1}{2\Pi r_3 H h_b}$$

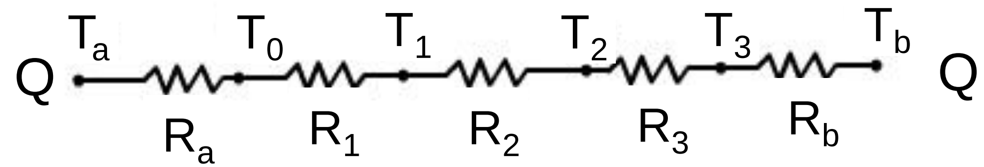
$$Q = \frac{T_a - T_b}{R} \quad W$$

$$R = R_a + R_1 + R_2 + R_3 + R_b$$

Composite Spheres

Composite Medium





$$Q = \frac{T_a - T_0}{R_a} = \frac{T_0 - T_1}{R_1} = \frac{T_1 - T_2}{R_2} = \frac{T_2 - T_3}{R_3} = \frac{T_3 - T_b}{R_b}$$

$$R_a = \frac{1}{4\Pi r_0^2 h_a}; R_1 = \frac{1}{4\Pi k_1} \frac{r_1 - r_0}{r_1 r_0}; R_2 = \frac{1}{4\Pi k_2} \frac{r_2 - r_1}{r_2 r_1}$$

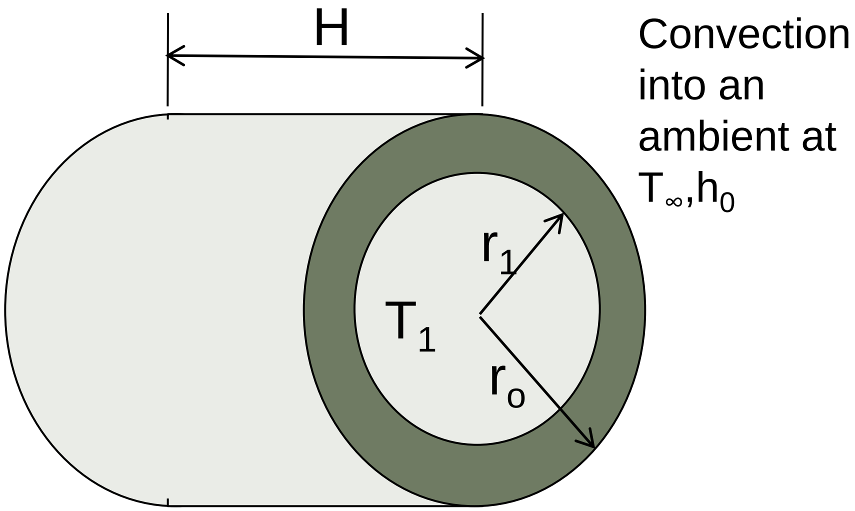
$$R_3 = \frac{1}{4\Pi k_3} \frac{r_3 - r_2}{r_3 r_2}; R_b = \frac{1}{4\Pi r_3^2 h_b}$$

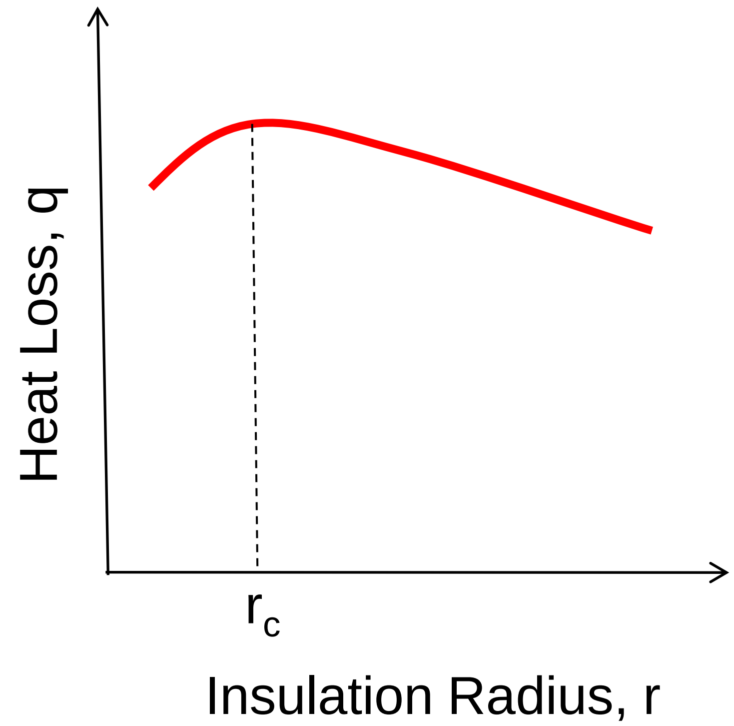
$$Q = \frac{T_a - T_b}{R} \quad W$$

$$R = R_a + R_1 + R_2 + R_3 + R_b$$

Critical Thickness of Insulation

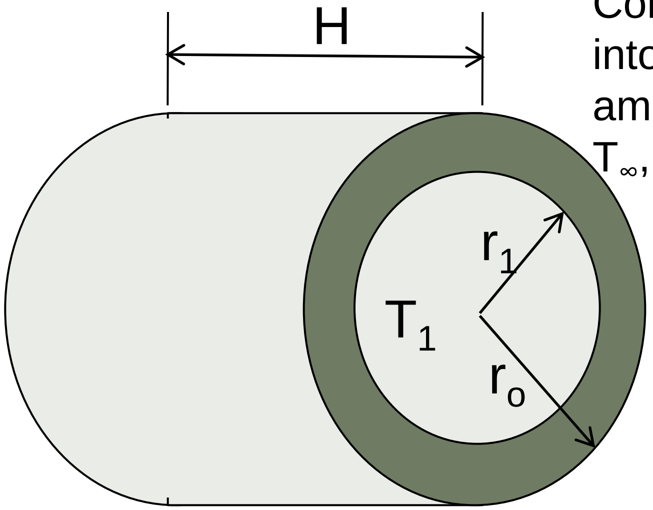
Composite Medium





Composite Medium

The rate of heat loss Q from the tube is given by



$$Q = \frac{T_i - T_\infty}{R_{ins} + R_0}$$

$$R_{ins} = \frac{1}{2\Pi kH} \ln \frac{r_0}{r_i} \qquad R_0 = \frac{1}{2\Pi r_0 H h_0}$$

$$\frac{dQ}{dr_0} = -\frac{2\Pi kH(T_i - T_\infty)}{[\ln(r_0/r_i) + k/(h_0 r_0)]^2}\left(\frac{1}{r_0} - \frac{k}{h_0 r_0^2}\right) = 0$$

For Cylinder

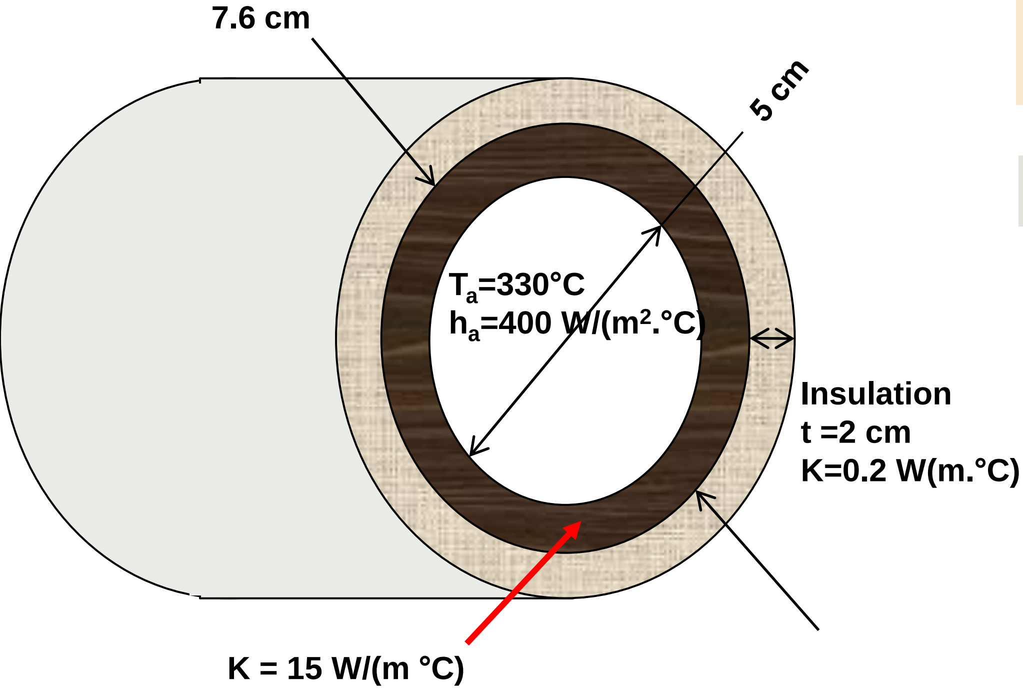
$$r_{0c} = \frac{k}{h_0}$$

For Sphere

$$r_{0c} = \frac{2k}{h_0}$$

Composite Medium

Solved Example (Composite Cylinder)



Ambient air
T_b=30°C
h_b= 60 W/(m².°C)

Calculate,

1. Heat loss from tube for length H=10m
2. Temperature drops resulting in thermal resistances

Determination of heat loss

$$Q = \frac{T_a - T_b}{R_a + R_1 + R_2 + R_b} \quad W$$

$$R_a = \frac{1}{2\Pi r_0 H h_a} = \frac{1}{2\Pi \times 0.025 \times 10 \times 400}$$

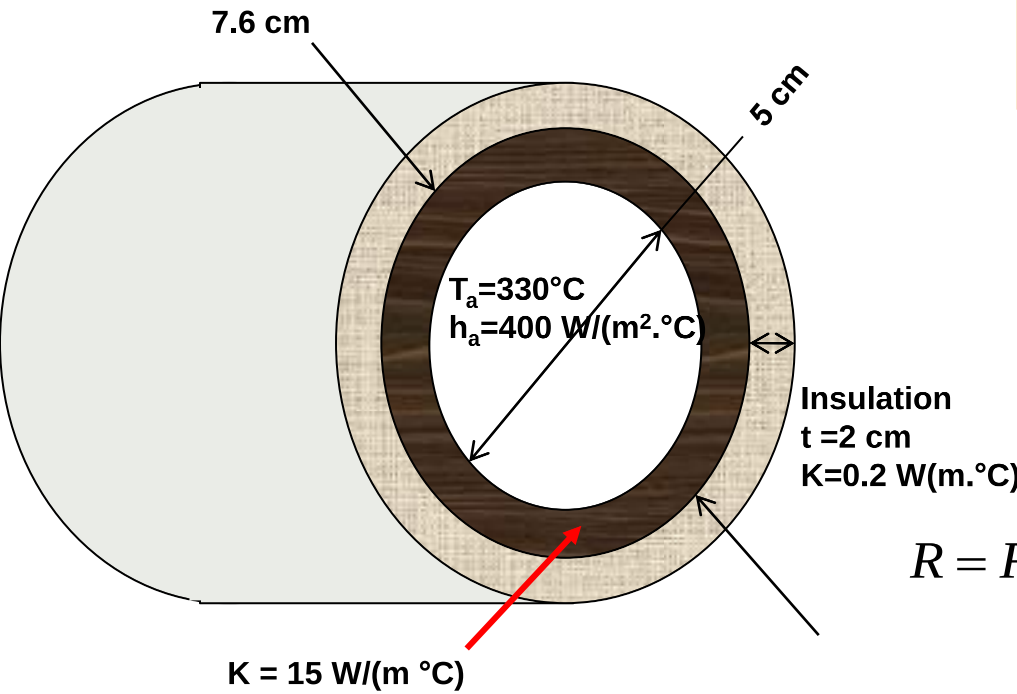
$$R_a = 1.59 \times 10^{-3} \ {}^\circ C / W$$

$$R_1 = \frac{1}{2\Pi H k_1} \ln \frac{r_1}{r_0} = \frac{1}{2\Pi \times 10 \times 15} \ln \frac{3.8}{2.5}$$

$$R_1 = 0.44 \times 10^{-3} \ {}^\circ C / W$$

Solved Example (Composite Cylinder)

Composite Medium



Ambient air
T_b=30°C
h_b= 60 W/(m².°C)

Calculate,

1. Heat loss from tube for length H=10m
2. Temperature drops resulting in thermal resistances

Determination of heat loss

$$R_2 = 33.65 \times 10^{-3} \ °C/W$$

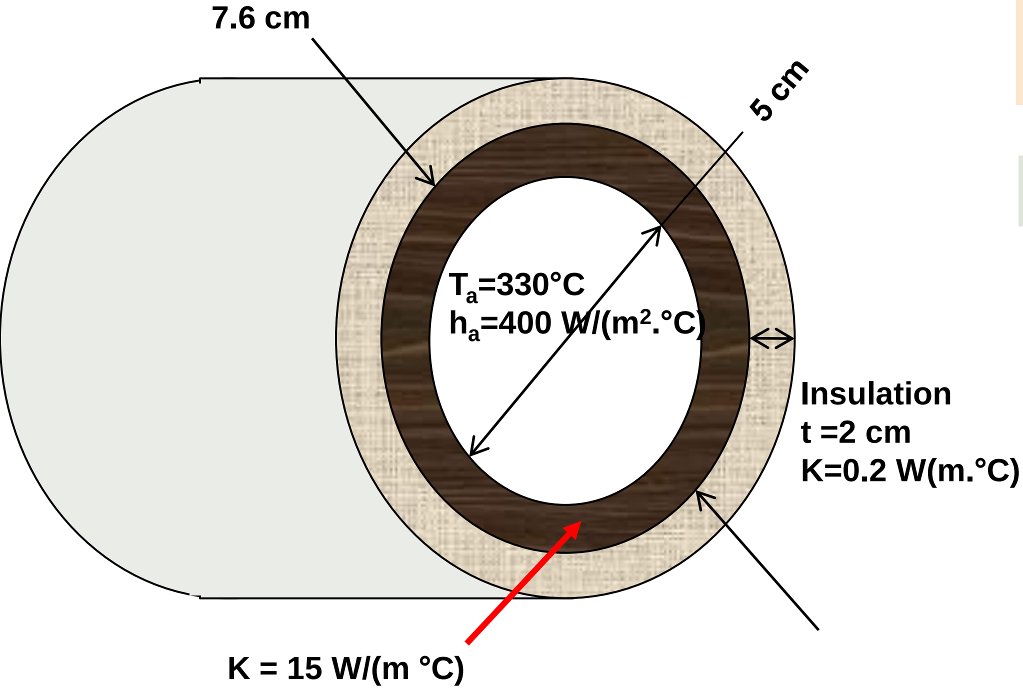
$$R_b = 4.21 \times 10^{-3} \ °C/W$$

$$R = R_a + R_1 + R_2 + R_3 + R_b = 39.89 \times 10^{-3} \ °C/W$$

$$Q = \frac{330 - 30}{39.89 \times 10^{-3}} = 7521 \ W$$

Solved Example (Composite Cylinder)

Composite Medium



Ambient air
T_b=30°C
h_b= 60 W/(m².°C)

Calculate,

1. Heat loss from tube for length H=10m
2. Temperature drops resulting in thermal resistances

Determination of temperature drops

$$Q = \frac{T_a - T_0}{R_a} = \frac{T_0 - T_1}{R_1} = \frac{T_1 - T_2}{R_2} = \frac{T_2 - T_b}{R_b}$$

$$\Delta T_{hotgas} = QR_a = 12.0°C$$

$$\Delta T_{tube} = QR_1 = 3.3°C$$

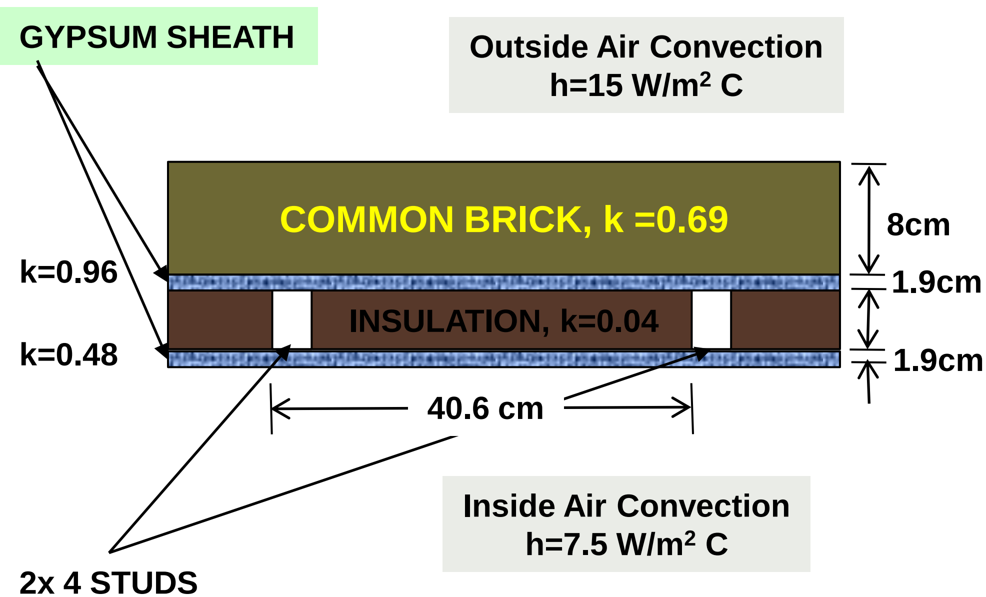
$$\Delta T_{insulation} = QR_2 = 253.0°C$$

$$\Delta T_{outside} = QR_b = 31.7°C$$

Solved Example (Composite Wall)

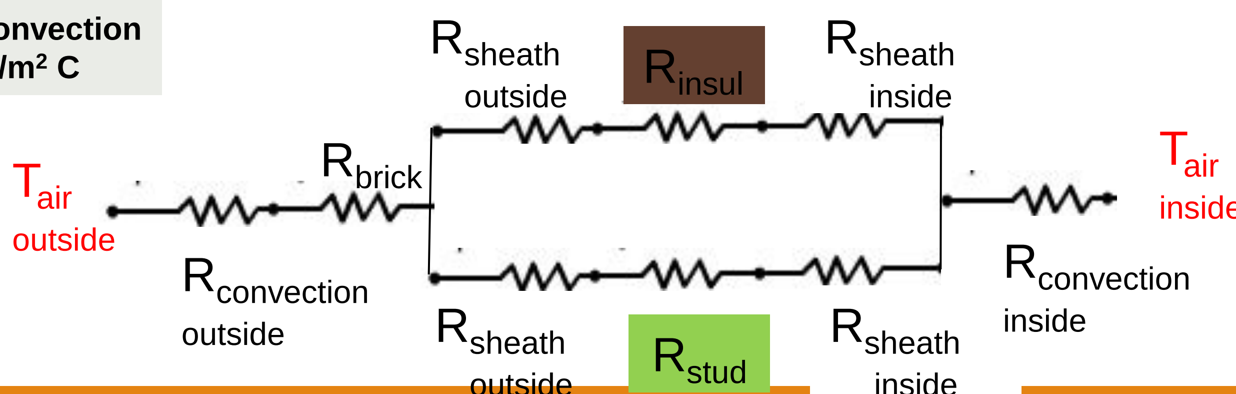
Composite Medium

2 x 4 wood studs have actual dimensions of 4.13 x 9.21 cm with k = 0.1 W/m.°C
Calculate,
1. Overall heat transfer coefficient
2. R value of the wall



Thermal resistance model

Two parallel heat flow paths are possible
1. Through the studs
2. Through the insulation

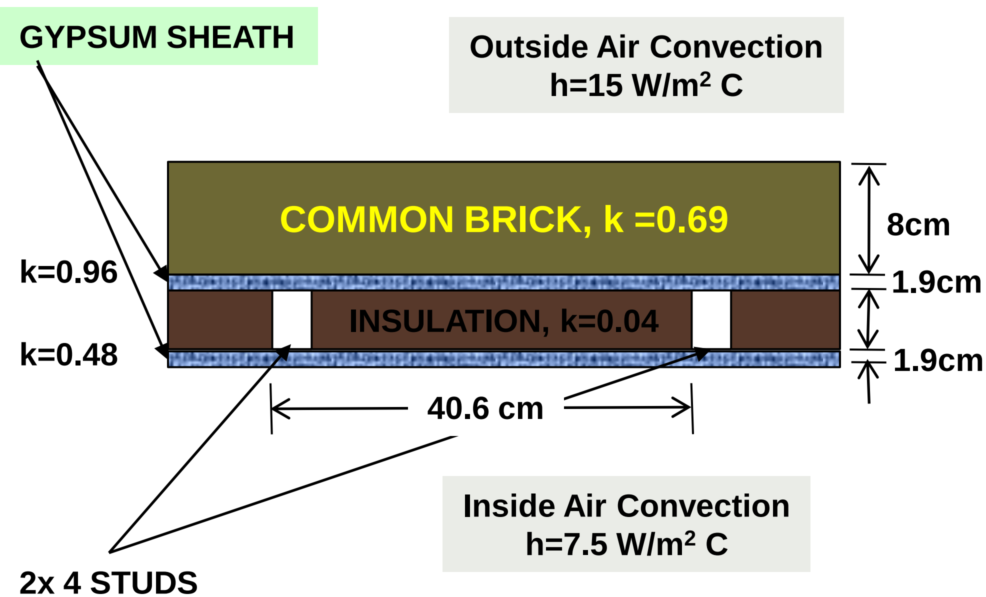


Note: 'k' is expressed in W/m °C

Solved Example (Composite Wall)

Composite Medium

Calculate,
1. Overall heat transfer coefficient
2. R value of the wall



Heat flow through the studs

Area = $0.0413 m^2$/unit depth

Heat flow occurs through **6** thermal resistances
1. Convection Resistance outside of brick
2. Conduction resistance in brick
3. Conduction resistance through outer sheet
4. Conduction resistance through wood stud
5. Conduction resistance through inner sheet
6. Convection resistance on inside

Recall,

$$R_{convection} = 1/hA \qquad R_{conduction} = \Delta x / kA$$

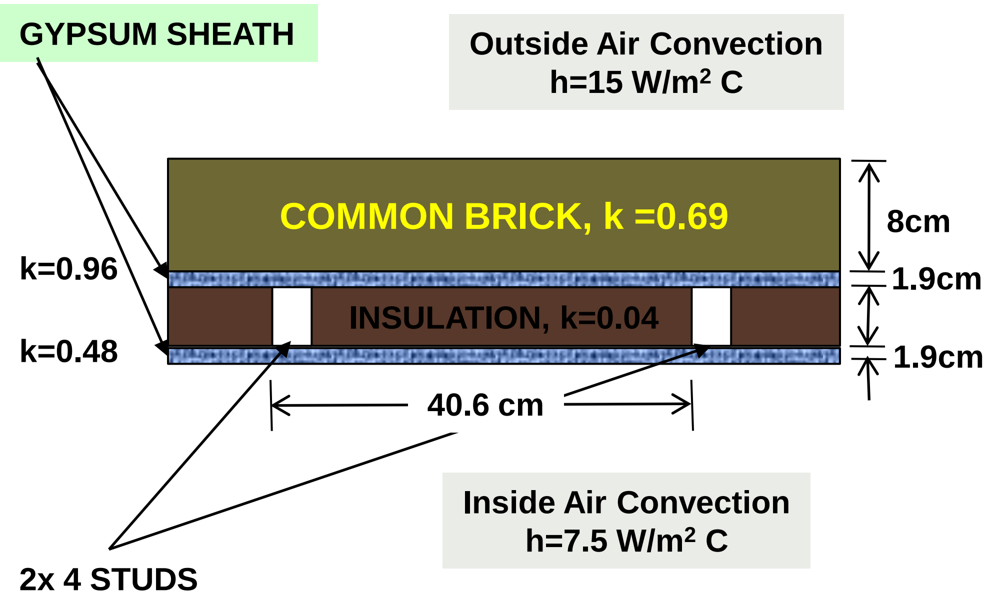
$$R_{total} = 1.614 + 2.807 + 0.48 + 22.3 + 0.96 + 3.23 = 31.39°C/W$$

Note: 'k' is expressed in W/m °C

Solved Example (Composite Wall)

Composite Medium

Calculate,
1. Overall heat transfer coefficient
2. R value of the wall



Heat flow through the insulation

The five of the materials are same, but the resistances involve different area terms, i.e., 40.6 - 4.13 cm instead of 4.13 cm.

Thus the total resistance of the insulation section is given below

$$R_{total} = 7.337\ °C/W$$

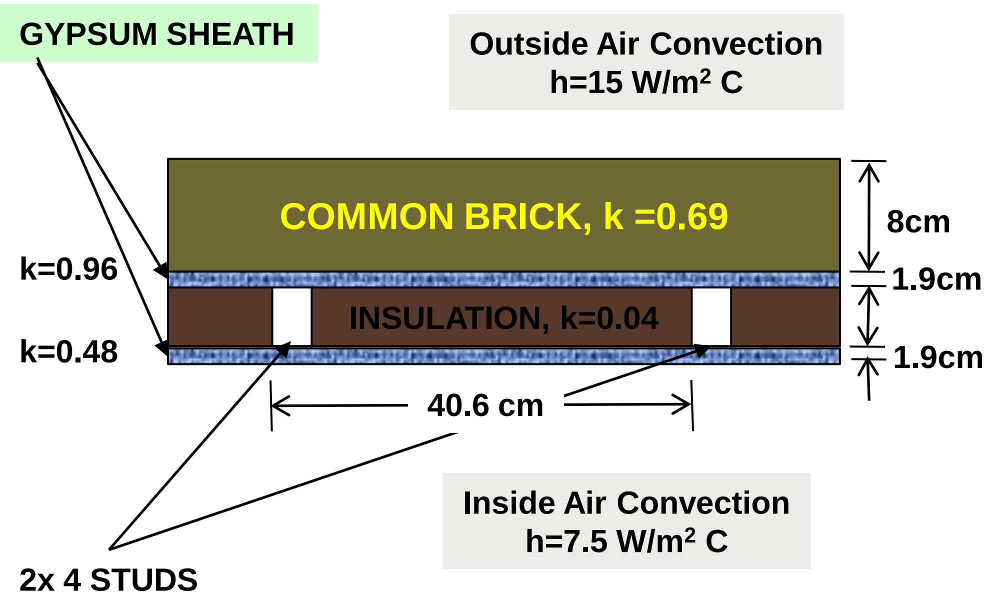
Note: 'k' is expressed in W/m °C

Solved Example (Composite Wall)

Composite Medium

Calculate,

1. Overall heat transfer coefficient
2. R value of the wall



1. Overall heat transfer coefficient

Overall resistance is obtained by combining the parallel resistances as calculated earlier.

$$R_{overall} = \frac{1}{(1/31.39)+(1/7.337)}$$

$$= 5.947°C/W$$

Overall heat transfer coefficient is found by,

$$q = UA\Delta T = \frac{\Delta T}{R_{overall}}$$ (here, A = 0.406m²)

$$U = \frac{1}{RA} = \frac{1}{(5.947)(0.406)} = 0.414 W/m^2°C$$

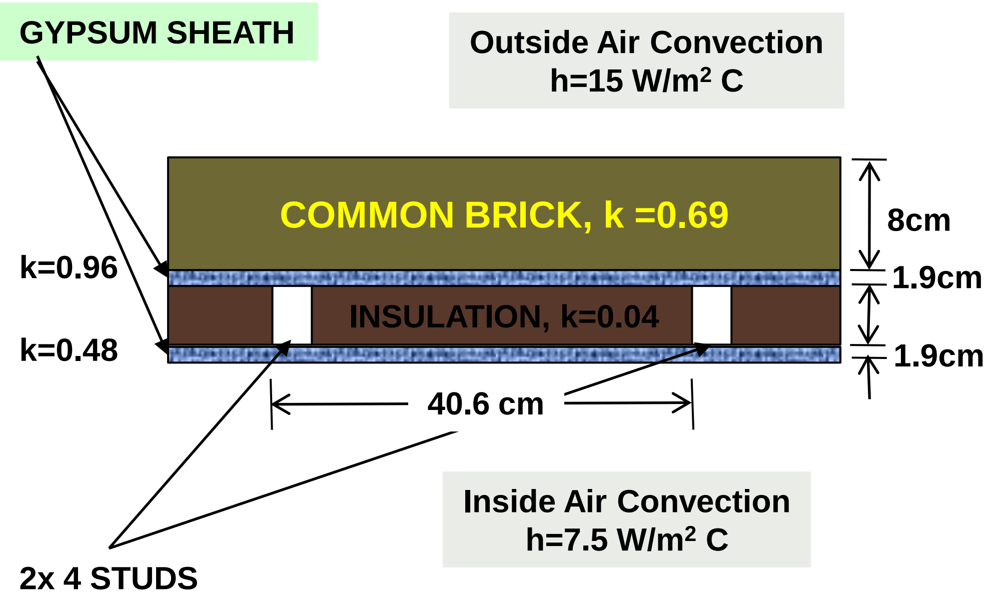
Note: 'k' is expressed in W/m °C

Solved Example (Composite Wall)

Composite Medium

Calculate,

1. Overall heat transfer coefficient
2. R value of the wall



2. R Value of the wall

The resistance of the wall is calculated using the overall heat transfer coefficient, as given below:

$$R_{value} = \frac{1}{U} = \frac{1}{0.414} = 2.414°C.m^2/W$$

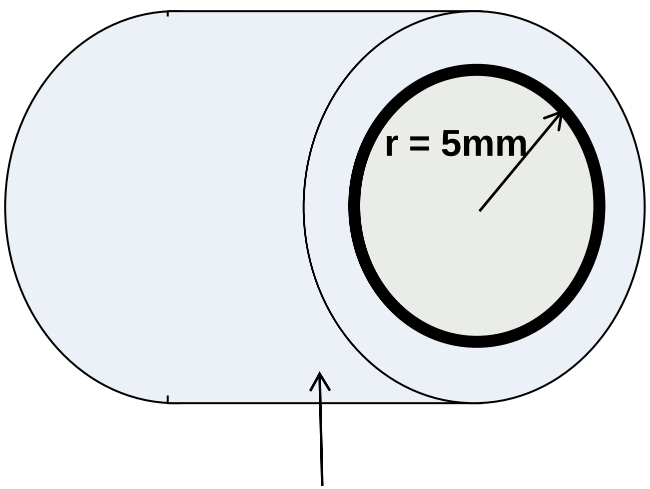
Note: 'k' is expressed in W/m °C

Solved Example (Critical Thickness of Insulation)

Composite Medium

Calculate, the critical thickness of rubber and the maximum heat transfer rate per metre length of conductor.
The temperature of rubber is not to exceed 65 °C (due to heat generated within).

Ambient at 30°C, 8.5 W/m²K



Rubber
k = 0.155 W/mK

Critical thickness

$$r_{0c} = k / h_o = 0.155 / 8.5 = 0.0182 \ m$$

Maximum heat transfer rate

$$R_{ins} = \frac{1}{2\Pi kH} \ln \frac{r_0}{r_i} = \frac{1}{2\Pi \times 0.155} \ln\left(\frac{0.0182}{0.005}\right) = 1.32 \ ^\circ C / W - m$$

$$R_0 = \frac{1}{2\Pi r_0 H h_0} = \frac{1}{2\Pi \times 0.0182 \times 8.5} = 1.02 \ ^\circ C / W - m$$

$$Q = \frac{T_i - T_\infty}{R_{ins} + R_0} = \frac{65 - 30}{1.32 + 1.02} = 14.89 \ W / m$$

Plane wall with heat generation

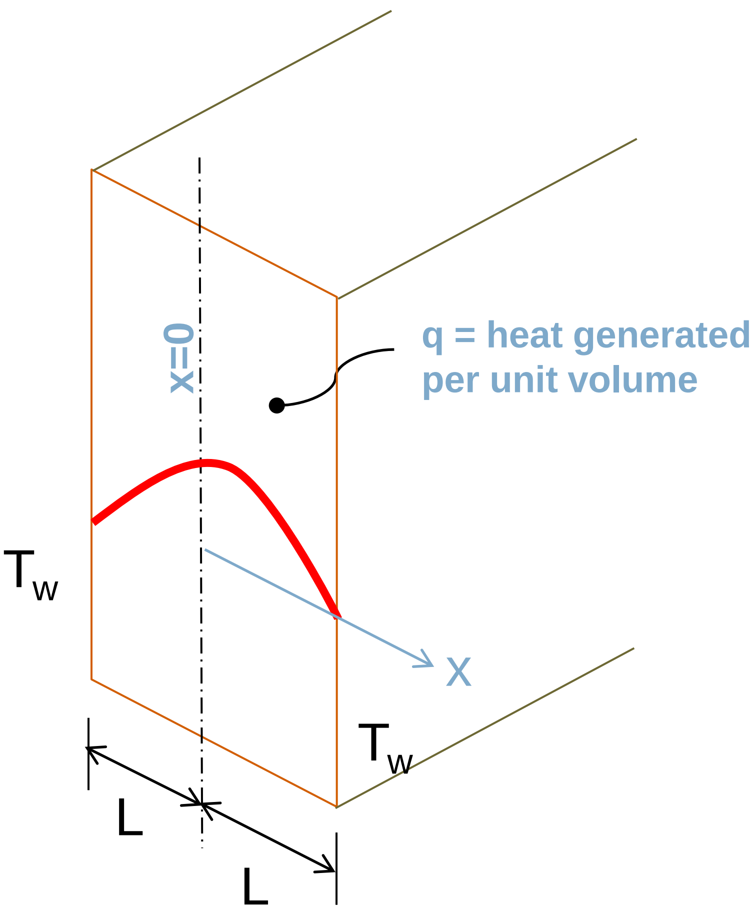
Heat Source Systems

Expression for mid plane temperature is given by,

$$T_0 = \frac{g}{2k} L^2 + T_w$$

The temperature distribution can also be written in alternative form as:

$$\frac{T - T_w}{T_0 - T_w} = 1 - \left(\frac{x}{L} \right)^2$$



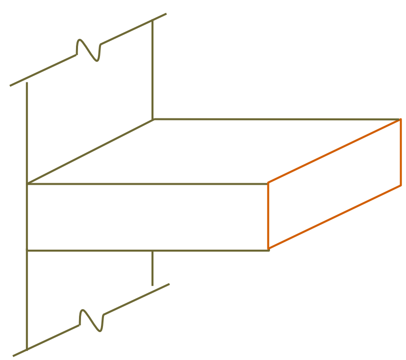
Conduction-Convection Systems

Fins / Extended Surfaces

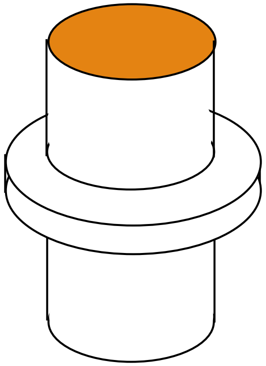
- **Necessity for fins**
- **Biot Number**

$$\frac{hx}{k} = \frac{(x/k)}{1/h} = \frac{\text{Internal Conductive resistance}}{\text{Surface Convective resistance}}$$

FIN TYPES



LONGITUDINAL RECTANGULAR FIN

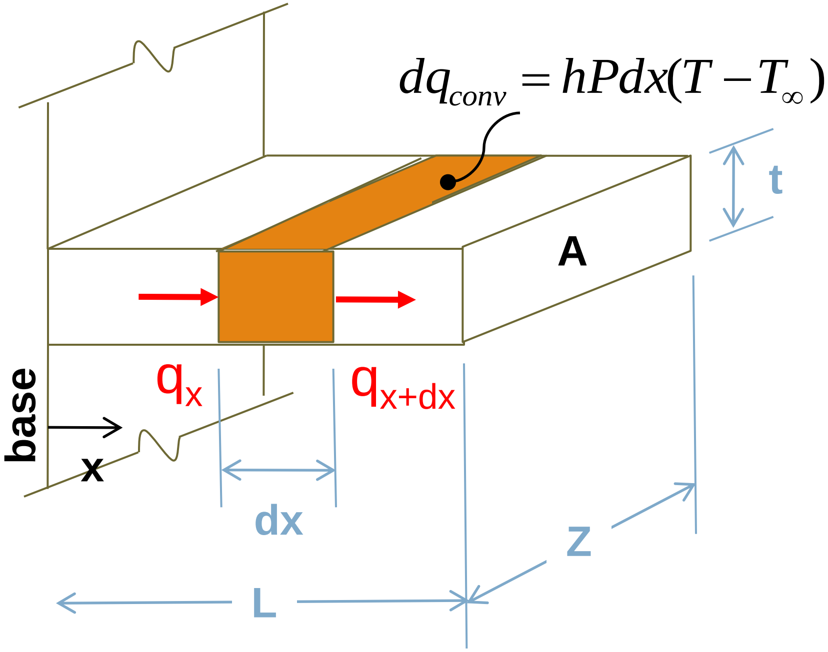


RADIAL FIN

Conduction-Convection Systems

Governing Equation (Rectangular Fin)

Net Heat Conducted – Heat Convected = 0



$$[Aq_x - Aq_{x+dx}] - hPdx(T - T_\infty) = 0$$

$$\frac{Aq_x - Aq_{x+dx}}{dx} - hP(T - T_\infty) = 0$$

$$-\frac{d}{dx}(Aq_x) - hP(T - T_\infty) = 0$$

$$-\frac{d}{dx}\left(-kA\frac{dT}{dx}\right) - hP(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where, $m = \frac{hP}{kA}$ & $\theta = T - T_\infty$

Conduction-Convection Systems

Boundary Conditions

LONG FIN

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad \text{in } x > 0$$

$$\theta(x) = T_0 - T_\infty \equiv \theta_0 \quad \text{at } x = 0$$

$$\theta(x) \to 0 \quad \text{as } x \to \infty$$

SHORT FIN
(end insulated)

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad \text{in } 0 \le x \le L$$

$$\theta(x) = T_0 - T_\infty \equiv \theta_0 \quad \text{at } x = 0$$

$$\frac{d\theta(x)}{dx} = 0 \quad \text{at } x = L$$

SHORT FIN
(end not insulated)

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \quad \text{in } 0 \le x \le L$$

$$\theta(x) = T_0 - T_\infty \equiv \theta_0 \quad \text{at } x = 0$$

$$k\frac{d\theta(x)}{dx} + h_e\theta(x) = 0 \quad \text{at } x = L$$

Conduction-Convection Systems

Types of Fin Boundaries

Type of FIN boundary	Temperature Distribution $\frac{T-T_\infty}{T_b-T_\infty}$	Heat transferred by fin Q
Long Fin ($T_L = T_\infty$)	e^{-mx}	$(T_b-T_\infty)(hPkA)^{0.5}$
Short Fin (end insulated)	$\frac{Cosh\ m(L-X)}{Cosh\ (mL)}$	$(hPkA)^{0.5}(T_b-T_\infty)\tanh(mL)$ *
Short Fin (end not insulated)	$\frac{Cosh[m(L-X)]+(h_L/mk)Sinh[m(L-X)]}{Cosh(mL)+(h_L/mk)Sinh(mL)}$	$(T_b-T_\infty)\frac{\tanh(mL)+(h_L/mk)}{1+(h_L/mk)\tanh(mL)}(hPkA)^{0.5}$
Specified End Temperature At x=L; $T=T_L$	$\frac{\frac{T_L-T_\infty}{T_b-T_\infty}Sinh(mx)+Sinh[m(L-x)]}{Sinh(mL)}$	$[(T_b-T_\infty)+(T_L-T_\infty)]\frac{Cosh(mL)-1}{Sinh(mL)}(hPkA)^{0.5}$

* For higher values of mL (i.e., m=4), tanh mL = 0.999 ≈ 1.
Thus $Q_{short\ fin} \rightarrow Q_{long\ fin}$ for higher values of mL

Conduction-Convection Systems

Performance Parameters

Fin Efficiency

$$\eta = \frac{\text{Actual heat transfer through fin}}{\text{Ideal heat transfer through fin if entire fin surface were at fin base temperature}, T_0} = \frac{Q_{fin}}{Q_{ideal}}$$

In practical applications, a finned heat transfer surface is composed of the ***finned surfaces*** and the ***unfinned portion***. In such cases total heat transfer is used.

$$Q_{total} = Q_{fin} + Q_{unfinned}$$
$$= \eta\, a_f\, h\, \theta_0 + (a - a_f)\, h\, \theta_0$$

Where, a = total heat transfer area (i.e., fin surface + unfinned surface)
a_f = heat transfer area of fins only

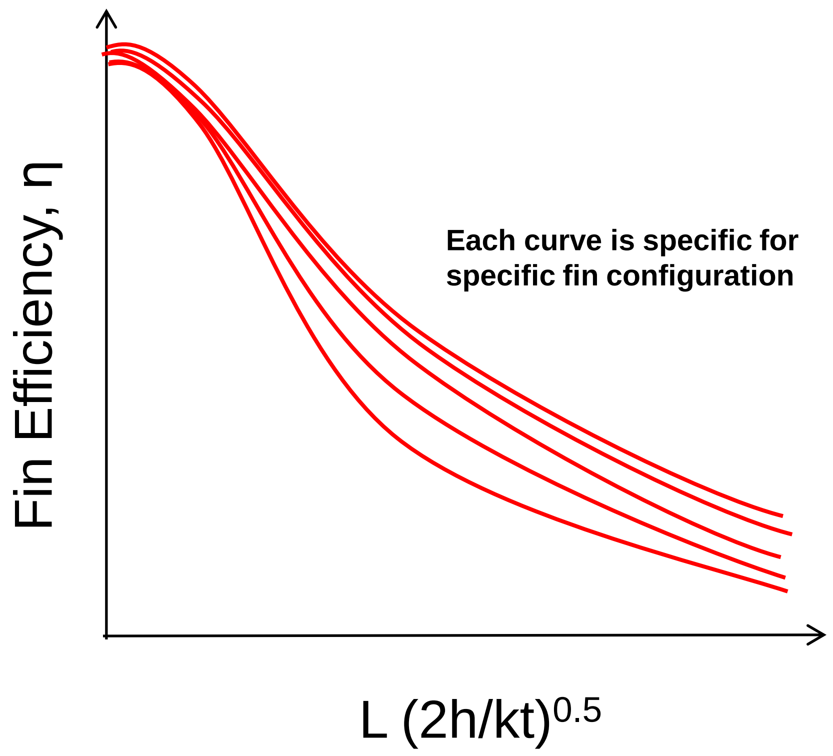
$$Q_{total} = [\eta\beta + (1-\beta)]\, a\, h\, \theta_0 \equiv \eta'\, a\, h\, \theta_0$$

Where, $\eta' = \beta\eta + 1 - \beta$ = area – weighted fin efficiency
$\beta = a_f / a$

Conduction-Convection Systems

Performance Parameters

Fin Efficiency



Fin efficiency curves are available for fins of various configuration (eg. Axial, circular disk fins of various length, thickness etc)

Conduction-Convection Systems

Performance Parameters

Fin Effectiveness

$$\text{Effectiveness} = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

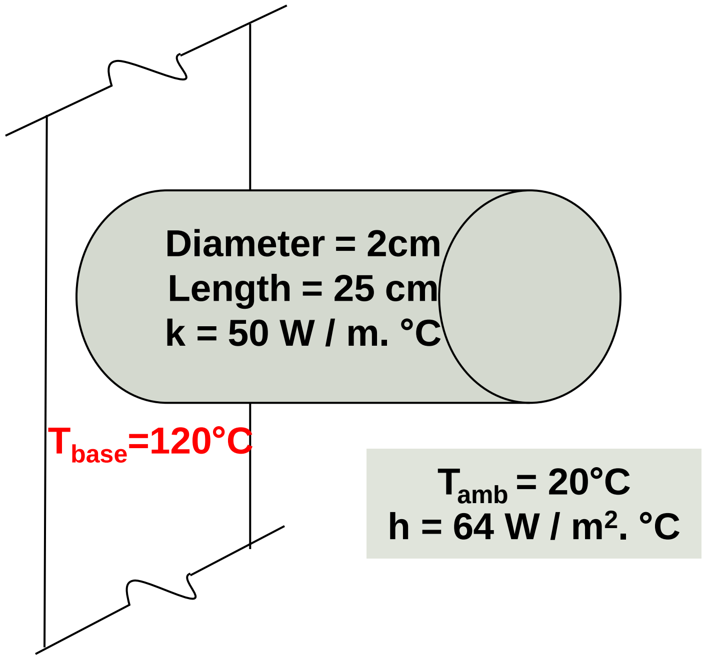
Although the addition of fins on a surface increases surface area, it also increases thermal resistance over the portion of the surface where fins are attached. Therefore there may be situations in which the addition of fins does not improve heat transfer.

$Pk / (Ah) > 1$ (to justify usage of fins)

Conduction-Convection Systems

Solved Example

A steel rod is exposed to ambient air. If one end of the rod is maintained at a temperature of 120 °C, calculate the heat loss from the rod



The condition for other end of the rod is not specified explicitly. By considering L/D ratio, it appears that a long fin assumption is applicable. Using the simplest analysis to solve, computing mL:

$$m^2 = \frac{hP}{kA} = \frac{h\pi D}{(\pi/4)D^2 k} = \frac{4h}{kD} = \frac{4\times 64}{50\times 0.02}$$

$$m = 16 \quad \text{and mL} = 16\times 0.25 = 4$$

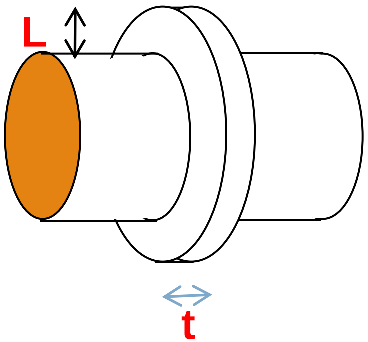
Therefore, expression for $Q_{\text{long fin}}$ can be used.

$$Q = (T_b - T_\infty)\sqrt{hPkA} = (T_b - T_\infty)\sqrt{(\pi D)\left(\frac{\pi}{4}D^2\right)kh}$$

$$= (120-20)\frac{\pi}{2}\sqrt{(0.02)^3\times 50\times 64} = 25.1\,\text{W}$$

Conduction-Convection Systems

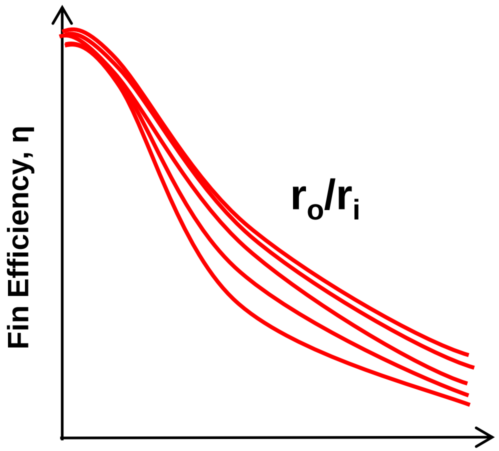
Solved Example (Fin Efficiency)



CIRCULAR DISK FIN

Circular disk fins of constant thickness are attached on a 2.5 cm OD tube with a spacing of 100 fins per 1m length of tube.
Fin Properties: Aluminium k = 160 W / m.°C, t = 1mm L = 1 cm
Tube wall temperature = 170 °C; Ambient temperature = 30 °C
Heat transfer coeff. of ambient , h = 200 W/m^2. °C.
Calculate,

1. Fin Efficiency and area weighted fin efficiency
2. Heat lost to the ambient air per 1m length of tube
3. Heat loss with that if there were no fins on tube



Fin Efficiency

Fin efficiency is determined using the graph shown aside.
The following parameters are calculated, firstly:

$$L\sqrt{\frac{2h}{kt}} = 1\times 10^{-2}\sqrt{\frac{2\times 200}{160\times 10^{-3}}} = 0.5$$

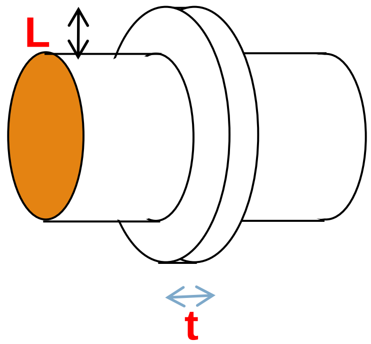
$$\frac{r_o}{r_i} = \frac{1.25+1}{1.25} = 1.8$$

The fin efficiency is determined from graph

$$\eta \cong 0.9$$

Conduction-Convection Systems

Solved Example (Fin Efficiency)



CIRCULAR DISK FIN

Tube OD = 2.5 cm
100 fins per 1m tube length
k_{fin} = 160 W/m°C
t = 1mm; L = 1cm
T_{tube} = 170°C; T_{amb} = 30°C
h_{amb} = 200 W/m^2. °C

Calculate,

1. Fin Efficiency and area weighted fin efficiency
2. Heat lost to the ambient air per 1m length of tube
3. Heat loss with that if there were no fins on tube

Area Weighted Fin Efficiency

Ratio of heat transfer area for fin to the total heat transfer area, β

Fin Surface per cm of tube length = $2\pi(r_0^2 - r_i^2) = 2\pi[2.25^2 - 1.25^2]$
$= 21.99\ cm^2$

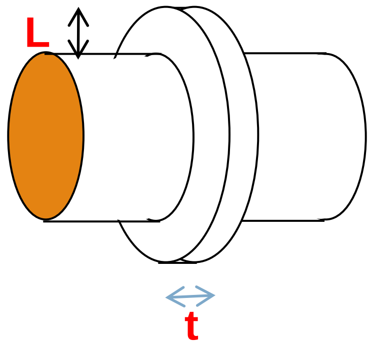
Total heat transfer surface per cm of tube length
$= 2\pi\ (r_0^2 - r_i^2) + 2\pi r_i\ (1 - t)$
$= 2\pi[2.25^2 - 1.25^2] + 2\pi(1.25)(1 - 0.1)$
$= 29.06\ cm^2$

$\beta = a_f / a = 21.99 / 29.06 = 0.757$

Area Weighted Fin Efficiency, $\eta' = \beta\eta + 1 - \beta = 0.757(0.9) + 0.243$
$= 0.924$

Conduction-Convection Systems

Solved Example (Fin Efficiency)



CIRCULAR DISK FIN

Tube OD = 2.5 cm
100 fins per 1m tube length
k_{fin} = 160 W/m°C
t = 1mm; L = 1cm
T_{tube} = 170°C; T_{amb} = 30°C
h_{amb} = 200 W/m². °C

Calculate,

1. Fin Efficiency and area weighted fin efficiency
2. Heat lost to the ambient air per 1m length of tube
3. Heat loss with that if there were no fins on tube

Heat lost to ambient per 1m length of tube

Total heat transfer surface ***a*** per 1m of tube length

$$a = 29.06 \times 100 \text{ cm}^2 = 0.29 \text{ m}^2$$

$Q = \eta' a h \theta_0 = 0.924 \times 0.29 \times 200 \, (170 - 30) =$ **7503 W**

Heat lost per 1m length of tube with no fins

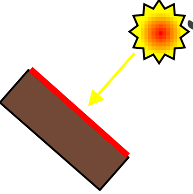
$Q_{no\ fin} = 2\pi r_i h \theta_0 = 2\pi \times 0.0125 \times 200 \times (170 - 30) =$ **2199 W**

Clearly, the addition of fins increases the heat dissipation by a factor of about 3.4

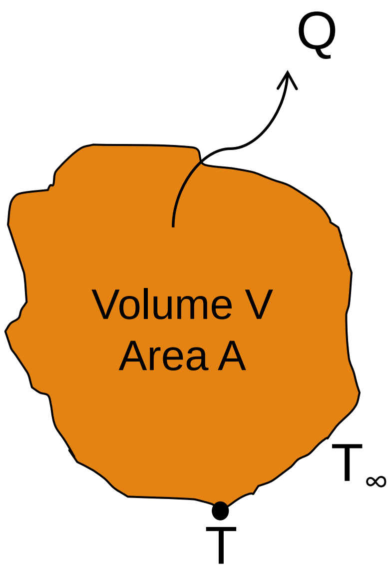
Transient Conduction

- If the surface temperature of a solid body is suddenly altered, the temperature within the body begins to change over time.
- Variation of temperature both with position and time makes determination of temperature distribution under transient condition more complicated.
- In some situations, variation of temperature with position is negligible under transient state, hence the temperature is considered to vary only with time.
- The analysis under the above assumption is called **lumped system analysis**.
- Biot Number, $Bi = (hx) / k$
- Lumped System Analysis is applicable only when $Bi < 0.1$

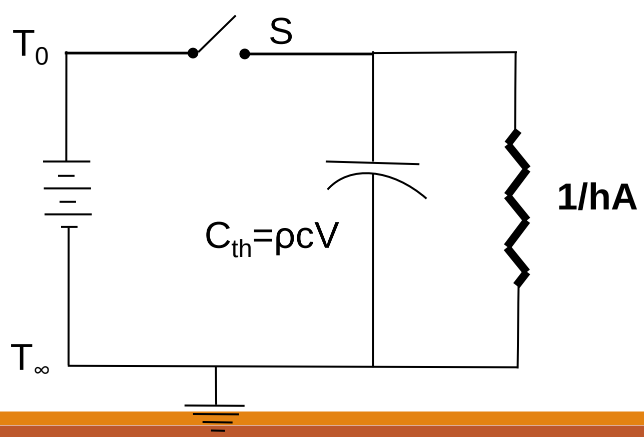
Systems with Negligible Internal Resistance



Lumped Heat Analysis



$T = T_0$ at $t = 0$



- The convective heat loss from the body (shown aside) has its magnitude equal to decrease in internal energy of solid.

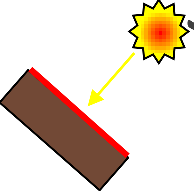
$$Q = -hA(T - T_\infty) = pcV\frac{dT}{dt}$$

$$\frac{dT}{T - T_\infty} = \frac{-hA}{pcV}dt$$

On Integration, $$\ln(T - T_\infty) = \frac{-hA}{pcV}t + C_1$$

Solving and rearranging,

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-(hA/pcV).t\right]$$



Systems with Negligible Internal Resistance

Biot Number

- It is a non-dimensional parameter used to test the *validity* of the lumped heat capacity approach.

$$Bi = \frac{\text{internal resistance}}{\text{convective resistance}} = \frac{hL_c}{k}$$

- The characteristic length (L_c) for some common shapes is given below:

Plane Wall (thickness 2L)

$$L_c = \frac{A.2L}{2.A} = L$$

Long cylinder (radius R)

$$L_c = \frac{\pi R^2.L}{2\pi R.L} = \frac{R}{2}$$

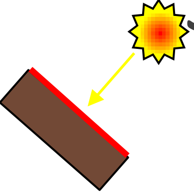
Sphere (radius R)

$$L_c = \frac{(4/3)\pi R^3}{4\pi R^2} = \frac{R}{3}$$

Cube (side L)

$$L_c = \frac{L^3}{6L^2} = \frac{L}{6}$$

- The lumped heat capacity approach for simple shapes such as plates, cylinders, spheres and cubes can be used if **$Bi < 0.1$**



Systems with Negligible Internal Resistance

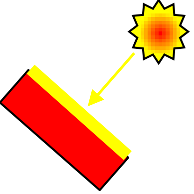
Response time of a Temperature measuring Instrument

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-(hA/\rho cV).t\right]$$

- For a rapid response of temperature measuring device, the index, (hAt/ρcV) should be large to make the exponential term reach zero faster.
- This can be achieved by decreasing wire diameter, density and specific heat or by increasing value of ‘h’.
- The quantity (ρcV/hA) has the units of time and is called ‘time constant’ of system. Hence at time $t=t^*$ (one time constant),

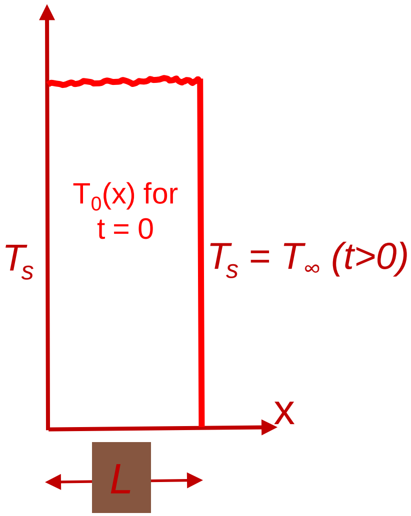
$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-1} = 0.368$$

- At the end of time period t^* the temperature difference between the body and ambient would be 0.368 of the initial temperature difference.
- In other words, the temperature difference would be reduced by 63.2 percent.
- This reduction in 63.2 percent of initial temperature difference is called ‘sensitivity’
- Lower the value of time constant, better the response of instrument.



Systems with Negligible Surface Resistance

Large Flat Plate with Negligible Surface Resistance



- When convective heat transfer coefficient at the surface is assumed to be infinite, the surface temperature remains constant at all the time (t>0) and its value is equal to that of ambient temperature.
- The systems exhibiting above said conditions are considered to have 'negligible surface resistance'
- An important application of this process is in heat treatment of metals by quenching, viz., the dropping of a metallic sphere initially at 300 °C into a 20 °C oil bath.
- Mathematical formulation of this case is :

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad 0 \le x \le L$$

$$\mathrm{T} = \mathrm{T}_0(\mathrm{x}) \text{ at } \mathrm{t} = 0 \text{ for } 0 \le \mathrm{x} \le \mathrm{L} \text{ (initial condition)}$$

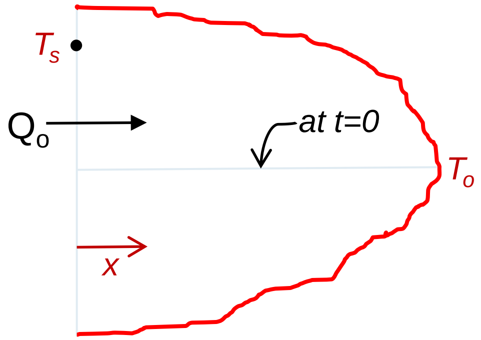
$$\left.\begin{aligned} \mathrm{T} &= \mathrm{T_s} \quad \text{at } \mathrm{x} = 0 \text{ for } \mathrm{t} > 0 \\ \mathrm{T} &= \mathrm{T_s} \quad \text{at } \mathrm{x} = \mathrm{L} \text{ for } \mathrm{t} > 0 \end{aligned}\right\}$$

Boundary Conditions

Heat flow in an Infinitely Thick Plate

Semi-infinite body

Semi-Infinite Plate



- A semi-infinite body is one in which at any instant of time there is always a point where the effect of heating / cooling at one of its boundaries is not felt at all.
- At this point the temperature remains unchanged.
- Mathematical formulation is :

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

with initial and boundary conditions,

$$T = T_0 \text{ at } t = 0 \text{ for all } x$$

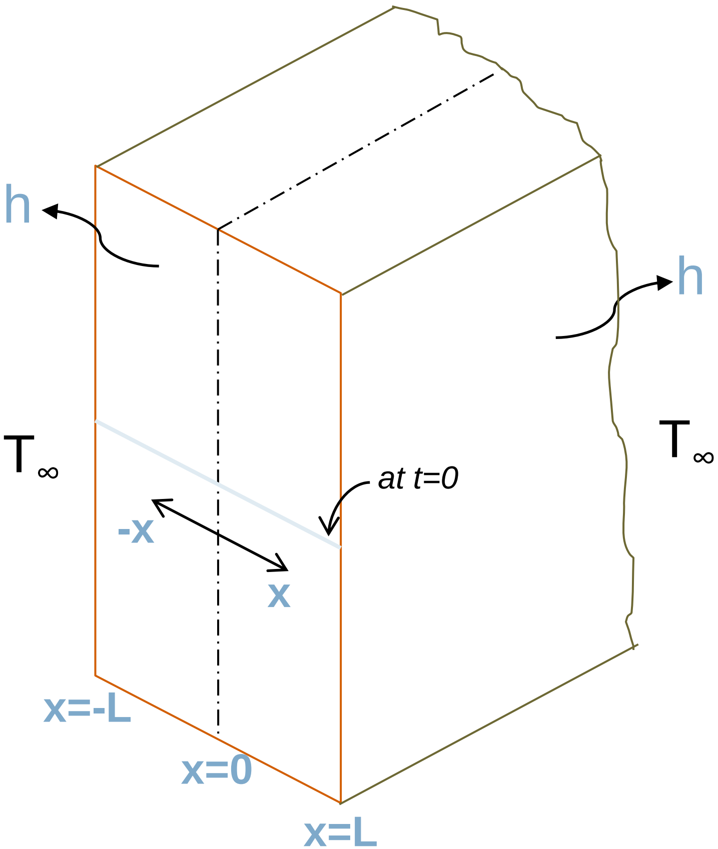
$$T = T_s \text{ at } x = 0 \text{ for all } t > 0$$

$$T \rightarrow T_0 \text{ as } x \rightarrow \infty \text{ for all } t > 0$$

Systems with Finite Surface and Internal Resistance

Mathematical formulation :

Infinitely Large Flat Plate of Finite Thickness (2L)



$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$T = T_0 \text{ at } t = 0 \text{ (for } -L \le x \le L)$$

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0 \text{ (centre line)}$$

$$\frac{-\partial T}{\partial x} = \frac{h}{k}(T - T_\infty) \text{ at } x = \pm L$$

Chart Solutions of Transient Heat Conduction Problems

Heisler Charts (by Heisler, 1947)

Infinite Plate

Time History
Mid Plane

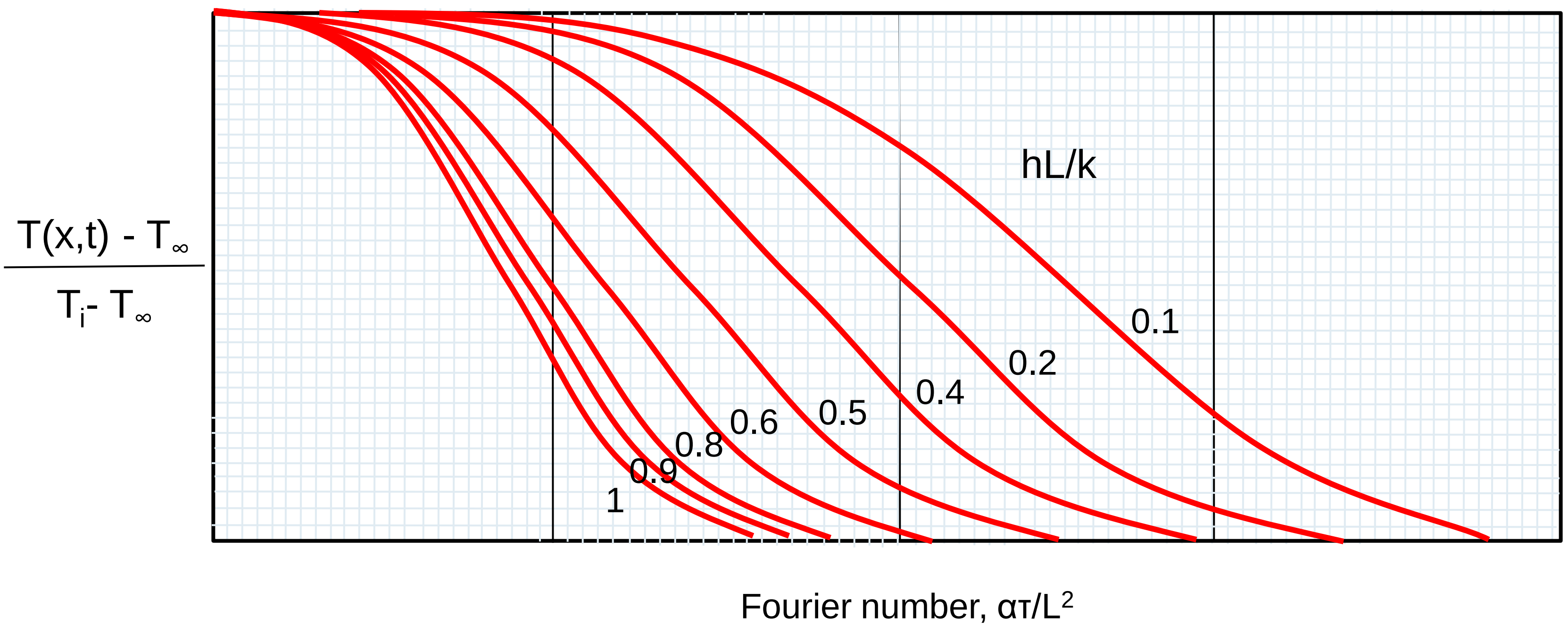


Chart Solutions of Transient Heat Conduction Problems

Heisler Charts (by Heisler, 1947)

Infinite Plate

Time History
Any Position, x

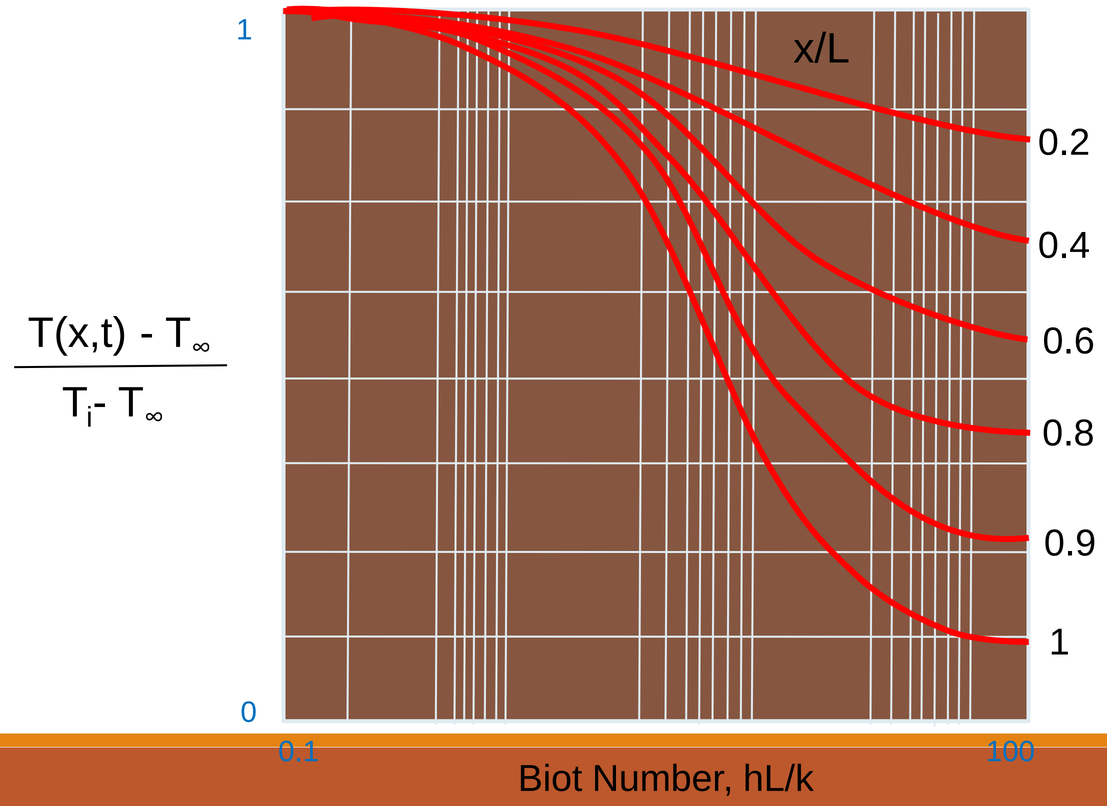
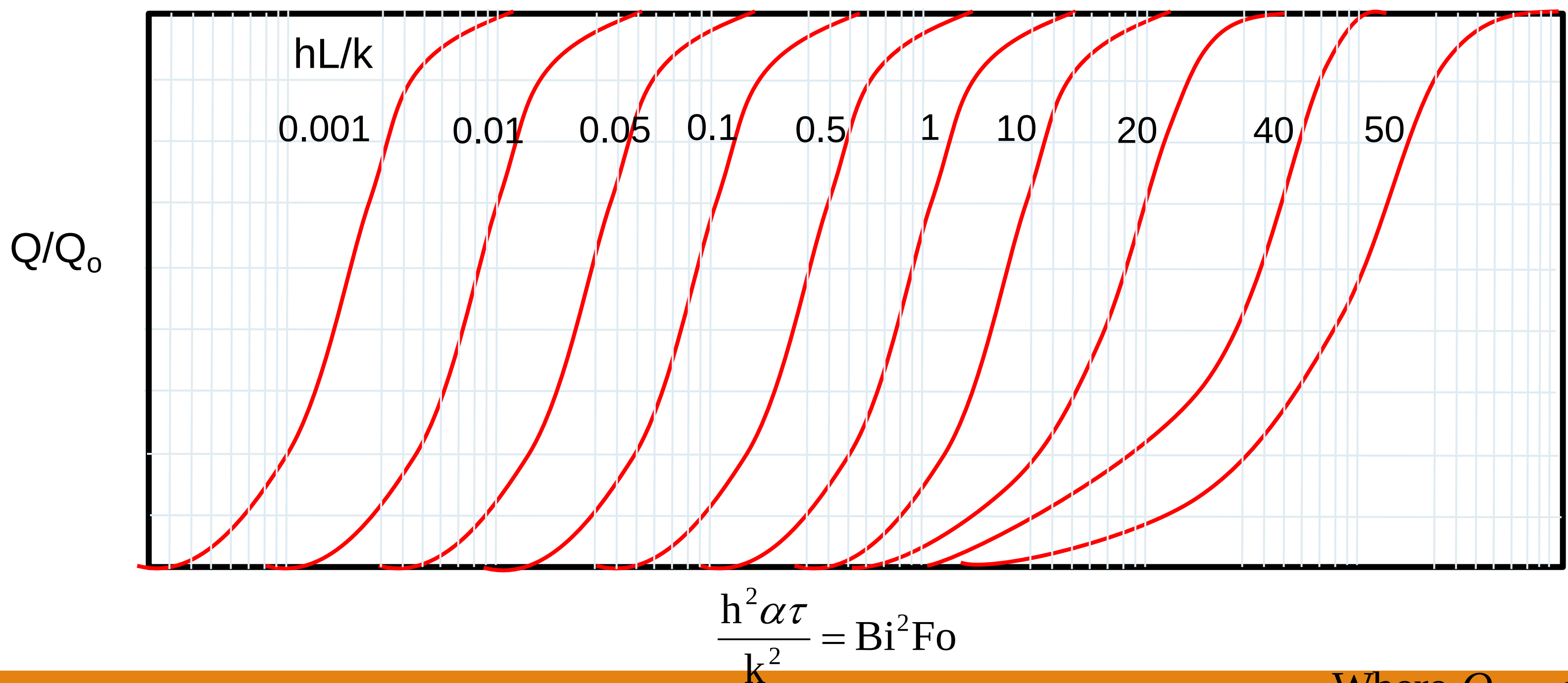


Chart Solutions of Transient Heat Conduction Problems

Heisler Charts (by Heisler, 1947)

Infinite Plate

Heat Flow



$$\text{Where, } Q_o = \rho c V (T_o - T_\infty)$$

Lumped System Analysis

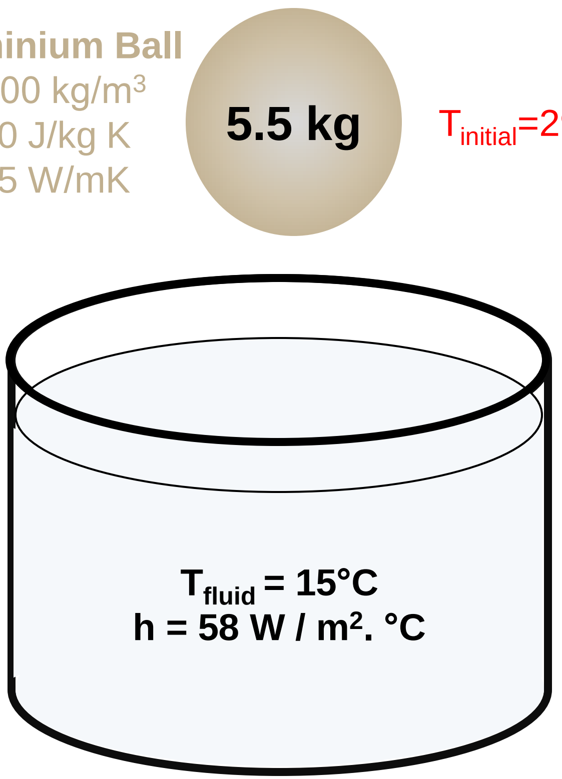
Solved Example

Aluminium Ball

ρ = 2700 kg/m³

c = 900 J/kg K

k = 205 W/mK



$T_{initial}$=290°C

Time required to cool the aluminium ball to **95°C** **?**

Determination of Time required to cool

Volume

$$V = \frac{4}{3}\pi R^3 = \frac{mass}{density} = \frac{5.5}{2700} = 2.037\times10^{-3}$$

Radius

$$R = (3V/4\pi)^{1/3} = 0.0786\text{m}$$

Characteristic Length

$$L_c = \frac{R}{3} = 0.0262\text{m}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{hA}{\rho cV}\right).t\right]$$

$$T = 95°C$$

$$T_\infty = 15°C$$

$$T_0 = 290°C$$

$$\frac{hA}{\rho cV} = \frac{3h}{\rho cR} = \frac{3\times58}{2700\times900\times0.0786} = 9.1\times10^{-4}\ /\ s$$

$$\frac{95-15}{290-15} = \frac{80}{275} = \exp(-9.1\times10^{-4}.t)$$

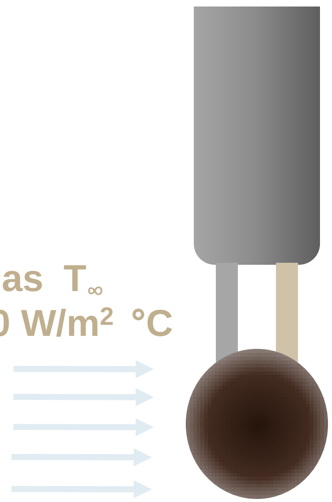
$$3.4375 = \exp(-9.1\times10^{-4}.t)$$

$$t = 1357\ s$$

Lumped System Analysis

Temperature Measurement by Thermocouples

Solved Example



How long will it take for the thermocouple to read **99 % of Initial Temperature difference** **?**

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1mm diameter sphere (shown aside)
Determine how long it will take for the thermocouple to read 99% of initial temperature difference

L_c = V/A_s = (1/6)D = (1/6)x0.001 = $1.67\text{x}10^{-4}$ m
Bi = hL/k = ($210\text{x } 1.67\text{x}10^{-4}$) x 35 = 0.001 **< 0.1**
Therefore, lumped system analysis is applicable.

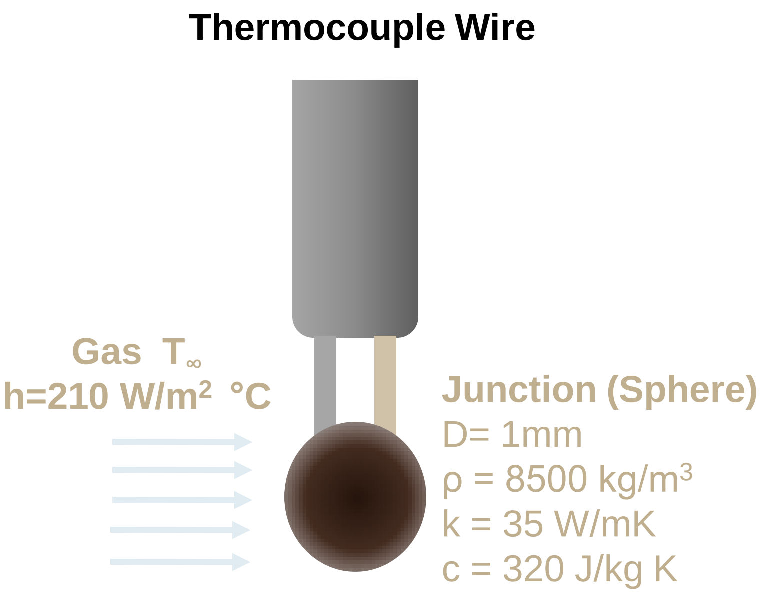
In order to read 99% of initial temperature difference $T_i - T_\infty$ between the junction and the gas, we must have

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.01$$

Lumped System Analysis

Temperature Measurement by Thermocouples

Solved Example



How long will it take for the thermocouple to read **99 % of Initial Temperature difference** ?

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1mm diameter sphere (shown aside)
Determine how long it will take for the thermocouple to read 99% of initial temperature difference

Time

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.01 = \exp[-(hA/pcV).t]$$

$$\frac{hA_s}{\rho cV} = \frac{h}{\rho cL_c} = \frac{210}{8500 \times 320 \times 1.67 \times 10^{-4}} = 0.462\ s^{-1}$$

$$\exp[-(0.462).t] = 0.01$$

t = 10s

Transient Conduction in Semi-infinite Solids

Solved Example

A water pipe is to be **buried** in soil at sufficient depth from the surface to prevent freezing in winter.

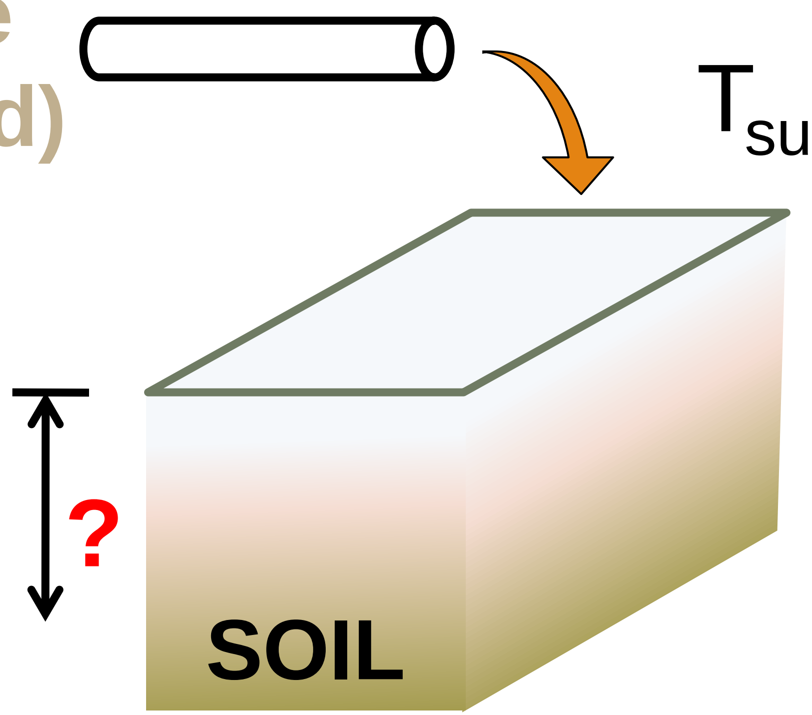
What **minimum depth** is required to prevent the **freezing** of pipe when soil is at uniform temperature of $T_i = 10$ °C, the surface is subjected to a uniform temperature of $T_0 = -15$ °C continuously for 50 days.

Also the pipe surface temperature should not fall below 0 °C.

Transient Conduction in Semi-infinite Solids

Solved Example

Water Pipe (to be buried)



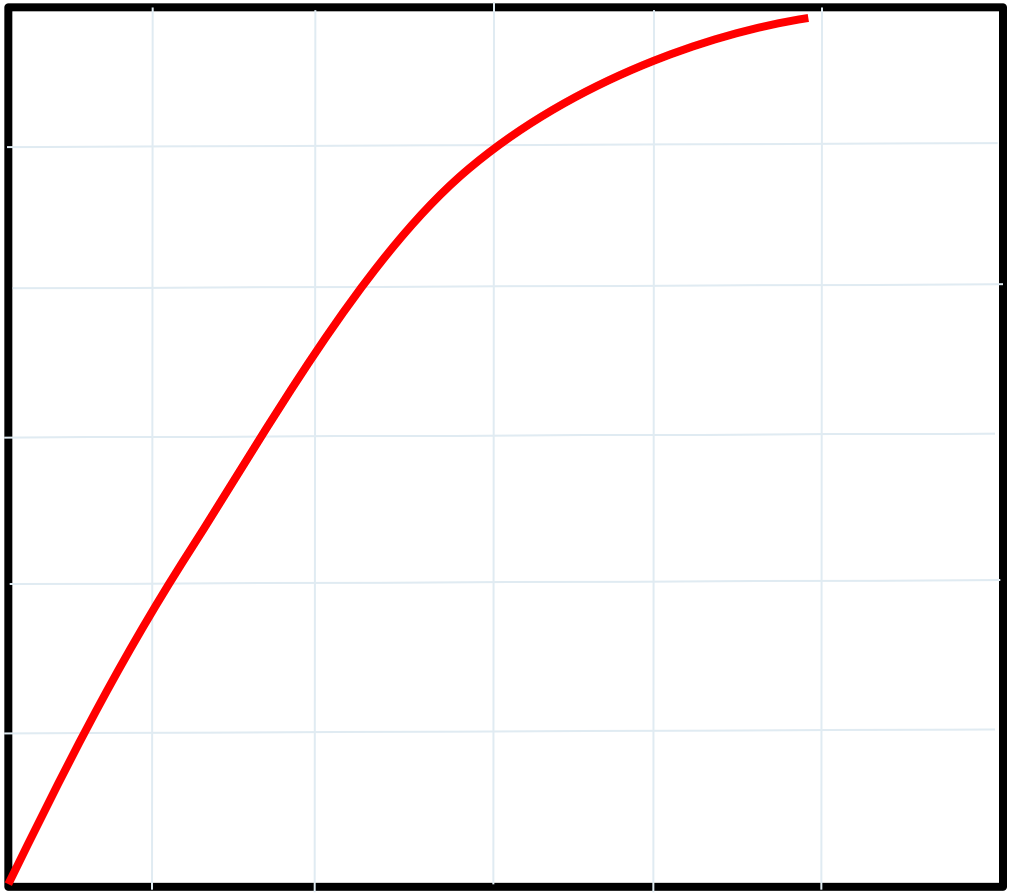
$T_{surface}$ = -15 °C

T_{soil} = 10 °C

Condition : $T_{pipe\ wall}$ should not fall below 0 °C

What **burial depth** is needed to prevent **freezing** of the pipe **?**

$$\frac{T(x,t) - T_{surface}}{T_{initial} - T_{surface}}$$



$$\text{Error Function}, \xi = \frac{x}{2\sqrt{\alpha t}}$$

$$\theta(x,t) = \frac{T(x,t) - T_0}{T_i - T_0} = \frac{0+15}{10+15} = 0.6$$

For $\theta(x,t) = 0.6, \xi = 0.6$ (from graph)

For $\alpha = 0.2 \times 10^{-6} m^2 / s$ and $\mathrm{t} = 50 \times 24 \times 3600\mathrm{s}$

the error function ξ is given by,

$$\xi = \frac{\mathrm{x}}{2\sqrt{\alpha \mathrm{t}}} = \frac{\mathrm{x}}{2\sqrt{0.2 \times 10^{-6} \times 50 \times 24 \times 3600}} = 0.538x$$

$$0.538x = 0.6$$

$$x = \frac{0.6}{0.538} = 1.12m$$

The pipe should be buried at least to a depth of **1.12 m** to prevent freezing.

Application of Heisler Charts

Aluminium Slab

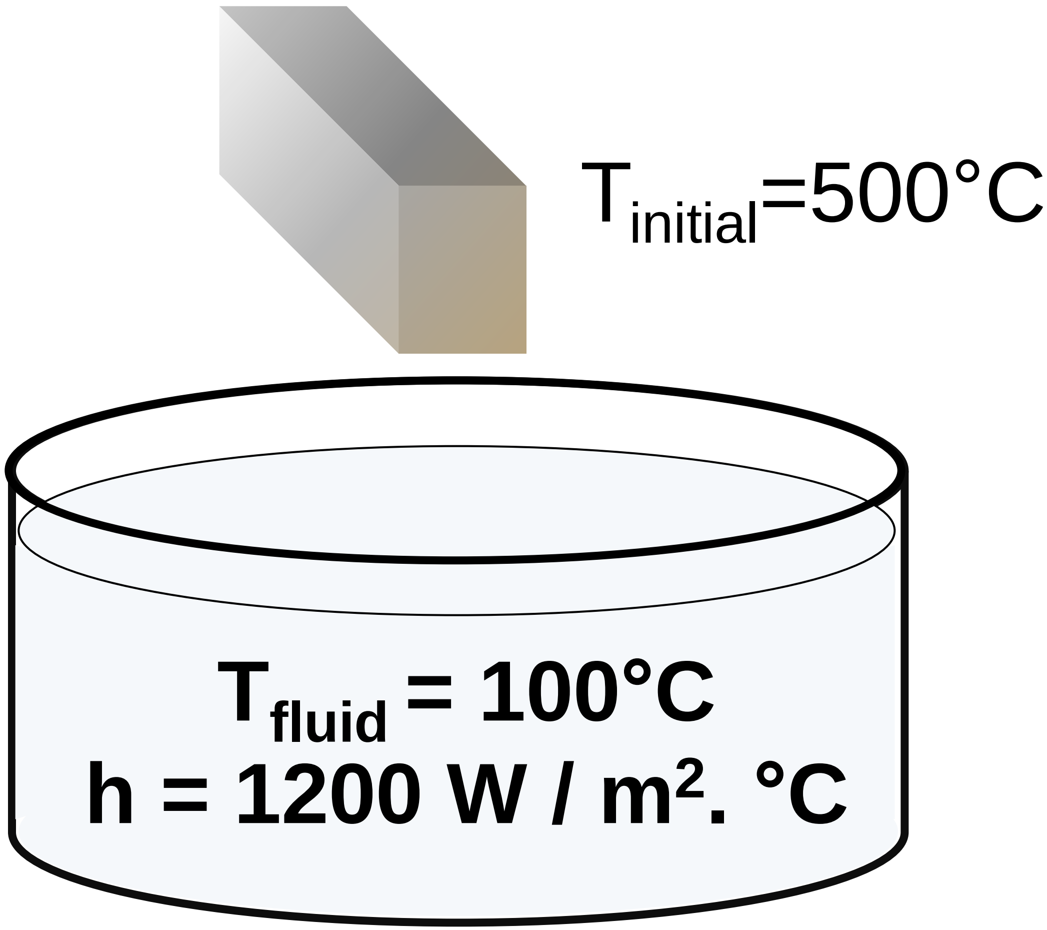
Thickness=10cm

$\alpha = 8.4 \times 10^{-5}\ m^2/s$

$\rho = 2700\ kg/m^3$

$c = 900\ J/kg\ K$

$k = 215\ W/mK$



Mid-plane Temperature and **Surface Temperature** after **1 min?**

Determination of Mid plane Temperature

2L=10 cm ; L = 5 cm ; t = 1min = 60 s

$\alpha t/L^2$ = $(8.4\text{x}10^{-5} \text{ x } 60) / 0.05^2$ = 2.016

Bi = hL/k = (1200x0.05) x 215 = 0.28

Using above two parameters in Heisler Chart,

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.68$$

$$\therefore T = 100 + 0.68(500 - 100) = 372°C$$

Determination of Surface Temperature

For x/L = 1 and Bi = 0.28,

$$\frac{T - T_\infty}{T_0 - T_\infty} = 0.88$$

$$\therefore T = 100 + 0.88(372 - 100) = 339.36°C$$

Energy Loss

$h^2\alpha t/k^2 \quad = (1200^2 x 8.4x10^{-5} \text{ x } 60) / 215^2 = 0.157$

$Bi = hL/k \quad = (1200x0.05) \text{ x } 215 \quad = 0.28$

Using above 2 parameters in Heisler Chart for

Heat flow, $Q/Q_0 = 0.32$

$$\frac{Q_0}{A} = \frac{\rho c V\left(T_0 - T_\infty\right)}{A} = \rho c(2L)(T_0 - T_\infty)$$

$$= \left(2700\right)\left(900\right)\left(0.1\right)\left(400\right)$$

$$= 97.2 \times 10^6 \, J / m^2$$

$$\frac{Q}{A} = 0.32 \times 97.2 \times 10^6$$

$$= 31.1 \times 10^6 \, J/m^2$$

UNIT III – CONVECTIVE HEAT TRANSFER

CONVECTION HEAT TRANSFER

Modes

- ***forced***
  flow induced by external agency e.g. pump
  eg. forced-draught air cooler, evaporators

- **natural**
  fluid motion caused by temperature-induced density gradients within fluid

Examples

air flow over hot steam pipe, fireplace circulation, cooling electronic devices

CONVECTION HEAT TRANSFER

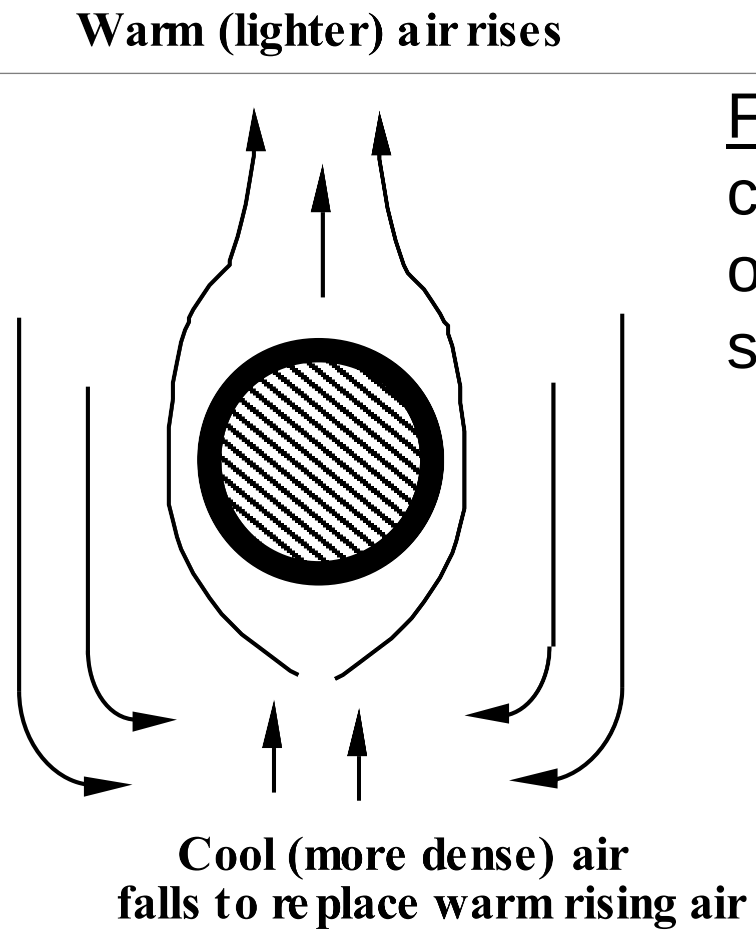
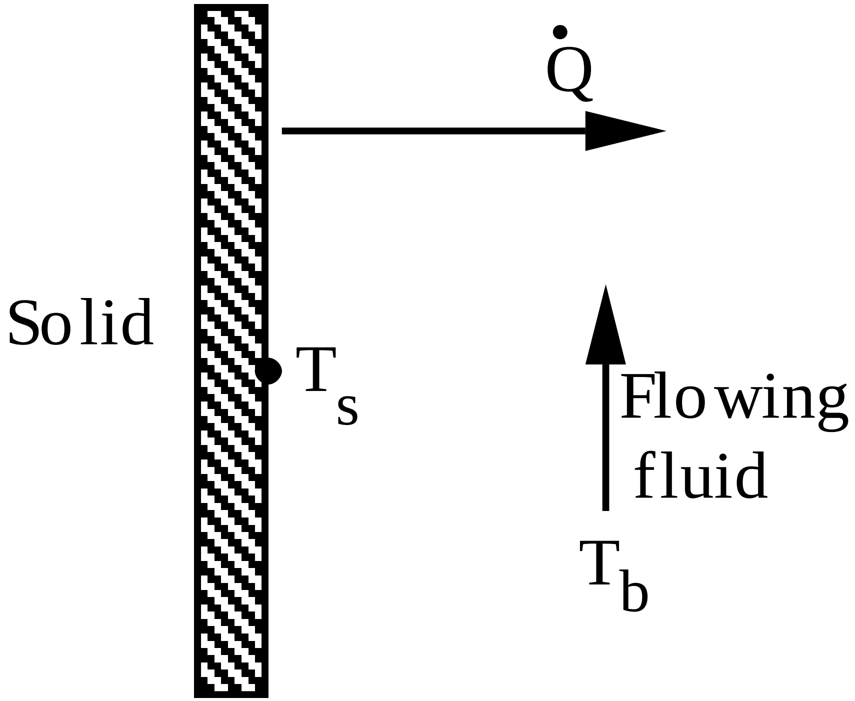


Figure: Natural convection flow over a heated steam pipe

Modelling Convection

Forced convection generally most-effective transport of energy from solid to fluid.



Engineer's prime concern

$\Downarrow$

rate of convection

$\Downarrow$

enables sizing of equipment

Modelling Convection

Experimentally found that:

$$\dot{Q} \; \alpha \; A\left(T_s - T_b\right)$$

$$\dot{Q} = hA\left(T_s - T_b\right)$$

h - convective heat transfer coefficient.

Main problem

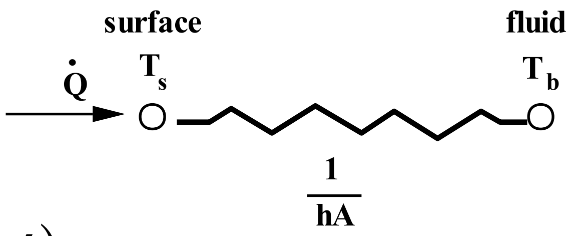
predict *h* value for:

- variety fluids & flow rates
- range of shapes

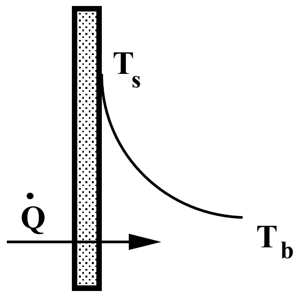
Resistance Concept

Rate equation

Written in same form as Ohm's Law:



$$\text{Current flow } (I) = \frac{\text{Potential Difference}(\Delta V)}{\text{Resistance}(R)}$$



$$\dot{Q} = hA\left(T_s - T_b\right) = \frac{\left(T_s - T_b\right)}{1/hA} = \frac{T_s - T_b}{R}$$

$(T_s\text{-}T_b)$= driving force

$(1/hA)$ – thermal resistance (R) for convection heat transfer.

TYPICAL UNITS FOR *h*

S.I.:	$W\ m^{-2}K^{-1}$ or $J\ s^{-1}m^{-2}K^{-1}$
British:	$Btu\ hr^{-1}ft^{-2}(F\ deg)^{-1}$
Conversion:	$1\ W\ m^{-2}K^{-1} = 0.176\ Btu\ hr^{-1}ft^{-2}(F\ deg)^{-1}$

Typical Values

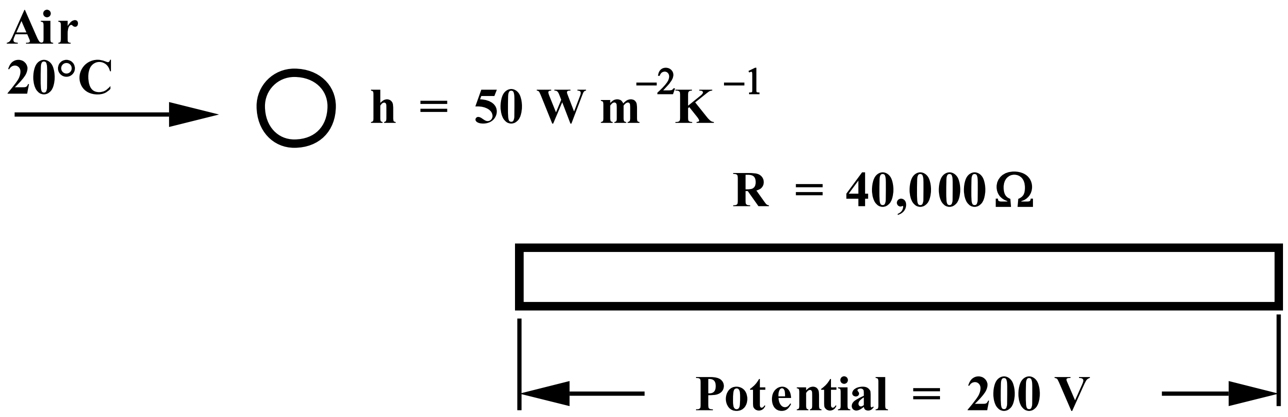
free convection (air)	5 - 60
forced convection (air)	25 - 300
forced convection (water)	200 - 10,000
boiling water	2,000 - 25,000
condensing steam	4,000 - 110,000

Illustration 27.1

Air at 20°C is blown over an electrical resistor to keep it cool. The resistor is rated at 40,000 ohm and has a potential difference of 200 volts applied across it.

The expected mean heat transfer coefficient between the resistor surface and the air is 50 W $m^{-2}K^{-1}$.

What will be the surface temperature of the resistor, which has a surface area of 2 cm^2?



SOLUTION

Energy Balance

Generation = heat loss by convection

Rate of heat generation

$$P = \dot{Q}$$

$$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

$$\dot{Q} = \frac{V^2}{R} = \frac{200^2}{4x10^4} = 1 \text{ watt}$$

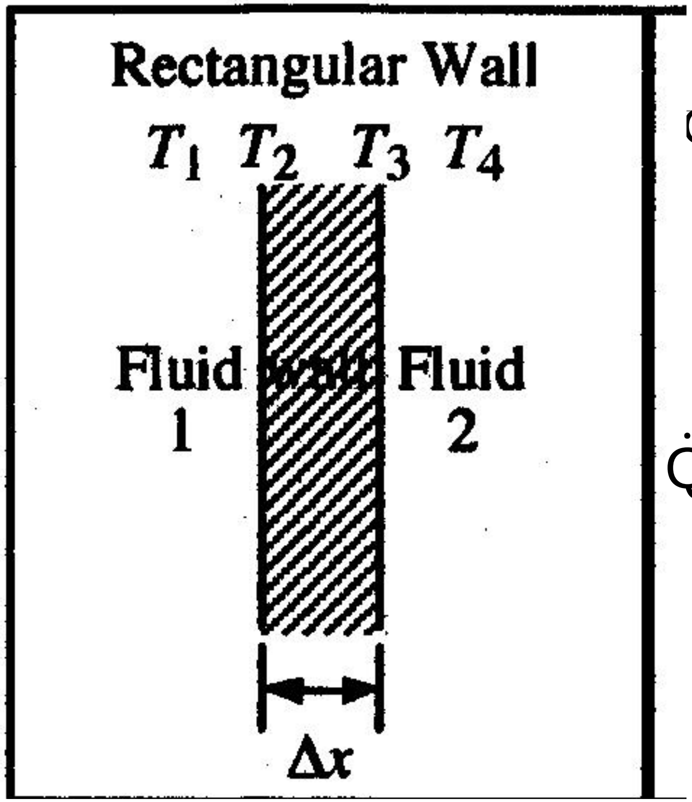
Convective loss

$$\dot{Q} = hA(T_s - T_b)$$

$$1 = (50)(2x10^{-4})(T_s - 20)$$

$$T_s = 120^o C$$

Determining the size (H/T area) of the exchanger



$$\dot{Q} = h_1 A (T_1 - T_2) = kA \frac{(T_2 - T_3)}{\Delta x} = h_2 A (T_3 - T_4)$$

(10.25)

$$\dot{Q} = \frac{(T_1 - T_4)}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}} = \frac{\text{overall driving force}}{\sum \text{resistances}}$$

(10.26)

Figure 1. Heat transfer between two flowing fluids separated by a rectangular

Determining the size (H/T area) of the exchanger

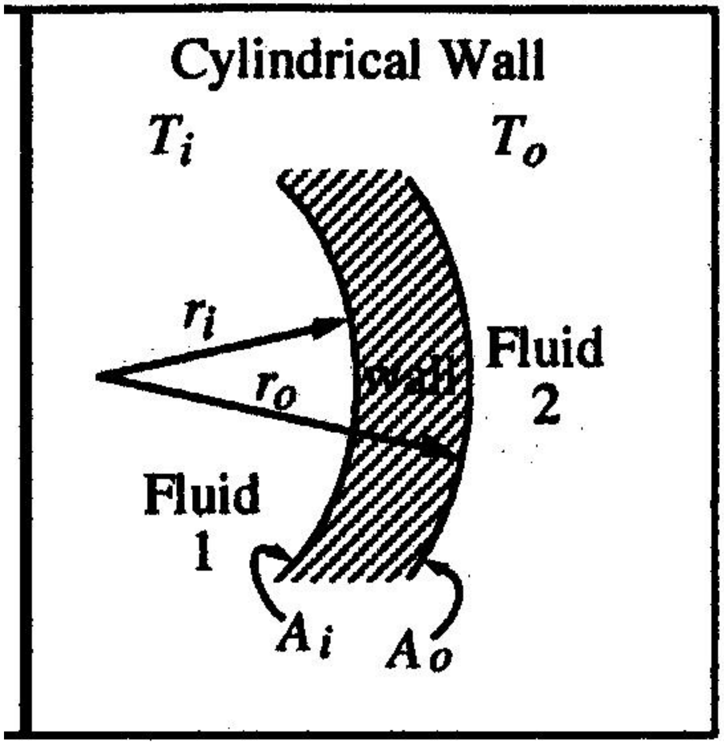


Figure 2: Heat transfer between two flowing fluids separated by a cylindrical wall

$$\dot{Q} = \frac{(T_i - T_o)}{\dfrac{1}{h_i A_i} + \dfrac{\ln\left(r_o / r_i\right)}{2\pi k L} + \dfrac{1}{h_o A_o}}$$

$$\dot{Q} = \frac{(T_i - T_o)}{R_1 + R_2 + R_3} \qquad (10.27)$$

Overall heat-transfer coefficient

As a short-hand method of describing heat-exchanger performance, we use the overall heat-transfer coefficient,

$$\dot{Q} = hA\left(T_s - T_b\right)$$

$$\dot{Q}_{duty} = U_o A \Delta T$$

$$\dot{Q}_{duty} = \frac{\Delta T}{\frac{1}{U_o A}} = \frac{\Delta T}{R} \qquad (10.28)$$

where

U_o = overall heat-transfer coefficient $\left(W/m^2K\right)$

Determining the size (H/T area) of the exchanger

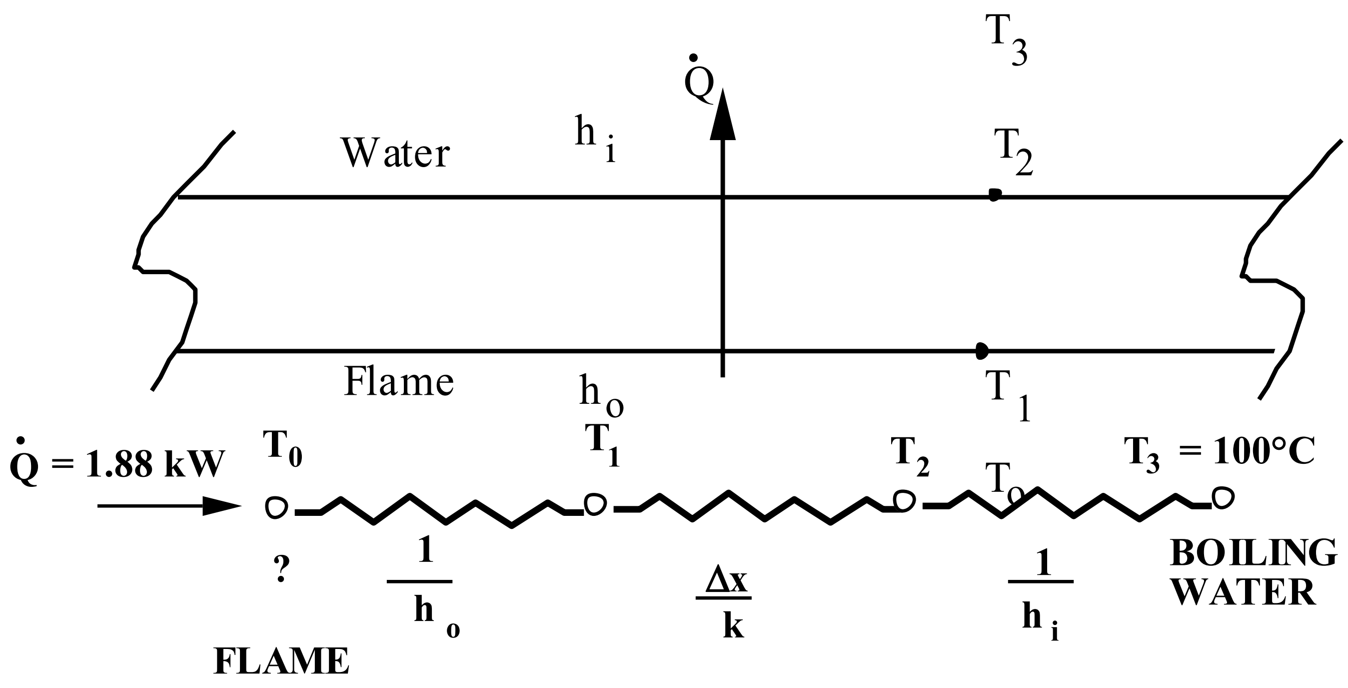
Table 10.5 Approximate Values of U_o (from Reference 1)

Hot Stream: Cold Stream	U_o, $Btu/hr\ ft^2\ °F$
Saturated vapor: Boiling liquid	250
Saturated vapor: Flowing liquid	150
Saturated vapor: Vapor	20
Liquid: Liquid	50
Liquid: Gas OR Gas: Liquid	20
Gas: Gas	10
Vapor (Partial condenser): Liquid	30

Illustration 27.2

Consider the kettle below. For the conditions given, find the flame temperature for the following values of the heat transfer coefficients:

h_i (boiling) = 4000 W m^{-2}K^{-1} h_o (gas flame) = 40 W m^{-2}K^{-1}



Solution

Plane slab- area constant, eliminate A:

$$\frac{1}{U_o A} = \frac{1}{h_o A} + \frac{\Delta x}{kA} + \frac{1}{h_1 A}$$

$$\frac{1}{U_o} = \frac{1}{40} + \frac{1.2x10^{-3}}{204} + \frac{1}{4000}$$

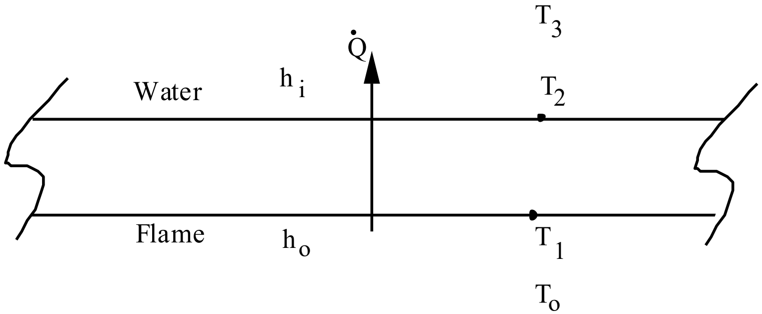
$$U_o = 39.6 \text{ W m}^{-2}\text{K}^{-1}$$

Solution

$$\dot{Q} = 1883 \text{ W (as before)}$$

$$A = 3.14\text{x}10^{-2} \text{ m}^2 \text{ (as before)}$$

$$\dot{Q}_{duty} = U_o A \left(T_o - T_3\right)$$



$$\left(T_o - T_3\right) = \frac{\dot{Q}_{duty}}{AU_o}$$

$$\left(T_o - T_3\right) = \frac{(1883)}{\left(3.14\text{x}10^{-2}\right)(39.6)} = 1514\text{K} = 1514^{o}\text{C}$$

$$T_o = 1514 + 100 = 1614^{o}\text{C}$$

Determining the size (H/T area) of the exchanger

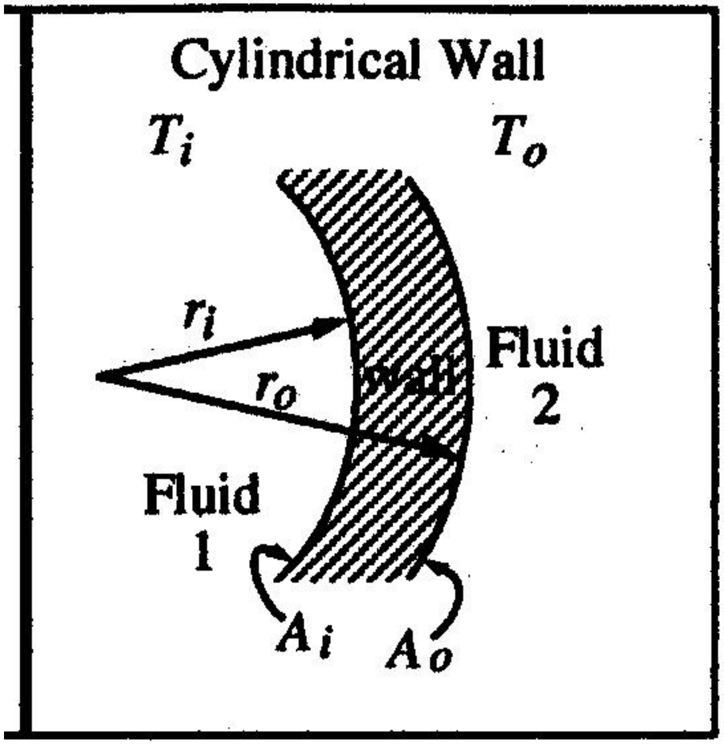
$$\dot{Q}_{duty} = U_o A \Delta T_{avg} \qquad (10.28)$$

- The term ΔT_{avg} in equation 10.28 represents the temperature difference between the hot and cold streams averaged.
- For single-pass exchangers, the appropriate form of ΔT_{avg} is the log-mean temperature difference, $\Delta T_{log\ mean}$ (often abbreviated LMTD), defined as

$$\text{Log mean temperature difference} = \Delta T_{log\ mean}$$

$$\Delta T_{log\ mean} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} \qquad (10.29)$$

Determining the size (H/T area) of the exchanger

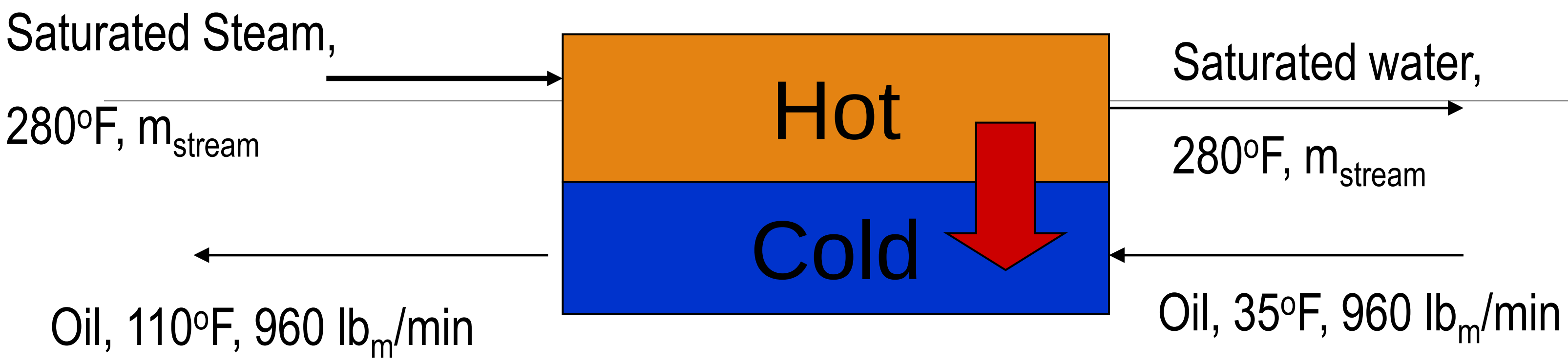


$$\dot{Q}_{duty} = U_o A \Delta T_{avg}$$

$$A = \frac{\dot{Q}_{duty}}{U_o \Delta T_{log\ mean}} \qquad (10.\ 30)$$

For shell-and-tube exchangers, the inside area (A_i) of the tubes is smaller than the outside area (A_o). However, the differences between A_i and A_o will be neglected.

Example



Balance on cold stream:

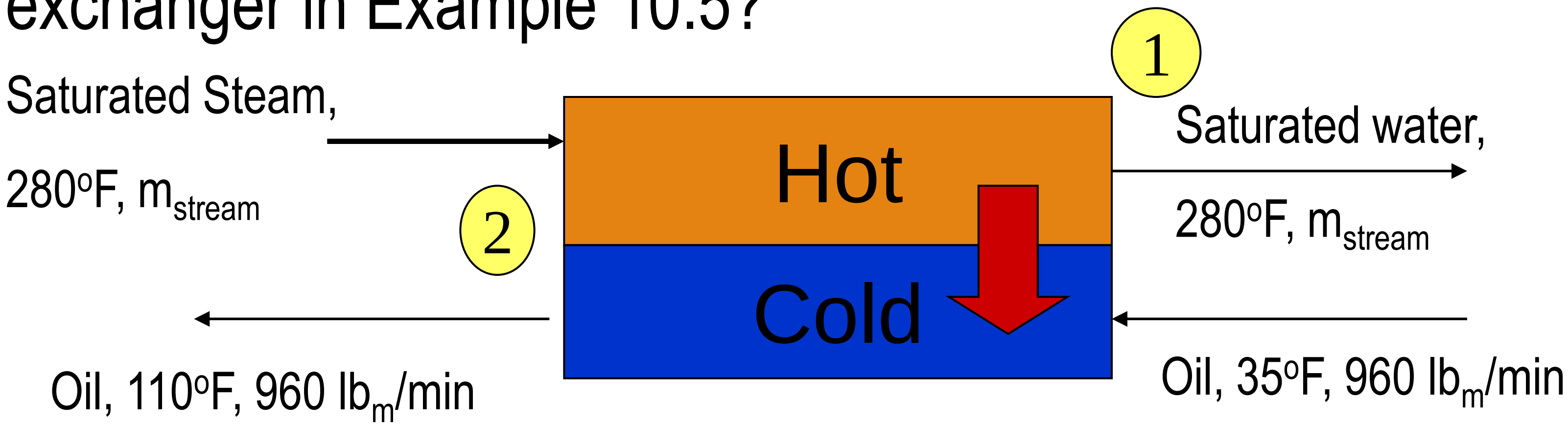
$$\left[\dot{m}\bar{C}_p\left(T_{out}-T_{in}\right)\right]_{cold}=\dot{Q}_{duty} \qquad (10.24b)$$

$$\left[\left(960\frac{lb_m}{min}\right)\left(0.74\frac{Btu}{lb_m{}^oF}\right)(110-35)^o F\right]_{cold}=\dot{Q}_{duty}$$

$$\dot{Q}_{duty}=53,280\frac{Btu}{min}$$

Example

How much area is required for the counter-current heat exchanger in Example 10.5?



$$\Delta T_1 = (280 - 35) = 245^oF$$
$$\Delta T_2 = (280 - 110) = 170^oF$$

$$\Delta T_{log\ mean} = \frac{245 - 170}{\ln\left(245/170\right)} = 205^oF$$

Example

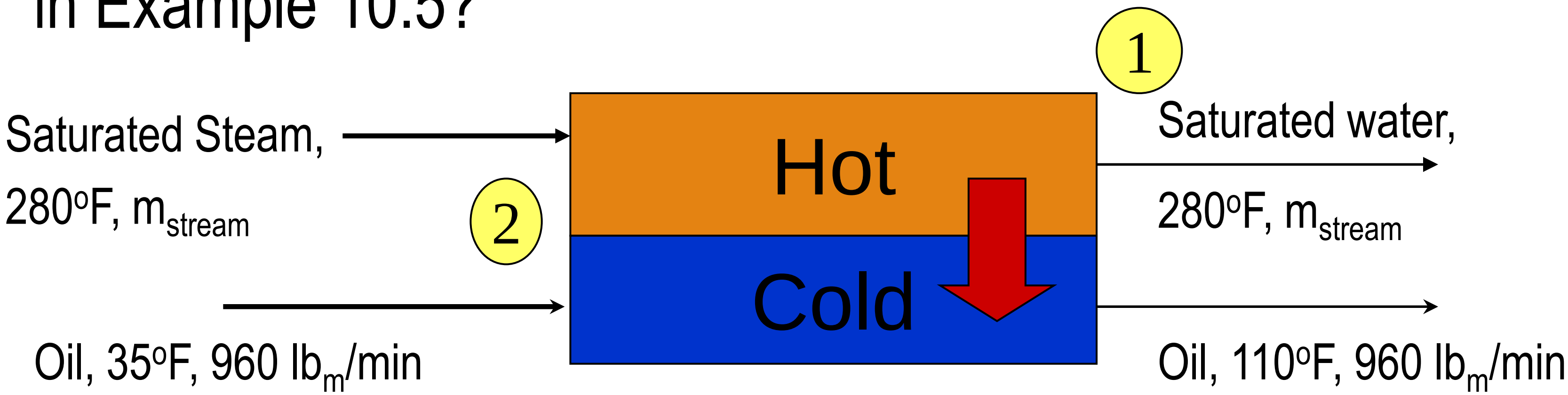
From table 10.5

$$U_o = 150 \text{ Btu}/\left(\text{hr ft}^2\ {}^o\text{F}\right)$$

$$A = \frac{53{,}280 \text{ Btu/min}}{\left(150 \frac{\text{Btu}}{\text{hr ft}^2\ {}^o\text{F}}\right)\left(205^o\text{F}\right)} \left| \frac{60 \text{ min}}{1 \text{hr}} \right. = 104 \text{ ft}^2$$

Example

How much area is required for the co-current heat exchanger in Example 10.5?



$$\Delta T_1 = (280 - 110) = 170^{o}F$$

$$\Delta T_2 = (280 - 35) = 245^{o}F$$

$$\Delta T_{log\ mean} = \frac{170 - 245}{\ln\left(170 \big/ 245\right)} = 205^{o}F$$

Example

From table 10.5

$$U_o = 150 \text{ Btu}/\left(\text{hr ft}^2\ {}^o\text{F}\right)$$

$$A = \frac{53{,}280 \text{ Btu/min}}{\left(150 \frac{\text{Btu}}{\text{hr ft}^2\ {}^o\text{F}}\right)\left(205^o\text{F}\right)}\left|\frac{60 \text{ min}}{1 \text{ hr}}\right. = 104 \text{ ft}^2$$

UNIT IV – RADIATION

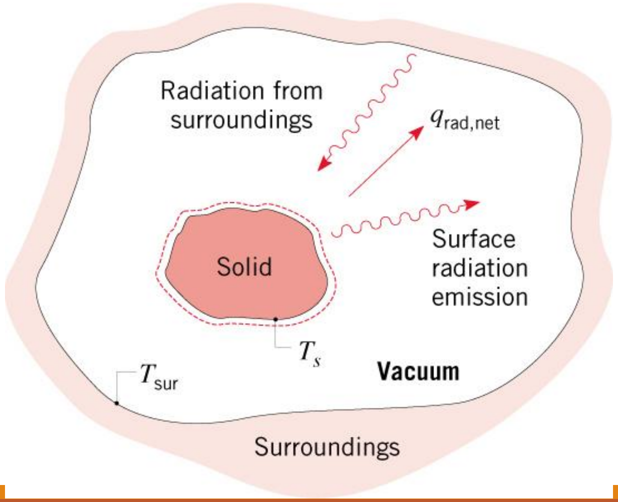
General Considerations

- Attention is focused on thermal radiation, whose origins are associated with emission from matter at an absolute temperature $T > 0$

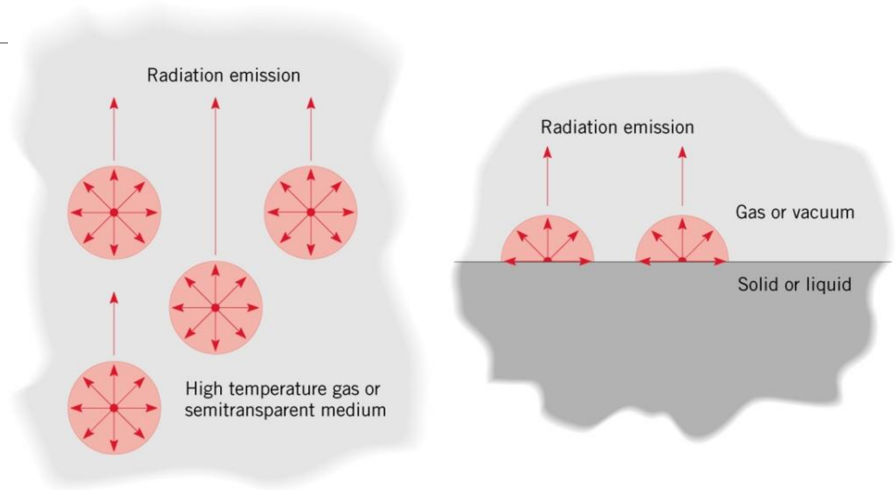
- Emission is due to oscillations and transitions of the many electrons that comprise matter, which are, in turn, sustained by the thermal energy of the matter.
- Emission corresponds to heat transfer from the matter and hence to a reduction in thermal energy stored by the matter.

- Radiation may also be intercepted and absorbed by matter.

- Absorption results in heat transfer to the matter and hence an increase in
thermal energy stored by the matter.
- Consider a solid of temperature T_s in an evacuated enclosure whose walls are at a fixed temperature T_{sur} :
  - What changes occur if $T_s > T_{sur}$? Why ?
  - What changes occur if $T_s < T_{sur}$? Why ?



- Emission from a gas or a semitransparent solid or liquid is a volumetric phenomenon. Emission from an opaque solid or liquid is a surface phenomenon.



(a) (b)

For an opaque solid or liquid, emission originates from atoms and molecules within 1 of the surface.

- The dual nature of radiation:
  - In some cases, the physical manifestations of radiation may be explained by viewing it as particles (aka photons or quanta).
  - In other cases, radiation behaves as an electromagnetic wave.

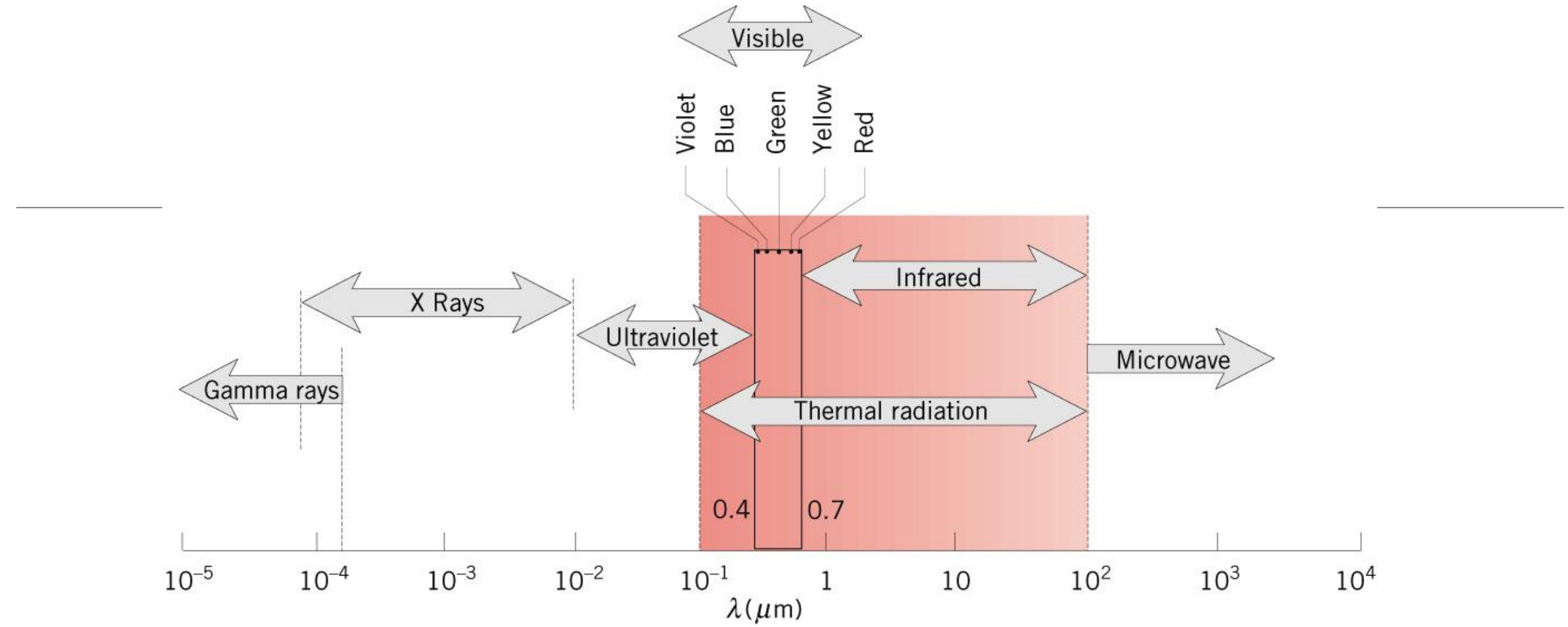
– In all cases, radiation is characterized by a wavelength ● and frequen
which are related through the speed at which radiation propagates in
medium of interest:

$$\lambda = \frac{c}{\upsilon}$$

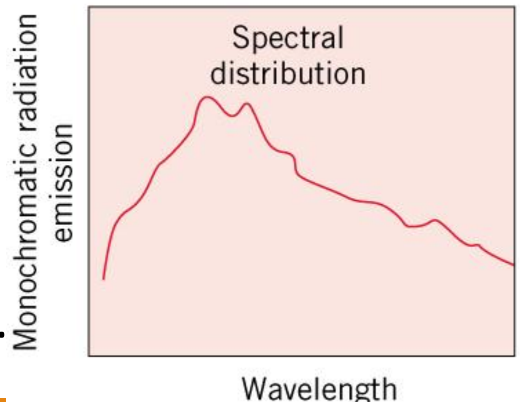
For propagation in a vacuum,

$$c = c_o = 2.998 \times 10^8 \text{ m/s}$$

The Electromagnetic Spectrum

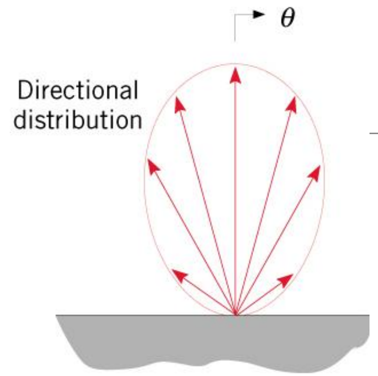


- Thermal radiation is confined to the <u>infrared</u>, <u>visible</u>, and <u>ultraviolet</u> regions of the spectrum $(0.1 < \lambda < 100\,\mu m)$
- The amount of radiation emitted by an opaque surface varies with wavelength, and we may speak of the <u>spectral distribution</u> over all wavelengths or of monochromatic/spectral components associated with particular wavelengths.

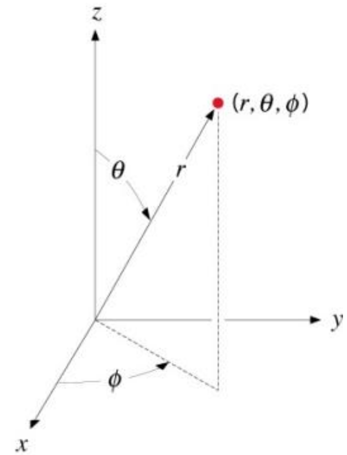


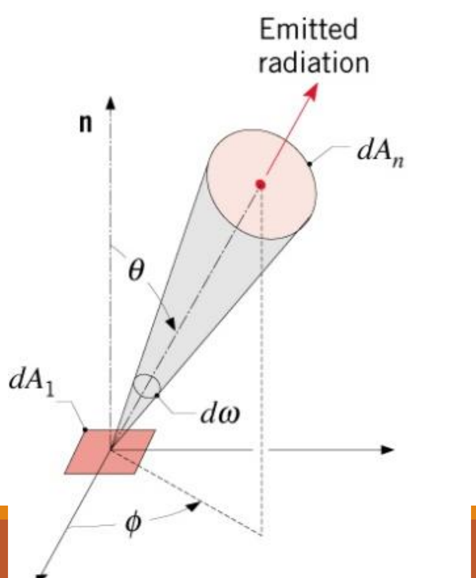
Directional Considerations and the Concept of Radiation Intensity

- Radiation emitted by a surface will be in al directions associated with a hypothetical hemisphere about the surface and is characterized by a directional distribution.



- Direction may be represented in a spherical coordinate system characterized by the zenith or polar angle ❑ and the azimuthal angle ↗.

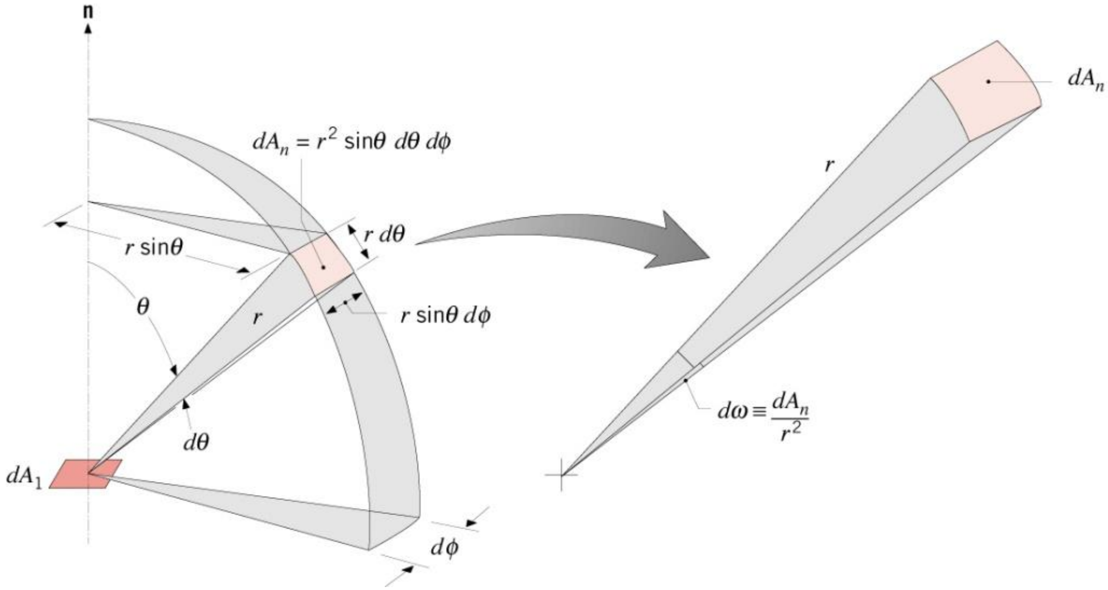




- The amount of radiation emitted from a surface, dA_1 and propagating in a particular direction, ❑, ↗, is quantified in terms of a differential solid angle associated with the direction.

$$d\omega \equiv \frac{dA_n}{r^2}$$

$dA_n \rightarrow$ unit element of surface on a hypothetical sphere and normal to the ❑, ↗ direction.



$$dA_n = r^2 \sin\theta d\theta d\phi$$

$$d\omega = \frac{dA_n}{r^2} = \sin\theta d\theta d\phi$$

– The solid angle ◆ has units of steradians (sr).

– The solid angle associated with a complete hemisphere is

$$\omega_{\text{hemi}} = \int_0^{2\pi}\int_0^{\pi/2} \sin\theta d\theta d\phi = 2\pi \text{ sr}$$

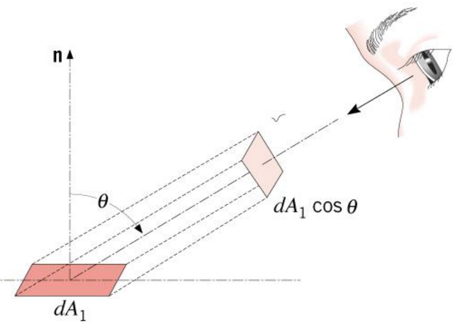
- Spectral Intensity: A quantity used to specify the radiant heat flux (W/m^2) within a unit solid angle about a prescribed direction (W/m^2⌚sr) and within a unit wavelength interval about a prescribed wavelength (W/m^2⌚sr ⌚❍m)

- The spectral intensity $I_{●,e}$ associated with emission from a surface element dA_1 in the solid angle d◆ about ❑, ⤯ and the wavelength interval d● about ● is defined as:

$$I_{\lambda,e}(\lambda,\theta,\phi) \equiv \frac{dq}{(dA_1 \cos\theta)\cdot d\omega \cdot d\lambda}$$

- The rationale for defining the radiation flux in terms of the projected surface area ($dA_1 \cos$ ❑) stems from the existence of surfaces for which, to a good approximation, $I_{\bullet,e}$ is independent of direction. Such surfaces are termed diffuse, and the radiation is said to be isotropic.

  - The projected area is how dA_1 would appear if observed along ❑, ↗.
    - What is the projected area for ❑ $= 0$?
    - What is the projected area for ❑ = ◘/2 ?



- The spectral heat rate and heat flux associated with emission from dA_1 are, respectively,

$$dq_\lambda \equiv \frac{dq}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)dA_1\cos\theta\cdot d\omega$$

$$dq''_\lambda = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\cdot d\omega = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

Relation of Intensity to Emissive Power, Irradiation, and Radiosity

- The spectral emissive power (W/m^2 $\cdot\mu$m) corresponds to spectral emission over all possible directions (hemispherical).

$$E_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

- The total emissive power (W/m^2) corresponds to emission over all directions and wavelengths (hemispherical).

$$E = \int_0^{\infty} E_\lambda(\lambda) d\lambda$$

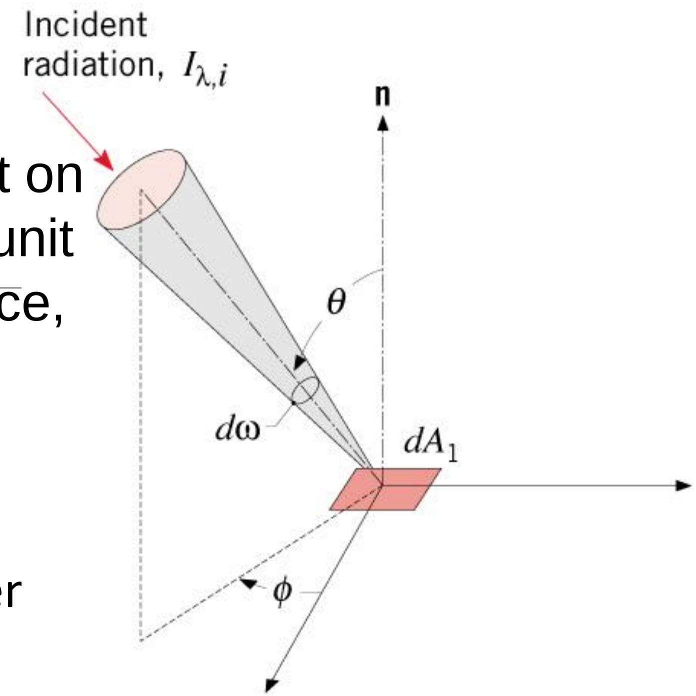
- For a diffuse surface, emission is isotropic and

$$E_\lambda(\lambda) = I_{\lambda,e}(\lambda)\int_0^{2\pi}\int_0^{\pi/2}\cos\theta\sin\theta d\theta d\phi = \pi I_{\lambda,e}(\lambda)$$

$$E = \int_0^{\infty} E_\lambda(\lambda) d\lambda = \pi\int_0^{\infty} I_{\lambda,e}(\lambda) d\lambda = \pi I_e$$

where I_e is called total intensity.

Irradiation



- The spectral intensity of radiation incident on a surface, $I_{●,i}$, is defined in terms of the unit solid angle about the direction of incidence, the wavelength interval d● about ●, and the projected area of the receiving surface, $dA_1 \cos$ ❑.

- The spectral irradiation (W/m^2 🕐❍m) is ther

$$G_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi) \cos\theta \sin\theta d\theta d\phi$$

and the total irradiation (W/m^2) is

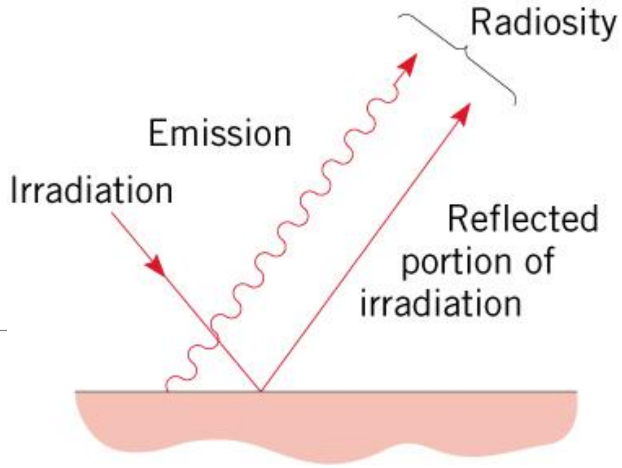
$$G = \int_0^\infty G_\lambda(\lambda) d\lambda$$

➢ How may $G_●$ and G be expressed if the incident radiation is diffuse?

$$G_\lambda(\lambda) = I_{\lambda,i}(\lambda) \int_0^{2\pi} \int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi = \pi I_{\lambda,i}(\lambda)$$

$$G = \int_0^\infty G_\lambda(\lambda) d\lambda = \pi \int_0^\infty I_{\lambda,i}(\lambda) d\lambda = \pi I_i$$

Radiosity



- The radiosity of an opaque surface accounts for all of the radiation leaving the surface in all directions and may include contributions to both reflection and emission.

- With $I_{\lambda,e+r}$ designating the spectral intensity associated with radiation emitted by the surface and the reflection of incident radiation, the spectral radiosity (W/m^2 ⋅ μm) is

$$J_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

and the total radiosity (W/m^2) is

$$J = \int_0^\infty J_\lambda(\lambda)\, d\lambda$$

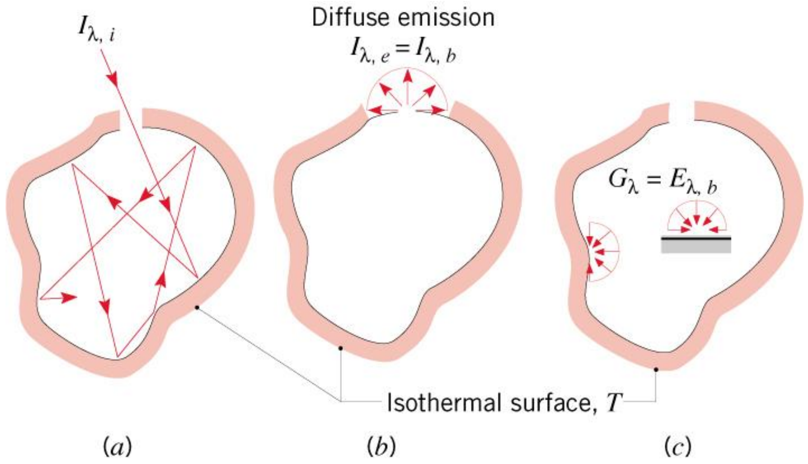
➢ How may J_λ and J be expressed if the surface emits and reflects diffusely?

$$J_\lambda(\lambda) = I_{\lambda,e+r}(\lambda)\int_0^{2\pi}\int_0^{\pi/2} \cos\theta\sin\theta\, d\theta\, d\phi = \pi I_{\lambda,e+r}(\lambda)$$

$$J = \int_0^\infty J_\lambda(\lambda)\, d\lambda = \pi\int_0^\infty I_{\lambda,e+r}(\lambda)\, d\lambda = \pi I_{e+r}$$

Blackbody Radiation

- The Blackbody
  - An idealization providing limits on radiation emission and absorption by matter.
    - For a prescribed temperature and wavelength, no surface can emit more radiation than a blackbody: the ideal emitter.
    - A blackbody is a diffuse emitter.
    - A blackbody absorbs all incident radiation: the ideal absorber.
- The Isothermal Cavity



(a) After multiple reflections, virtually all radiation entering the cavity is absorbed

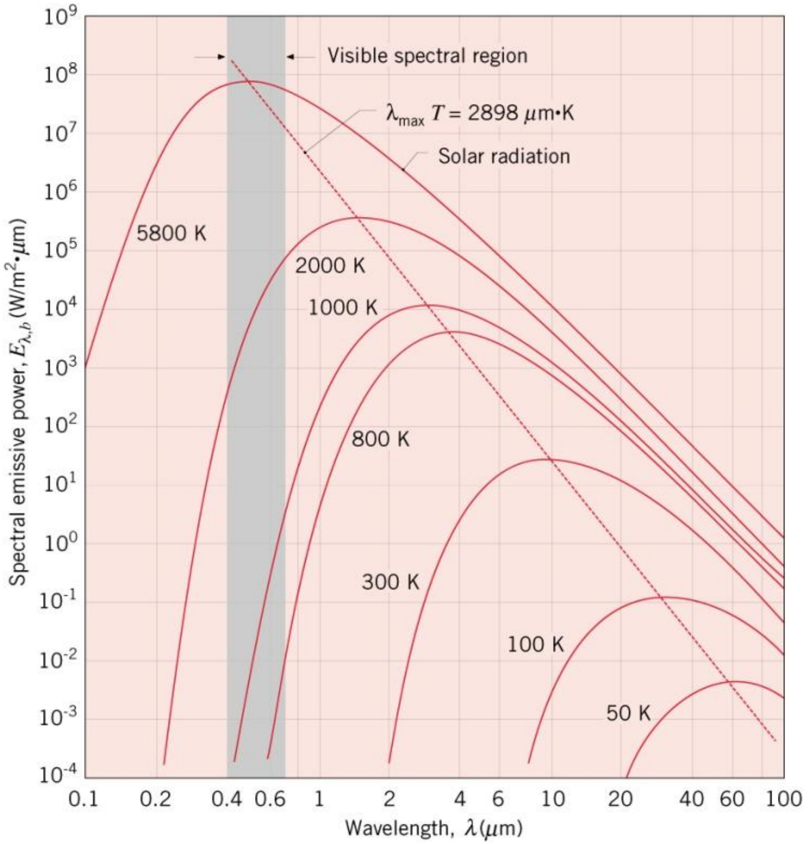
(b) Emission from the aperture is the maximum possible emission achievable for the temperature associated with the cavity and is diffuse.

(c) The cumulative effect of radiation emission from and reflection off the cavity wall is to provide diffuse irradiation corresponding to emission from a blackbody ($G_{\bullet} = E_{\bullet,b}$) for any small surface in the cavity.

– Does this condition depend on whether the cavity surface is highly reflecting or absorbing?

The Spectral (Planck) Distribution of Blackbody Radiation

- The spectral distribution of the blackbody emissive power (determined theoretically and confirmed experimentally) is



$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left[\exp\left(C_2/\lambda T\right)-1\right]}$$

$$C_1 = 3.742\times10^8\,\mathrm{W}\cdot\mu\mathrm{m}^4/\mathrm{m}^2,\ C_2 = 1.439\times10^4\,\mu\mathrm{m}\cdot\mathrm{K}$$

- ➢ $E_{\bullet,b}$ varies continuously with ● and increases with T.
- ➢ The distribution is characterized by a maximum for which $\bullet_{max}$ is given by Wien's displacement law: $\lambda_{max}T = C_3 = 2898\,\mu\mathrm{m}\cdot\mathrm{K}$
- ➢ The fractional amount of total blackbody emission appearing at lower wavelengths increases with increasing T.

The Stefan-Boltzmann Law and Band Emission

- The total emissive power of a blackbody is obtained by integrating the Planck distribution over all possible wavelengths.

$$E_\lambda = \pi I_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

→ the Stefan-Boltzmann law, where

$$\sigma = 5.670 \times 10^{-8} \mathrm{W/m^2 \cdot K^4} \rightarrow \text{the Stefan-Boltzmann constant}$$

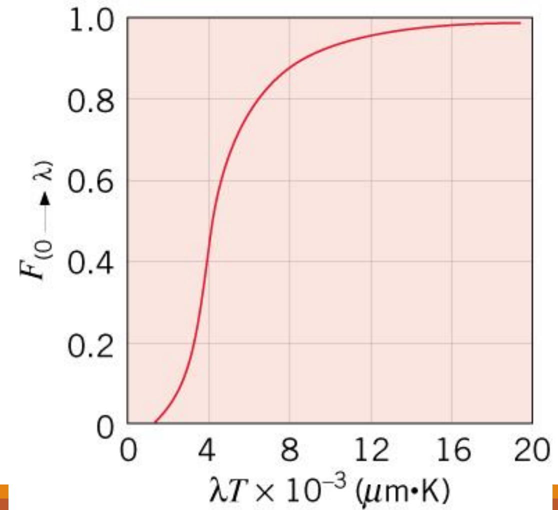
- The fraction of total blackbody emission that is *in a prescribed wavelength interval* or band $(\lambda_1 < \lambda < \lambda_2)$ is

$$F_{(\lambda_1 \rightarrow \lambda_2)} = F_{(0 \rightarrow \lambda_2)} - F_{(0 \rightarrow \lambda_1)} = \frac{\int_0^{\lambda_2} E_{\lambda,b} d\lambda - \int_0^{\lambda_1} E_{\lambda,b} d\lambda}{\sigma T^4}$$

where, in general,

$$F_{(0 \rightarrow \lambda)} = \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\sigma T^4} = f(\lambda T)$$

and numerical results are given in Table 12.1



- Table 12.1

TABLE 12.1 Blackbody Radiation Functions

λT ($\mu m \cdot K$)	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291

Note ability to readily determine $I_{\bullet,b}$ and its relation to the maximum intensity from the 3rd and 4th columns, respectively.

- If emission from the sun may be approximated as that from a blackbody at 5800K, at what wavelength does peak emission occur?
- Would you expect radiation emitted by a blackbody at 800K to be discernible by the naked eye?
- As the temperature of a blackbody is increased, what color would be the first to be discerned by the naked eye?

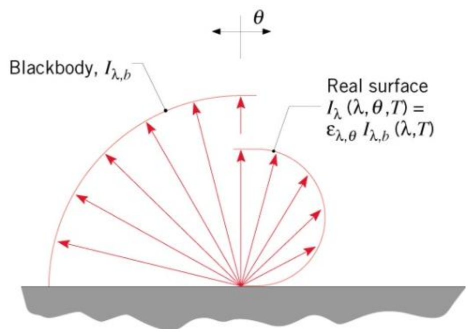
Surface Emissivity

- Radiation emitted by a surface may be determined by introducing a property (the emissivity) that contrasts its emission with the ideal behavior of a blackbody at the same temperature.

- The definition of the emissivity depends upon one's interest in resolving directional and/or spectral features of the emitted radiation, in contrast to averages over all directions (hemispherical and/or wavelengths (total).

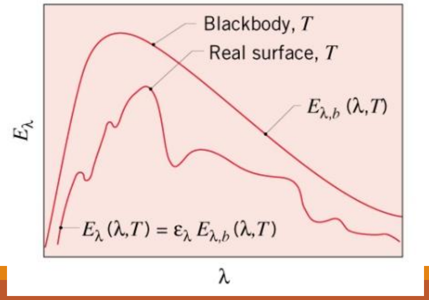
- The spectral, directional emissivity:

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) \equiv \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,T)}$$



- The spectral, hemispherical emissivity (a directional average):

$$\varepsilon_\lambda(\lambda,T) \equiv \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)} = \frac{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi,T)\cos\theta\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,b}(\lambda,T)\cos\theta\sin\theta\, d\theta\, d\phi}$$



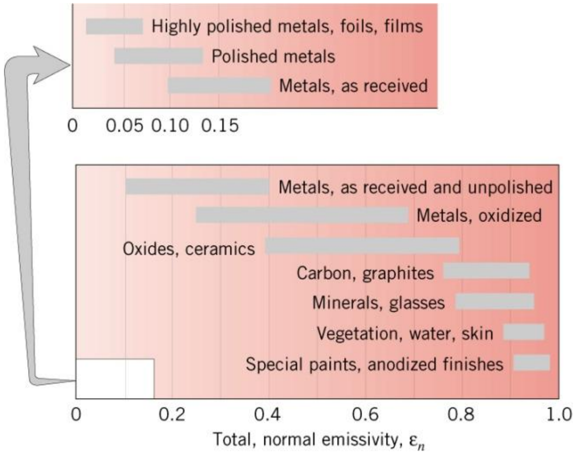
- The <u>total, hemispherical emissivity</u> (a directional and spectral average):

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)d\lambda}{E_b(T)}$$

- To a reasonable approximation, the hemispherical emissivity is equal to the normal emissivity.

$$\varepsilon = \varepsilon_n$$

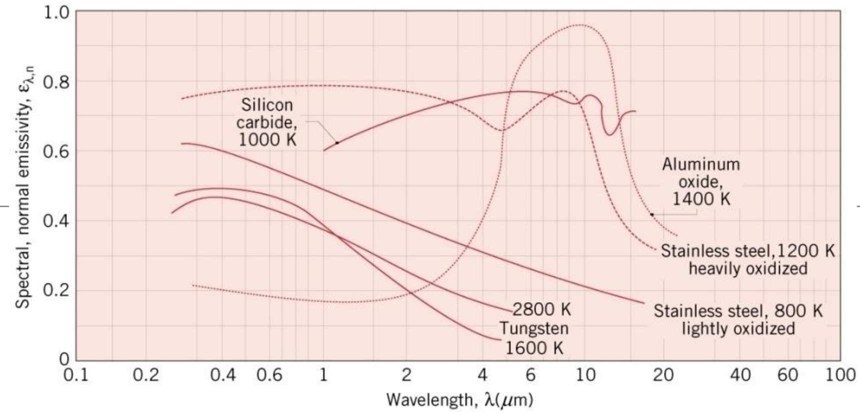
- Representative values of the total, normal emissivity:



Note:

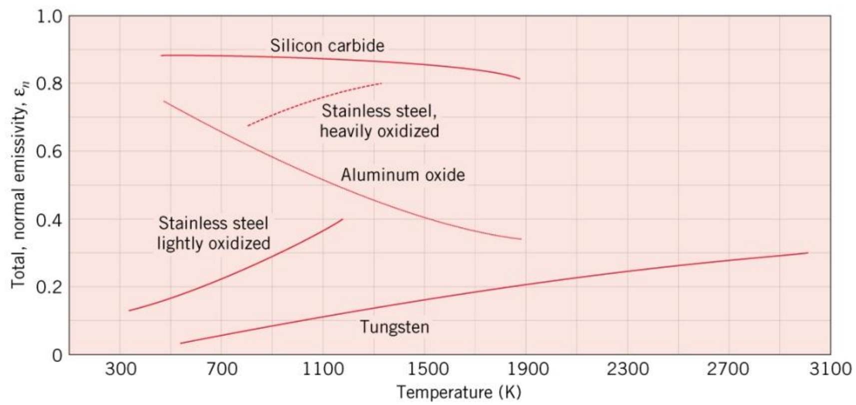
- Low emissivity of polished metals and increasing emissivity for unpolished and oxidized surfaces.
- Comparatively large emissivities of nonconductors.

- Representative spectral variations:



Note decreasing $\varepsilon_{\lambda,n}$ with increasing λ for metals and different behavior for nonmetals.

- Representative temperature variations:

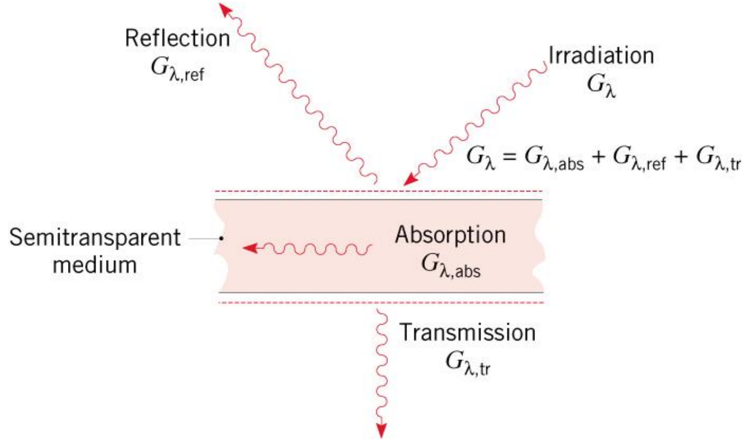


Why does ε_n increase with increasing T for tungsten and not for aluminum oxide?

Response to Surface Irradiation: Absorption, Reflection and Transmission

- There may be three responses of a semitransparent medium to irradiation:

  - Reflection from the medium
  - Absorption within the medium
  - Transmission through the medium



Radiation balance →

- In contrast to the foregoing volumetric effects, the response of an opaque mater to irradiation is governed by surface phenomena and

- The wavelength of the incident radiation, as well as the nature of the material, determine whether the material is semitransparent or opaque.
  - Are glass and water semitransparent or opaque?

- Unless an opaque material is at a sufficiently high temperature to emit visible radiation, its *color* is determined by the spectral dependence of reflection in response to visible irradiation.
  - What may be said about reflection for a white surface? A black surface?
  - Why are leaves green?

Absorptivity of an Opaque Material

- The spectral, directional absorptivity: Assuming negligible temperature dependence,

$$\alpha_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,abs}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

- The spectral, hemispherical absorptivity:

$$\alpha_{\lambda}(\lambda) \equiv \frac{E_{\lambda,abs}(\lambda)}{E_{\lambda}(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta}(\lambda,\theta,\phi) I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi}$$

  - ➢ To what does the foregoing result simplify, if the irradiation is diffuse? If the surface is diffuse?

- The total, hemispherical absorptivity:

$$\alpha \equiv \frac{G_{abs}}{G} = \frac{\int_0^{\infty} \alpha_{\lambda}(\lambda) G_{\lambda}(\lambda)\, d\lambda}{\int_0^{\infty} G_{\lambda}(\lambda)\, d\lambda}$$

  - ➢ If the irradiation corresponds to emission from a blackbody, how may the above expression be rewritten?
  - ➢ The absorptivity is approximately independent of the surface temperature, but if the irradiation corresponds to emission from a blackbody, why does depend on the temperature of the blackbody?

Reflectivity of an Opaque Material

- The spectral, directional reflectivity: Assuming negligible temperature dependence:

$$\rho_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,ref}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

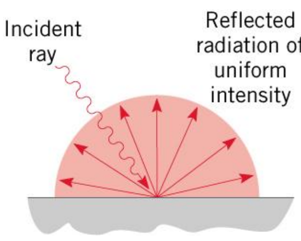
- The spectral, hemispherical reflectivity:

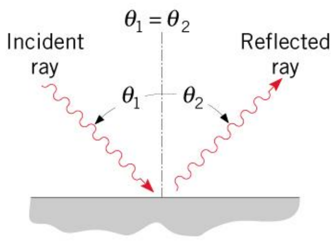
$$\rho_{\lambda}(\lambda) \equiv \frac{G_{\lambda,ref}(\lambda)}{E_{\lambda}(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\pi/2} \rho_{\lambda,\theta}(\lambda,\theta,\phi) I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta \sin\theta d\theta d\phi}$$

  - ➢ To what does the foregoing result simplify if the irradiation is diffuse? If the surface is diffuse?

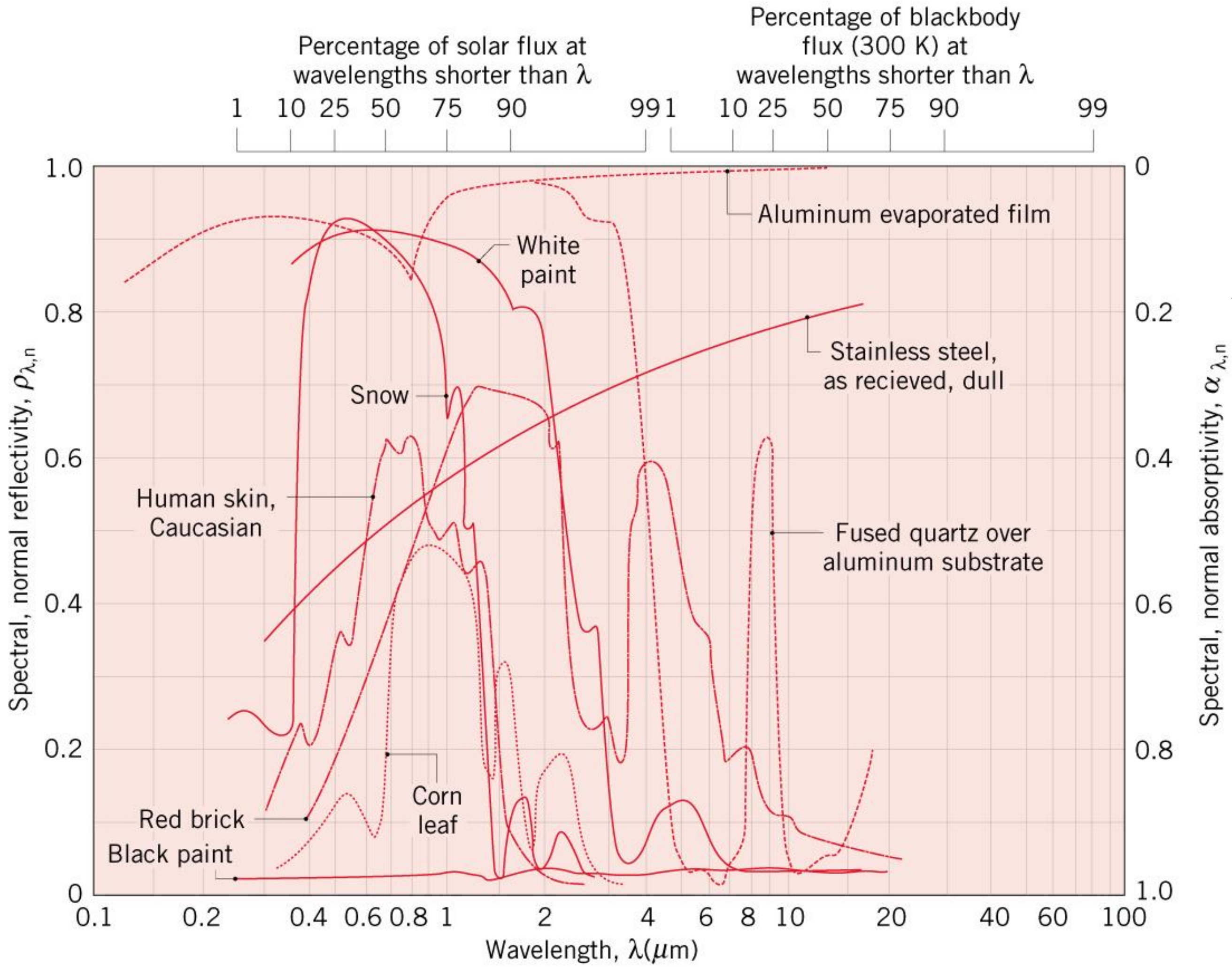
- The total, hemispherical reflectivity:

$$\rho \equiv \frac{G_{ref}}{G} = \frac{\int_0^{\infty} \rho_{\lambda}(\lambda) G_{\lambda}(\lambda) d\lambda}{\int_0^{\infty} G_{\lambda}(\lambda) d\lambda}$$





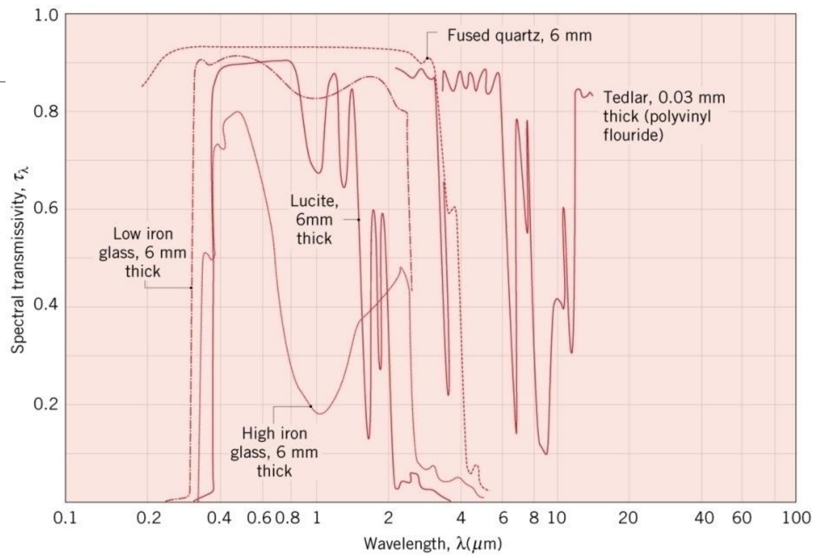
- Limiting conditions of diffuse and spectral reflection. Polished and rough surfaces.



- Note strong dependence of
- Is snow a highly reflective substance? White paint?

Transmissivity

The spectral, hemispherical transmissivity: Assuming negligible temperature dependence,



Note shift from semitransparent to opaque conditions at large and small wavelengths.

- The total, hemispherical transmissivity:

- For a semitransparent medium,

$$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$$

$$\alpha + \rho + \tau = 1$$

- For a gray medium,

$$\alpha_\lambda + \rho_\lambda = 1$$

$$\alpha + \rho = 1$$

- The demo experiment shows that the black material absorbs more irradiation than the aluminum does.

Kirchhoff's Law

- Kirchhoff's law equates the total, hemispherical emissivity of a surface to its total, hemispherical absorptivity:

$$\varepsilon = \alpha$$

However, conditions associated with its derivation are highly restrictive:

Irradiation of the surface corresponds to emission from a blackbody at the same temperature as the surface.

- However, Kirchhoff's law may be applied to the spectral, directional properties without restriction:

$$\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$$

Why are there no restrictions on use of the foregoing equation?

Diffuse/Gray Surfaces

- With

$$\varepsilon_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\lambda,\theta} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi}$$

and

$$\alpha_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi}$$

Under what conditions may we equate

Note: informally, a poor reflector (a good absorber) is a good emitter, and a good reflector (a poor absorber) is a poor emitter.

- With

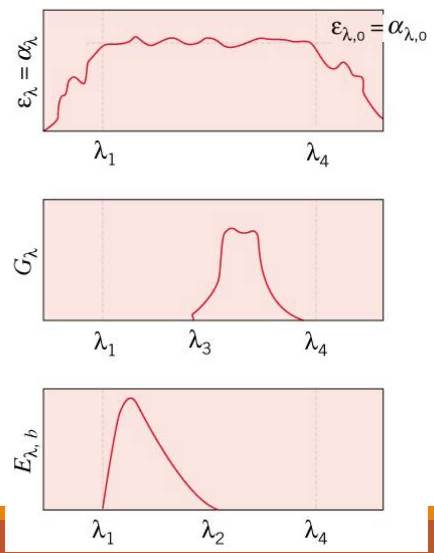
$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda) d\lambda}{E_b(T)}$$

and

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda) d\lambda}{G}$$

Under what conditions may we equate ε to α ?

- Conditions associated with assuming a gray surface:



Radiation Exchange Between Surfaces:
Enclosures with Nonparticipating Media

CHAPTER 13

SECTIONS 13.1 THROUGH 13.4

Basic Concepts

- Enclosures consist of two or more surfaces that envelop a region of space (typically gas-filled) and between which there is radiation transfer. Virtual, as well as real, surfaces may be introduced to form an enclosure.

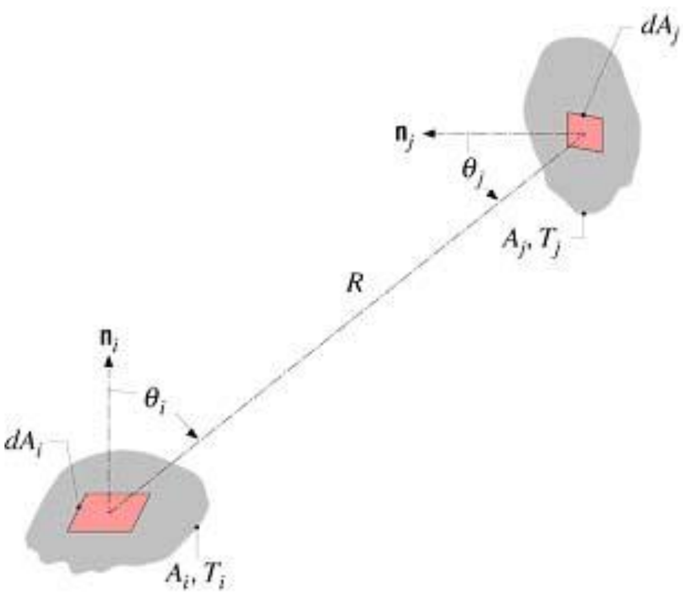
- A nonparticipating medium within the enclosure neither emits, absorbs, nor scatters radiation and hence has no effect on radiation exchange between the surfaces.

- Each surface of the enclosure is assumed to be isothermal, opaque, diffuse and gray, and to be characterized by uniform radiosity and irradiation.

The View Factor (also Configuration or Shape Factor)

- The view factor, is a geometrical quantity corresponding to the *fraction of the radiation leaving surface* i *that is intercepted by surface* j.

The view factor integral provides a general expression for Consider exchang between differential areas

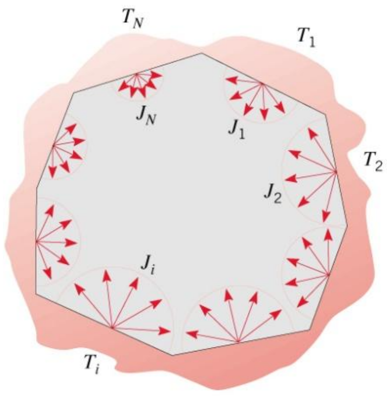


Surfaces are diffuse emittters and reflectors and have uniform radiosity.

View Factor Relations

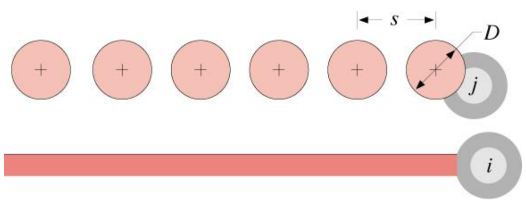
- Reciprocity Relation. With

- Summation Rule for Enclosures.



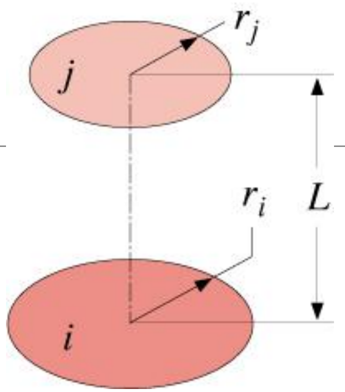
- Two-Dimensional Geometries (Table 13.1) for example,

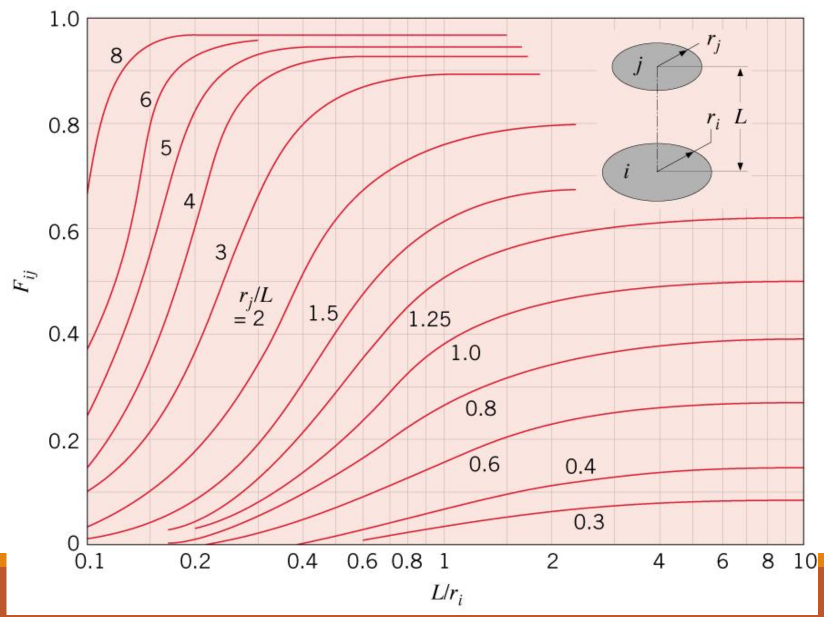
An Infinite Plane and a Row of Cylinders



- Three-Dimensional Geometries (Table 13.2).

For example, *Coaxial Parallel Disks*





UNIT V – HEAT EXCHANGERS

Classification of heat exchangers

Heat exchangers are devices that provide the flow of thermal energy between 2 or more fluids at different temperatures. They are used in a wide variety of applications. These include power production, process, chemical, food and manufacturing industries, electronics, environmental engg. , waste heat recovery, air conditioning, reefer and space applications.

Heat Exchangers may be classified according to the following criteria.

- Recuperators/ regenerators
- Transfer process: direct and indirect contact
- Geometry of construction; tubes, plates, and extended surfaces.
- Heat transfer mechanism: single phase and two phase
- Flow arrangement: Parallel, counter, cross flow.

Recuperation/regeneration HE

~~Conventional heat exchangers with heat transfer between~~ 2 fluids. The heat transfer occurs thro a separating wall or an interface.

In regenerators or storage type heat exchangers, the same flow passage alternately occupied by one of the two fluids. Here thermal energy is not transferred thro' a wall as in direct transfer type but thro' the cyclic passage of 2 fluid thro the same matrix.

Example is the ones used for pre heating air in large coal fired power plant or steel mill ovens.

Regenerators are further classified as fixed and rotary.

Transfer process

According to transfer process heat exchangers are classified as direct contact type and indirect contact type.

In direct contact type, heat is transferred between cold and hot fluids through direct contact of the fluids (eg. Cooling towers, spray and tray condensers)

In indirect heat exchanger, heat energy is transferred thro’ a heat transfer surface,

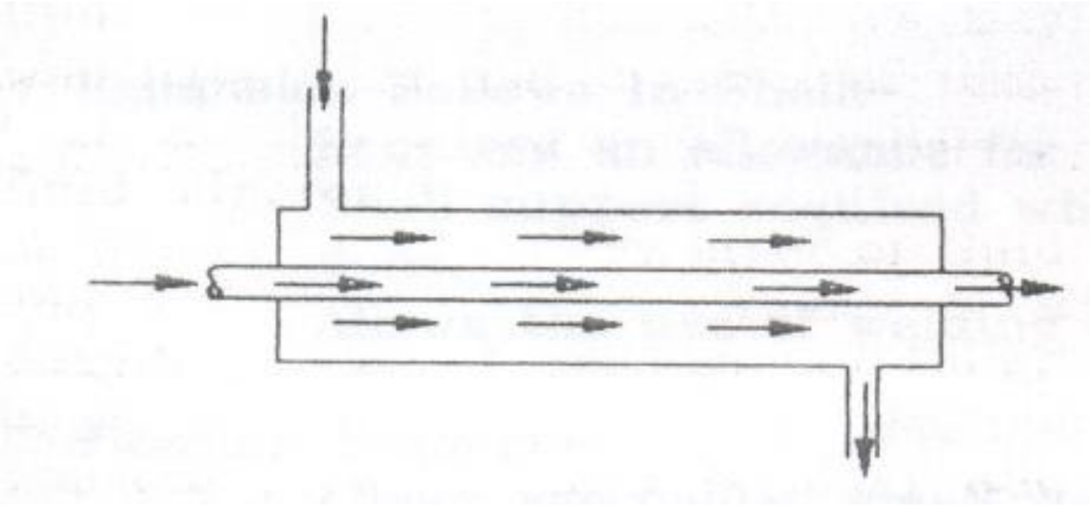
Direct Contact Heat Exchangers

In the majority of heat exchangers heat is transferred through the metal surfaces, from one fluid to another.
The fluid flow is invariably turbulent.
The transfer of heat has to overcome several thermal resistances that are in "Series“
Under normal service conditions tubes may well have a deposit of scale or dirt. Next to this a layer of stationary fluid adheres.
Between this stationary layer of fluid and the general flow there is a boundary (or buffer) layer of fluid
The thickness of the stationary and boundary layer depends on the flow velocity and the type of surface.
In all cases the thermal resistance of these "films“ is considerably greater than the resistance of the metal.

Heat Exchangers

The classic thermodynamic heat exchangers are classified as either:

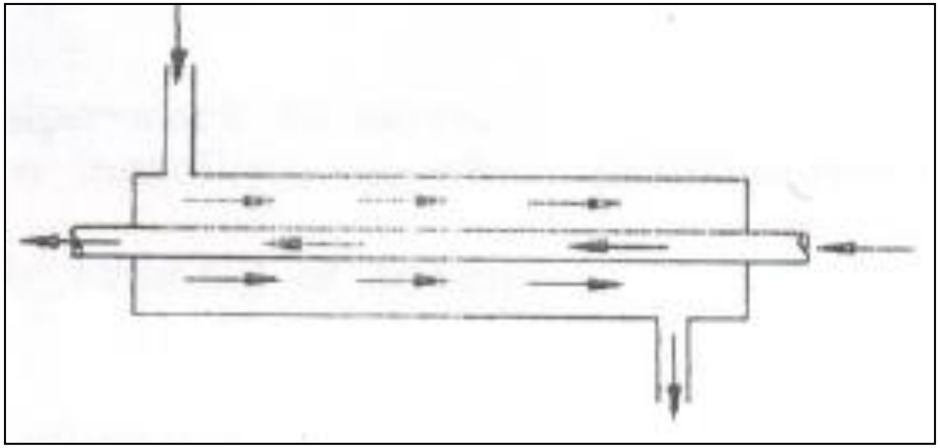
PARALLEL FLOW



Heat Exchangers

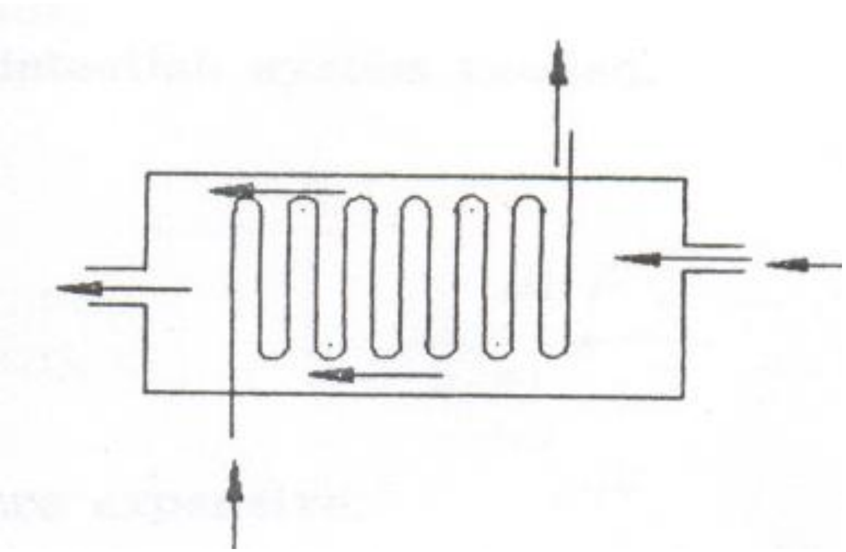
or

CONTRA FLOW.



Heat Exchangers

Most practical heat exchangers are a mixture of both types of flow. Some multi-pass arrangements try to approximate to the contra-flow. The greater the number of passes the closer the approximation.



TUBULAR HEAT EXCHANGERS

Shell:-

Generally cast iron or fabricated steel.

Tubes:-

Very often are of aluminum-brass, for more advanced heat exchangers cupro-nickel or even stainless steel may be used. The tubes are often expanded in to the tube plate but can be soldered, brazed or welded. In the tube stack the tubes pass through alternate baffles that support the tubes and also direct the fluid so that all the tube surfaces are swept, making maximum use of heat transfer area.

The number of tubes always has a fouling allowance. After final assembly the tube stack is machined to fit in the shell bore (the shell is also machined) to allow easy withdrawal.

TUBULAR HEAT EXCHANGERS

Tube-Plates:-

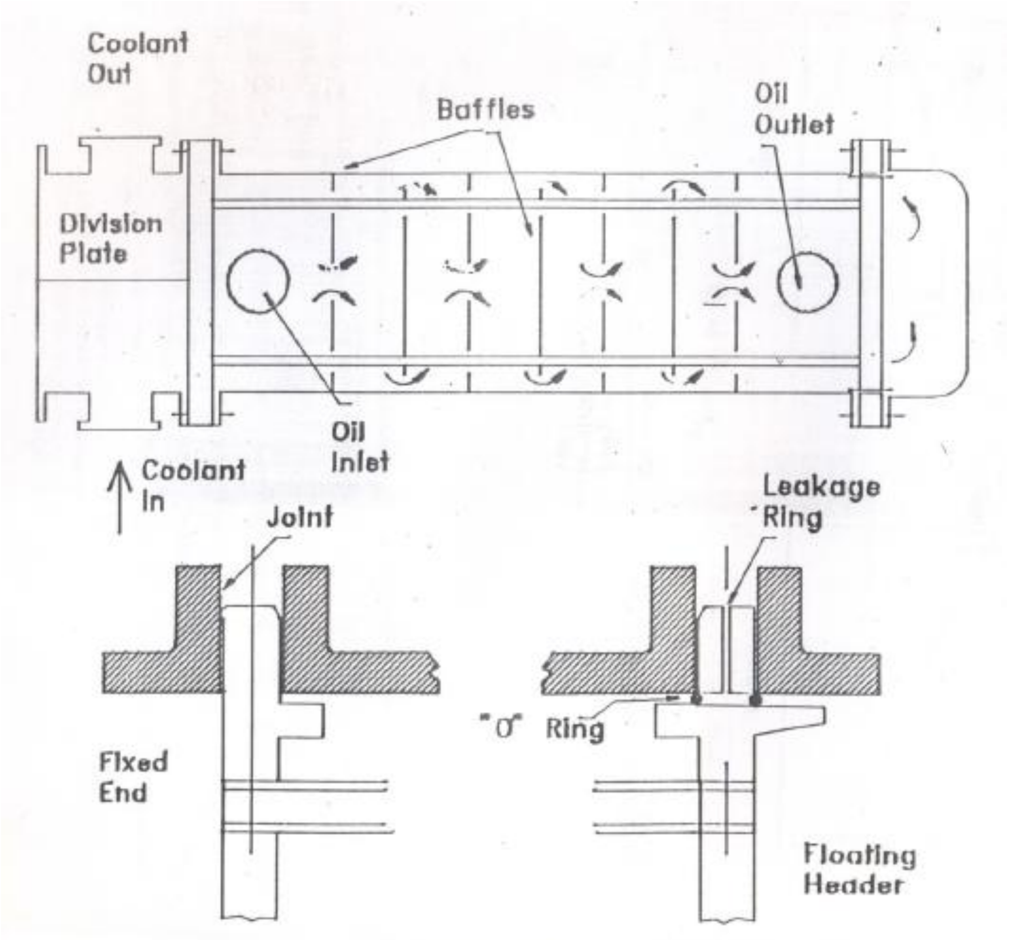
Material would be to suit the tube material and method of fixing. Usually assembled so that the water boxes can be removed without disturbing the tube fastening.

Water Boxes:-

Cast iron or. fabricated steel, always designed to keep turbulence and pressure loss at a minimum. Coated for corrosion protection.

Expansion arrangements can be either, 'u' tube, 'Floating Head', Bayonet Tube, or may have an Expansion Bellows in the shell.

TYPICAL OIL COOLER



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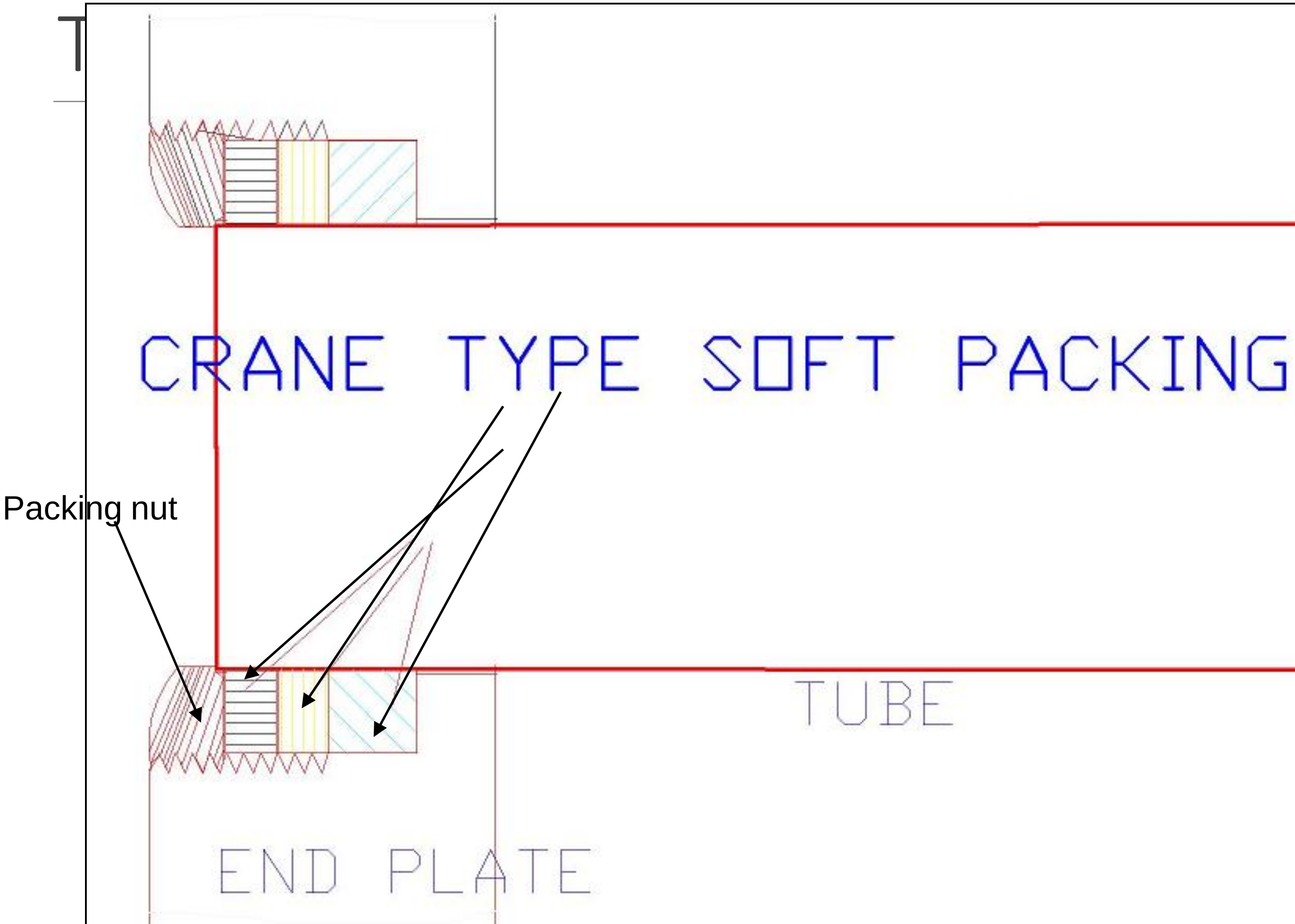


TUBULAR HEAT EXCHANGERS ;Types

- Expansion Bellows In Shell:-

Requires an allowance for pipe-work to move. Shell support required when installing or when maintenance work is done. Allows the use of welding or brazing of tubes.

- Floating Header:-

Allows removal of stack for cleaning. Requires a machined shell interior. Possibility of leaks so a leak detection system needed. This is Expensive

- Fixed 'u' Tube:-

Cheapest to manufacture Tube stack easily removed. Only uses one tube plate.

Can not be single pass. Non-standard tube so spares are expensive.

- Bayonet Tubes.

Usually used for sophisticated fuel oil heaters

Gasketed PHEs

Gasketed plate heat exchangers (the plate and frame) were introduced in the 30s mainly for the food industries because of their ease of cleaning, and their design reached maturity in the 60s with the development of more active plate geometries, assemblies and improved gasket materials. The range of possible applications has widened considerably and, at present, under specific and appropriate conditions, overlap and competes in areas historically considered the domain of tubular heat exchangers. They are capable of meeting an extremely wide range of duties in as many industries. Therefore they can be used as an alternative to shell and tube type heat exchangers for low and medium pressure liquid to liquid heat transfer applications.

Gasketed PHEs

Design of plate heat exchangers is highly specialized in nature considering the variety of designs available for plate and arrangement that possibly suits various duties. Unlike tubular heat exchangers for which design data and methods are easily available, plate heat exchanger design continues to be proprietary in nature. Manufacturers have developed their own computerized design procedures applicable to their exchangers they market.

Gasketed PHEs

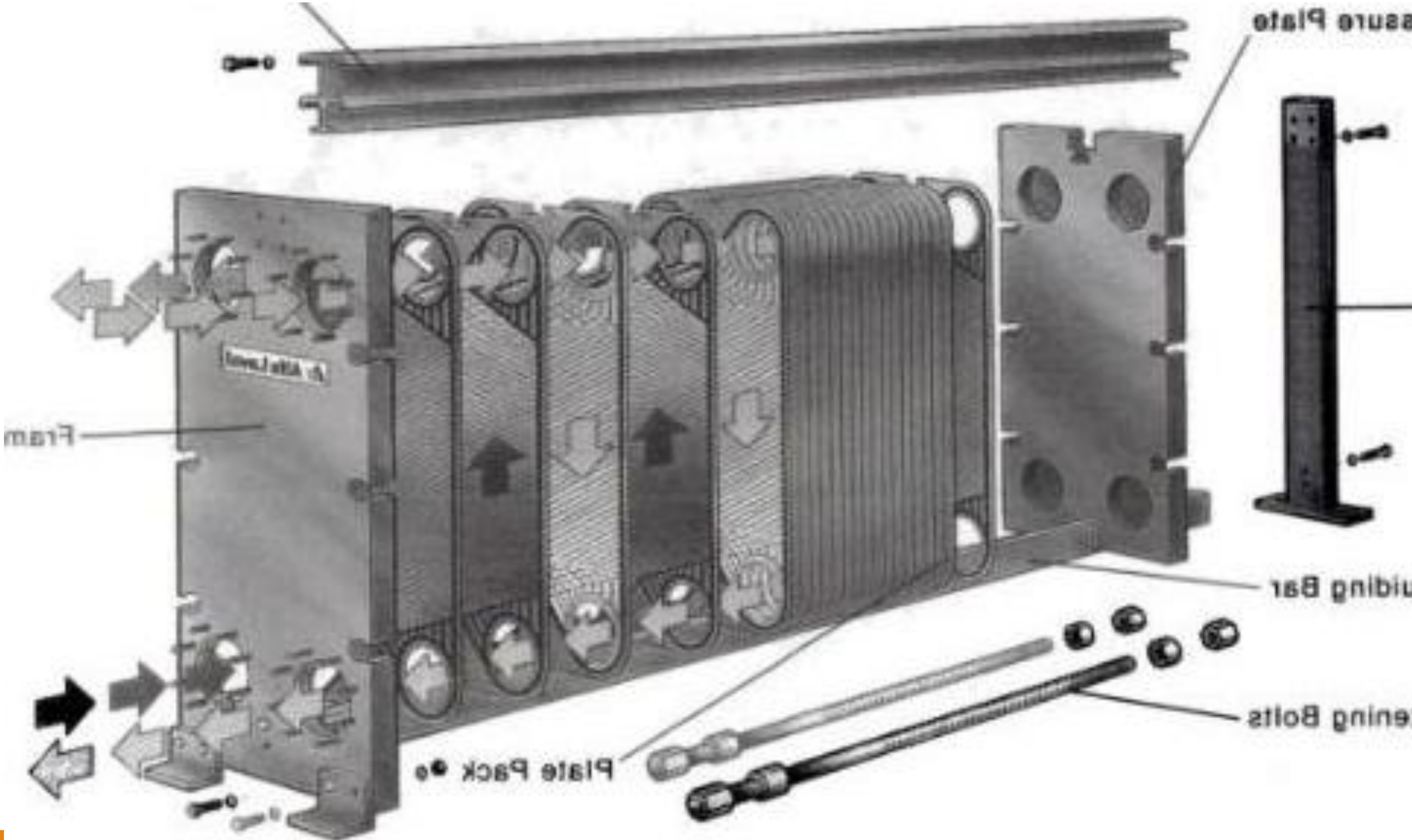


PLATE HEAT EXCHANGERS

Allow the use of contra-flow design, reducing heat transfer area.

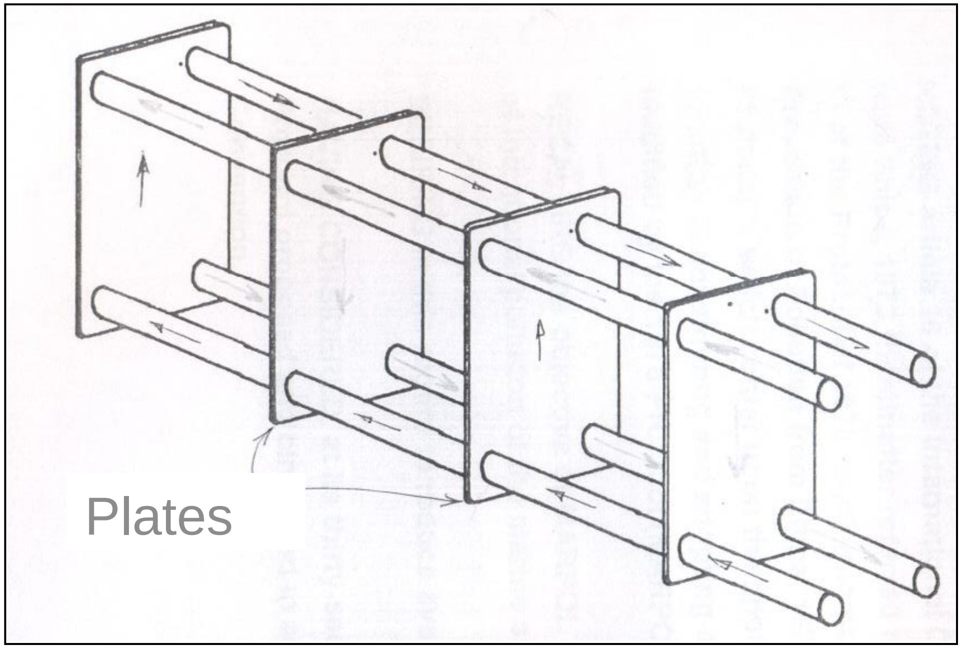
The liquid flows in thin streams between the plates. Troughs, pressed into the plates produce extremely high turbulence, this combined with large heat transfer areas result in a compact unit. The plate form can produce turbulent flow with Reynolds Numbers as low as ten. This type of flow produces a very low fouling rate in the heat exchanger when compared with the tubular type. The heaters are suitable for circulatory cleaning in place, (CIP) as there are no dead areas.

Only the plate edges are exposed to the atmosphere so heat loss is negligible, no insulation is required.

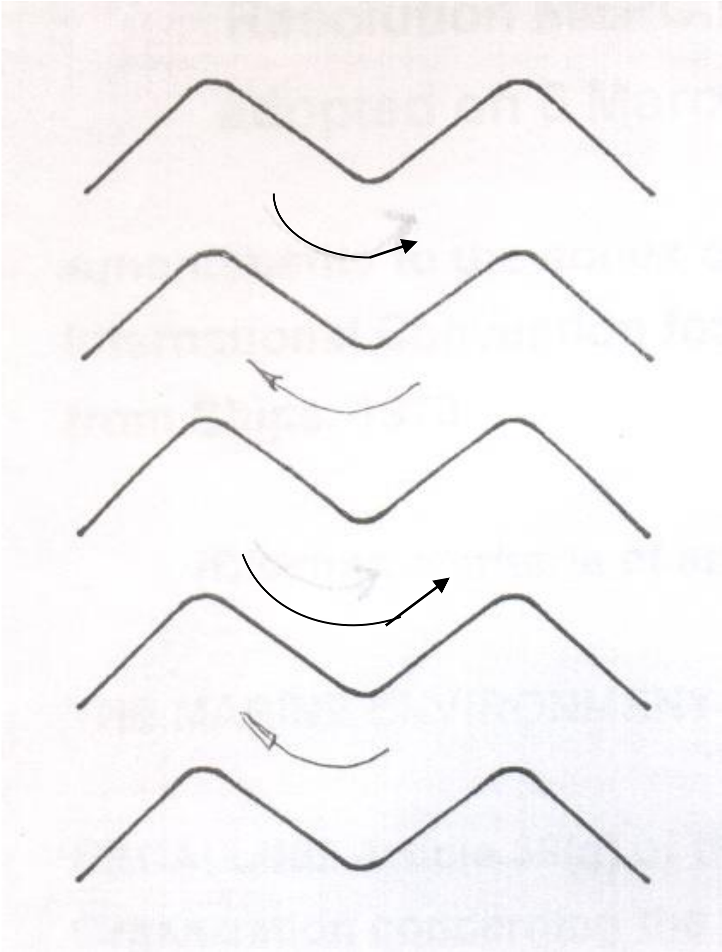
PHEs

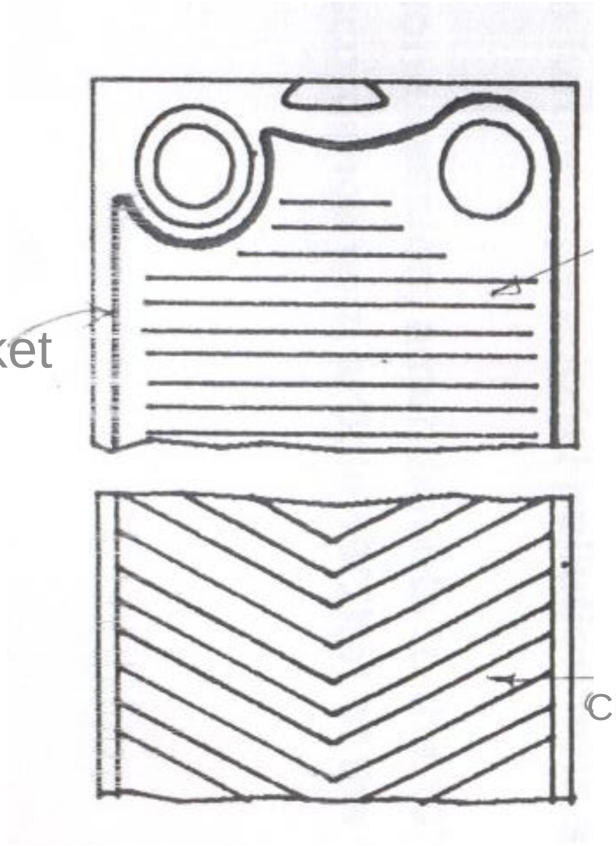
The plates are available in different versions of trough geometry, this gives flexibility in the "thermal length".
For instance, when a washboard type plate is assembled adjacent to a chevron type the thermal length will depend on the chevron angle. Plate heat exchangers can not deal with high pressures due to the requirement for plate gaskets. They can not deal with the large volume flows associated with low pressure vapours and gasses.
For same distance of travel larger time of heat exchange between the two fluids : “high thermal length”

PHEs



PHEs

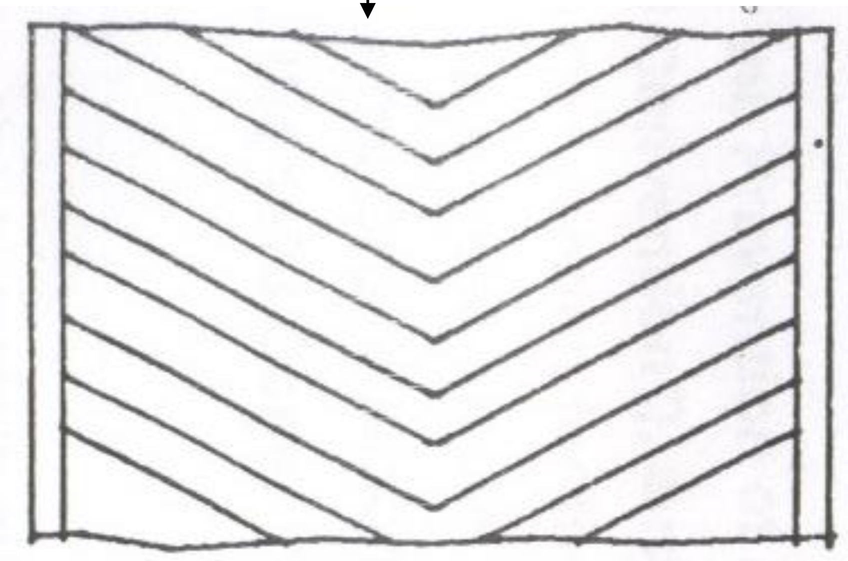


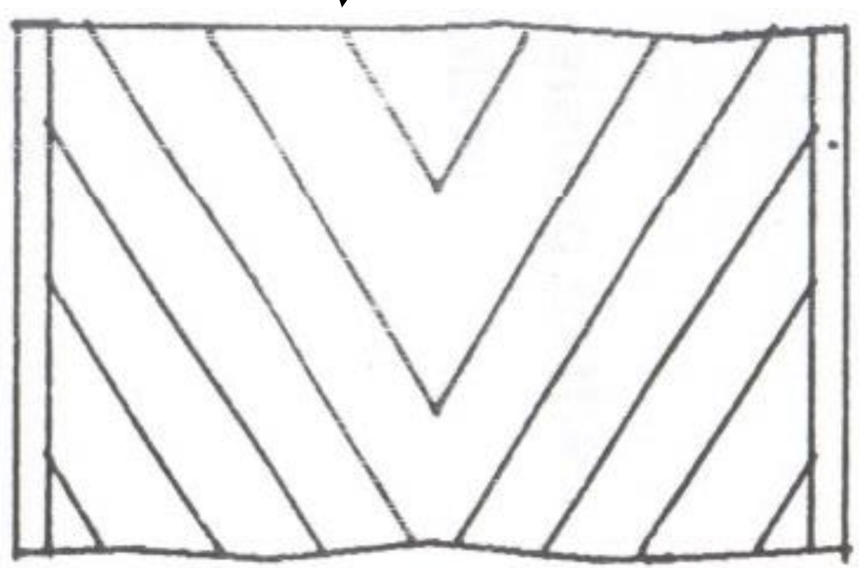


PHEs

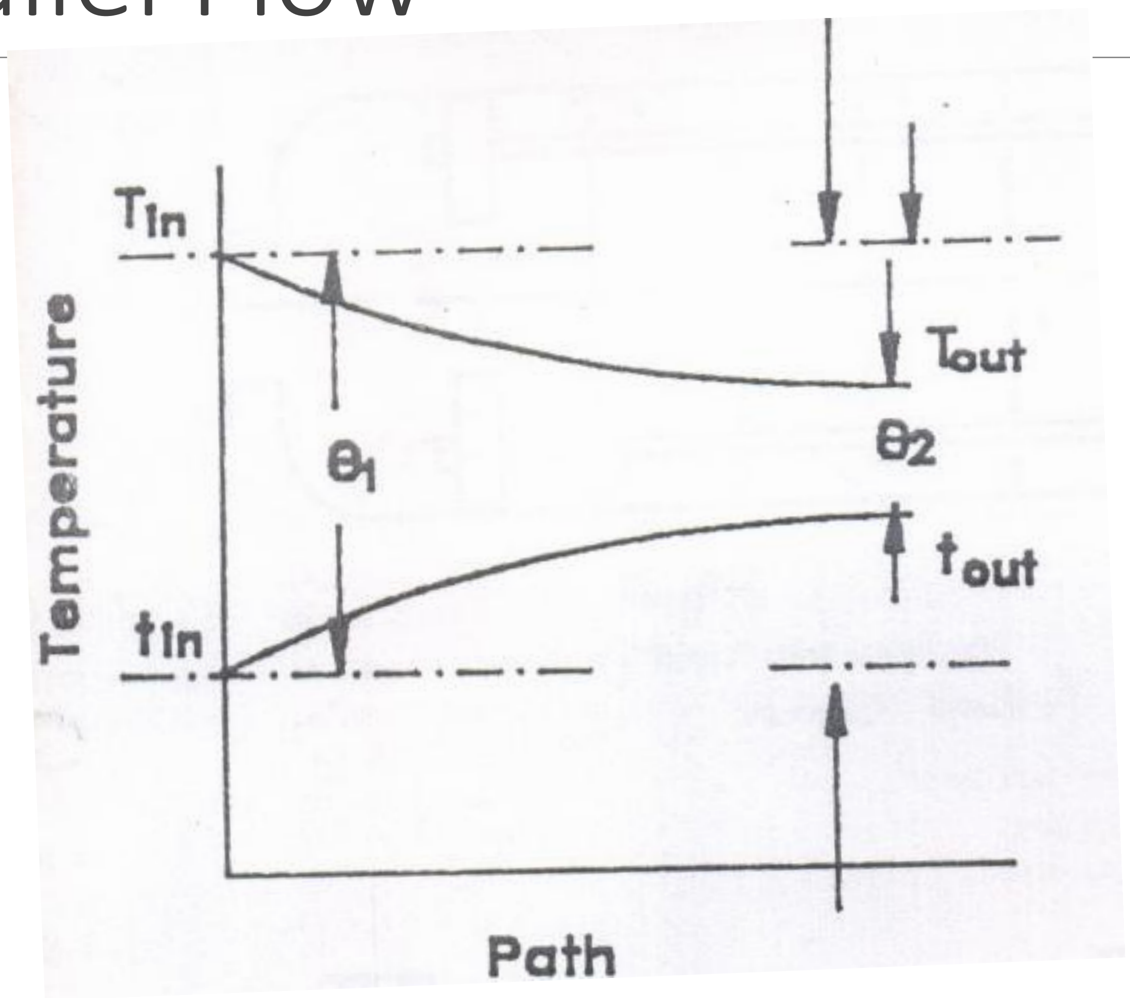
High Thermal length

Low thermal length

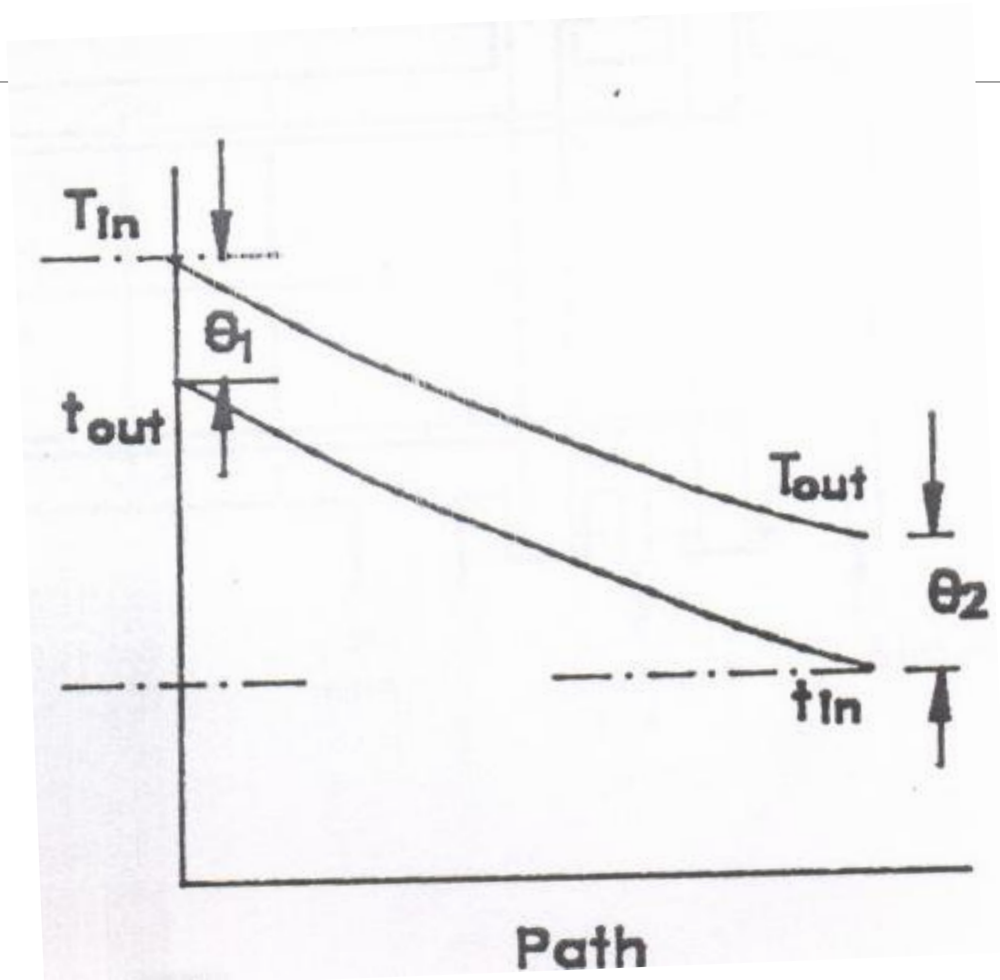




Parallel Flow



Contra-Flow

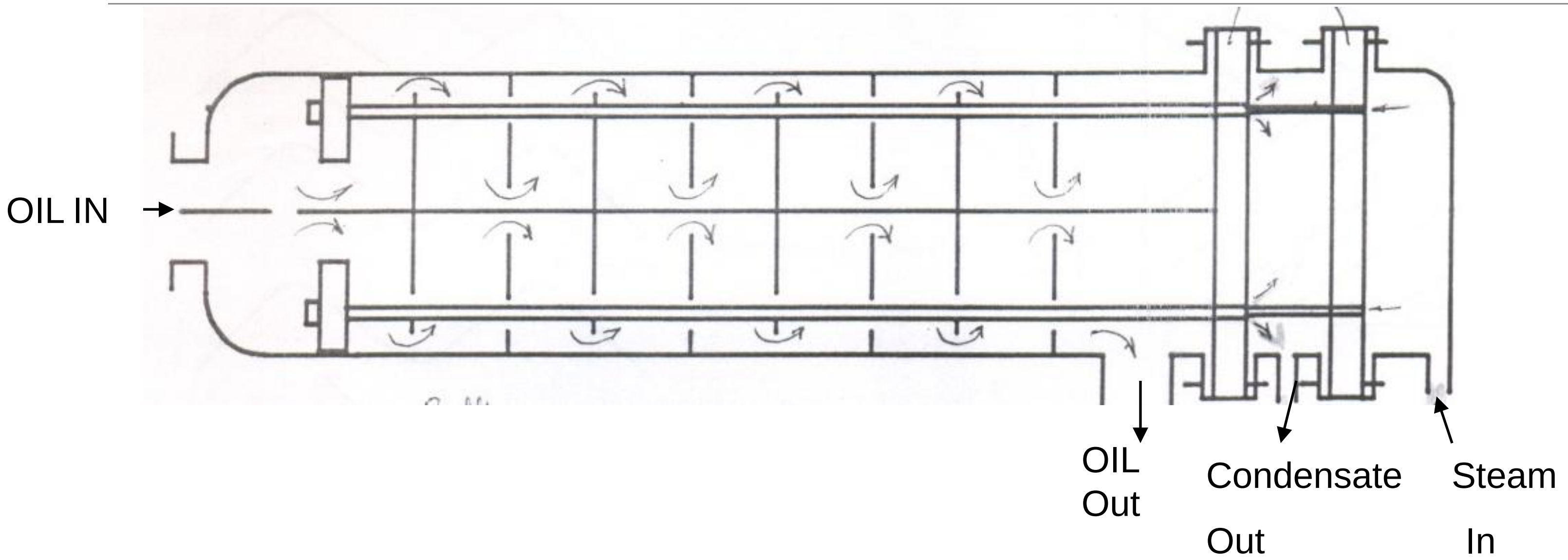


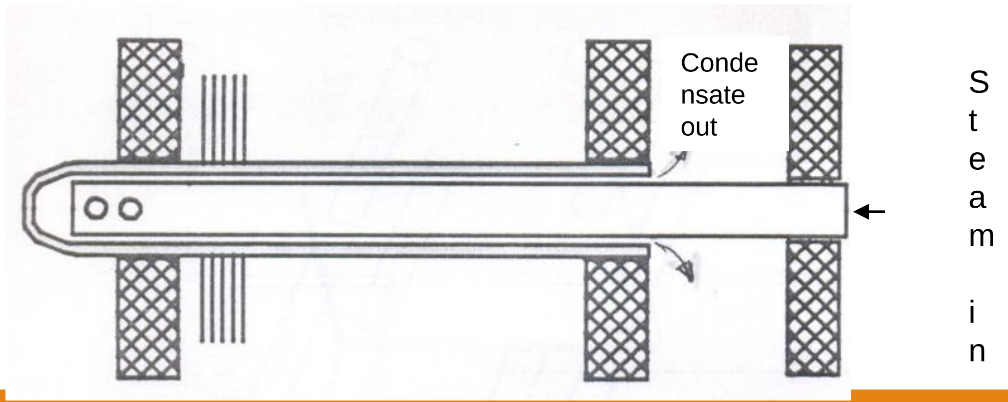
Para Flo Vs Contra Flo

In a parallel flow heating system, ‘t’ out of heated liquid< ‘T’ out of the heating liquid.

The use of a contra flo heat exchanger is usually more desirable thermodynamically as there is a reduction in area compared with parallel flo HE. With the contra flo (assuming a heating process) the final temperature of the heated fluid can be higher than the outlet temperature of heating medium. This is not possible with parallel flow. If it is necessary to restrict the temperature of the heated fluid, para flow can be chosen. This concept is used in some thermal heating fluids.

Guided Flow Fuel Oil Heater





Guided Flow Fuel 0il Heater

The guided flow heater uses a bayonet tube arrangement to

a. Limit tube wall temperature

b. Prevent tube wall distortion.

The oil flow is guided by baffle plates to ensure all surfaces are swept by oil with no dead pockets. The extended heating surface obtained by the fins, results in small volume heat exchanger.

Due to lower metal temperature in contact with oil there is less damage of oil cracking or carbonizing

Heat transfer analysis assumptions

- Heat exchanger operates under steady state condition
- Heat losses to/from the surroundings are negligible
- There are no thermal energy sources or sinks in the heat echangers are fluids as a heater, chemical reaction etc.
- Temperature of each fluid is uniform over every cross section of the counter and para flo HE. Ie. Proper transverse mixing and no gradient normal to the flo direction.
- Wall thermal resistance is disributed uniformaly in the entire heat exchanger.
- There are no phase change in the fluid streams flowing thro' the exchanger or phase change occurs at single temperature

Heat transfer analysis assumptions contd.

- Longitudinal heat conduction in the fluids and in the walls is negligible
(In heat exchangers temperature gradient exists in both fluids and in the separating wall in the fluid flow direction. This results in heat conduction in the wall and in fluids from hotter to colder regions which will affect the heat transfer rate, but generally not very critical other than in special appln., where proper allowances are made while designing the exchanger.)

- Individual and overall heat transfer coefficients are constant independent of time, temperature and position.

HE pressure drop analysis. assumptions

In any medium, fluid pumping load is proportional to the pressure drop, which is associated with fluid friction and other pressure drop contributions from the flow path of fluids.

Fluid pressure drop has direct relations with exchanger heat transfer, operation, size, mechanical characteristics, economy and other factors.

Heat transfer rate can be influenced by the saturation temperature change in the condensing /evaporating fluid if large pressure drop is associated with the flow.

Fluid pressure drop associated with the heat exchanger is the sum of the pressure drop across the core /matrix, and the distribution devices like pipe , header , manifolds etc.

Hence , ideally most of the pressure drop available shall be utilized in the core as this shall improve the uniform flow distribution thro' the core.

Pressure drop analysis. Assumptions contd.

Flow is steady and isothermal and fluid properties are independent of time.

Fluid density is dependant on local temperature only

The pressure at a point in the fluid is independent of direction

Body forces are caused by gravity only (magnetic, electric etc not to contribute)

The Bernoulli equation shall be valid only for stream line flow.

There are no energy sinks or sources in the stream line flow which can contribute or extract energy internally.

Friction is considered as constant along the length of flow

Fouling of heat exchanger

This refers to undesirable substance on the exchanger surface. Fouling causes lower heat transfer and increased pressure drop.

If liquids are used for sensible heat exchanging, fouling may substantially increase the required surface area .

For critical appln., chances of the rate of fouling dictates the design of the heat exchanger.

Strangely more heat exchangers are opened for cleaning due to excessive pressure drop than for an inability to meet the heat transfer requirement.

Condenser

A vessel in which a vapor is removed of its Latent Heat of Vaporization by cooling at constant pressure. In surface condensers steam enters at an upper level, passes over tubes in which cold water passes, condenses and falls as water to the bottom and is removed by a pump.

Construction is similar to tubular HE with sizes varying from small u - tube type double pass to large regenerative condensers for propulsion turbines.

Straight tubes can be (1)expanded into the end plates,
(2) expanded at the outlet and fitted with soft packing at the other, (3) or fitted with soft packing at both ends.
Tubes are supported at many places by support plates to prevent tubes from sagging and a baffle at the steam entrance prevent damage due to steam impingement.

Corrosion is inhibited by sacrificial anodes or impressed current system

Tube failures can be due to
a.. Impingement
b.. Corrosion/erosion due to entrapped air,
c.. Excessive water flow.
D.. Stress/corrosion crack or dezincification etc.

BASIC FUNCTION

- Remove latent heat from exhaust steam and hence allowing the distilled water to be pumped back to system, Create vacuum conditions assisting flow of exh stm. and also allowing for low saturation tempo and hence increasing recoverable heat energy from the stm.
- Deaerate

Only latent heat should be removed as this increases thermal efficiency

Even when the steam is expanded to vacuum conditions some 60% of the initial enthalpy at boiler conditions is thrown away in the condenser

Air must be removed from the condenser because;

-it dissolves in water and subsequent reasons for corrosion
-it destroys the vacuum
-poor conductor of heat and forms a thin film on pipes
-increases under cooling due to the following circumstances
The stm quantity reduces and hence it is responsible for less of the total pressure. Hence it is at a lower pressure ,has lower saturation temperature and so is under cooled with respect to the actual pressure within the

Condensers

Daltons law of partial pressure

Each constituent of a gas mix exerts a partial pressure equivalent to that, if it occupied the space alone.

Condensate falling through the lower cooler regions containing the high air content is further cooled and re absorbs gases.
Cross flow is adopted for ease of manufacture, this allied to the change of state gives a cooling efficiency approaching that of counter flow

Taking into account tube material ,

max sea water flow rate should be maintained so as to;

- maintain a sufficient steam/ coolant tempo difference across the material along the tube length
- prevent silting

Circulating system should offer no undue resistance to flow and supply water equally to all tubes.

The tube batches should be so arranged so as to provide no resistance to the flow of steam. There is normally a narrowing inlet space within or surrounding the bank so as the passage area remains constant as the steam condensers.

Failure to provide even flow leads to ;

- reduced efficiency
- pockets on non-condensable gasses being formed in the tube banks.

Allowance in the design should be made for some expanding arrangement.

PROTECTION OF CONDENSERS

Avoid low water speeds which causes silting. Too high a speed leads to erosion.

Cathodic protection for plates and tubes by using soft iron / mild steel anodes.

The effect can be increased with the use of impressed current using anodes of larger size and different material.

Alternately coating of the tubes with a 10% ferrous sulphate solution too helps.

Rubber bonding of water boxes.

Marine growth prevention

- -chlorine dosage
- -Electro chlorine generator making sodium hypochlorite (switched off when dosing with ferrous sulphate)

Erosion protection

- -Inlet of tubes streamlined to smooth flow by expanding and bell mouthing
- - the fitting of plastic ferrules
- -for aluminum-brass inserts fitted and glued

When laying up the following procedures should be carried out to prevent damage:

CONDENSER CLEANING

Before draining ensure no special chocking arrangements are necessary to prevent loading on springs or damage to the LP exhaust inlet gasket.

Water side

- General inspection before cleaning
- Place boards to protect the rubber lining
- Use water jets or balls blown by compressed air through the tubes
- Only brushes or canes as a last resort
- When plastic inserts are fitted work from the inlet end
- Test for leaks on completion
- Clean or renew the sacrificial anodes
- Remove the boards and clean vents and drains clear

Steam side

- Inspect the steam side for deposits, clean with a chemical solvent where required
- Examine the baffles, tube plates and deflectors
- Look for vibration erosion damage of the tubes
- Inspect for possible air leakage
- Box up and remove chocks

Leakage

The indications that a leak is in existence is that of high salinity measured in the condensate and boiler combined with a rapid drop in pH.

(In the earlier versions the first aid used to be the injection of sawdust followed by a shut down at the soonest possible time).

There are three methods for leak detection;

Ultrasonic-Here, electric tone speakers are fitted in the steam space, and a microphone passed down the tubes. Alternately, instead of speakers a vacuum can be drawn with the microphone picking up air leakage.

Fluorescent-The water side is cleaned and dried, chocks are fitted and the steam side filled with water containing a quantity of fluorescence. A UV lamp is then used on the water side.

Vacuum test- Draw a vacuum and cover the tube plate with plastic or use the ultrasound microphone

Maintenance

The only attention that heat exchangers require is to ensure that the heat transfer surfaces remain clean and flow passages are clear of obstruction

Electrical continuity is essential; in sea water circulating pipe work where sacrificial anodes are used.

To avoid the impingement attack care must be taken with water flow velocities thro' the tubes. For the cheap aluminum brass the upper limit is 2.5 m/s. It is equally bad to have a sea water velocity of less than 1m/s.

The practice of removing the tube stack and replace this after rotating the stack by 180' is followed for long time for a steady performance of the condenser.

While installing the shell and tube heat exchangers, clearance space is essential for the withdrawal of the stack

Regenerative Condenser

As it expands thro' the turbine maximum available useful work is extracted from the steam by maintaining vacuum condition in the condenser. Part of the duty of the condenser is to condense the steam from the low pressure end of the turbine at as low a pressure as possible. For effective operation sea water need be cooler than the saturation temperature of the steam and this means that there will be under cooling with waste of energy. To avoid this, part of the steam is admitted at the lower part of the condenser where the condensing water is met with and part of the heat transferred to the condensate dripping from the top there by imparting negligible under cooling of the water.

Under cooling can also affect the oxygen presence in the condensate water . Theoretically , if water droplet is at saturation temperature then no dissolved gasses shall be present in the water.

Charge air coolers

- The charge air coolers fitted to reduce the air temperature after the turbocharger but before entering the engine cylinder, are provided with fins on the heat transfer surfaces to compensate for the relatively poor heat transfer properties of the medium. Solid drawn tubes with semi flattened cross section is favored. These are threaded thro' the thin copper fin plates and bonded to them with solder for maximum heat transfer. The tube ends are fixed into the tube plate by expanding and then soldering.
  Cooling of air results in precipitation of moisture which is removed by water eliminators fitted at the air side. A change of direction in the flow of air is used in some charge air coolers to assist water removal.

Sp heat capacity of some metals

Metal	Specific Heat	Thermal Conductivity	Density	Electrical Conductivity
	c_p cal/g° C	k watt/cm K	g/cm^3	1E6/Ωm
Brass	0.09	1.09	8.5	
Iron	0.11	0.803	7.87	11.2
Nickel	0.106	0.905	8.9	14.6
Copper	0.093	3.98	8.95	60.7
Aluminum	0.217	2.37	2.7	37.7
Lead	0.0305	0.352	11.2	

alpha-beta brass - a brass that has more zinc and is stronger than alpha brass; used in making castings and hot-worked products

Reynolds Number

The Reynolds number for a flow through a pipe is defined as (1) the ratio of inertial forces ($vs\rho$) to viscous forces (μ/L)

It is also used to identify and predict different flow regimes, such as laminar or turbulent flow. Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion, while turbulent flow, on the other hand, occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce random eddies, vortices and other flow fluctuations.

Typically it is given as Re= Inertial Forces/Viscous Forces

At larger Reynolds numbers, flow becomes turbulent.

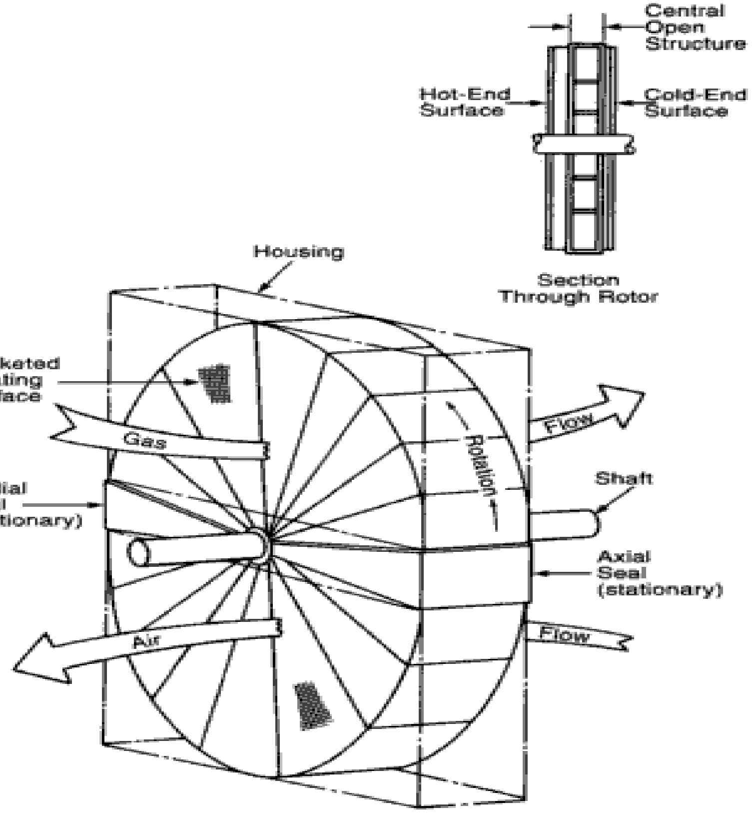

Central
Open
Structure
Hot-End
Surface
Cold-End
Surface
Housing
Section
Through Rotor
Basketed
Heating
Surface
Gas
Flow
Rotation
Radial
Seal
(stationary)
Shaft
Axial
Seal
(stationary)
Air
Flow

FIGURE 1.4