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AUTOMATA AND COMPILER DESIGN

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UNIT-1

Formal Language and Regular Expressions: Languages, Definition Language regular expressions, Finite Automata-DFA, NFA. Conversion of regular expression to NFA, NFA to DFA, Applications of Finite Automata to lexical analysis, lex tools.

Context Free grammars and parsing: Context free grammars, derivation, parse trees, ambiguity LL (K) grammars and LL (1) parsing.

COMPILER

A compiler is a program that reads a program in one language, the source language and

Translates into an equivalent program in another language, the target language.

The translation process should also report the presence of errors in the source program

COMPILER

Source

Program → Compiler → Target

Program

↓

Error

Messages

There are two parts of compilation.

.

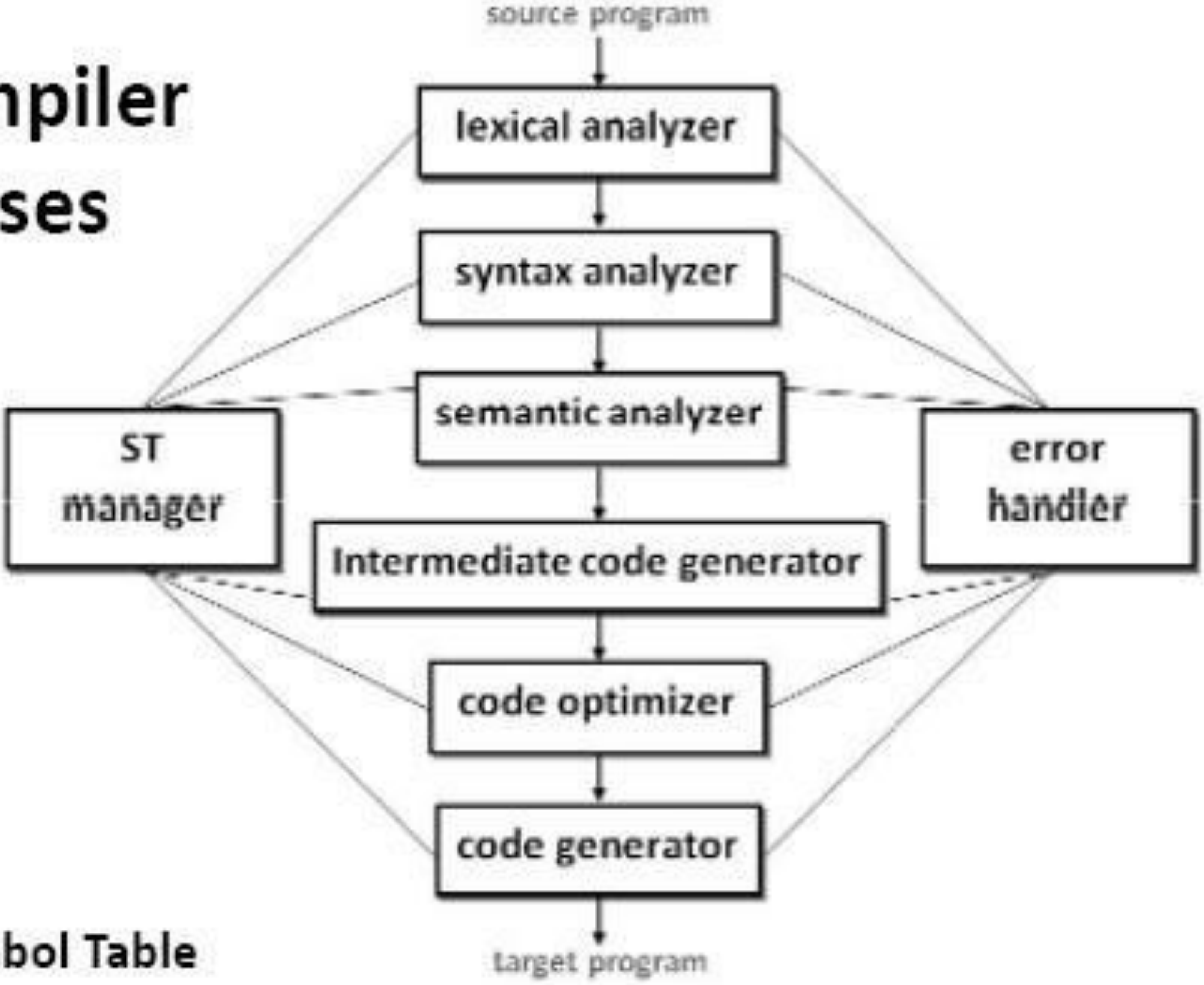
The analysis part breaks up the source program into constant piece and creates an

intermediate representation of the source program.

The synthesis part constructs the desired target program from the intermediate representation.

Phases of Compiler

Compiler Phases



ST = Symbol Table

Lexical analyzer

- **Lexical Analyzer reads the source program character by character and returns the**
- ***tokens of the source program.***
- **• A *token describes a pattern of characters having same meaning in the source***
- **program. (such as identifiers, operators, keywords, numbers, delimeters and so**
- **on)**
- **Ex: newval := oldval + 12 => tokens: newval identifier**
- **:= assignment operator**
- **oldval identifier**

Lexical analyzer

+

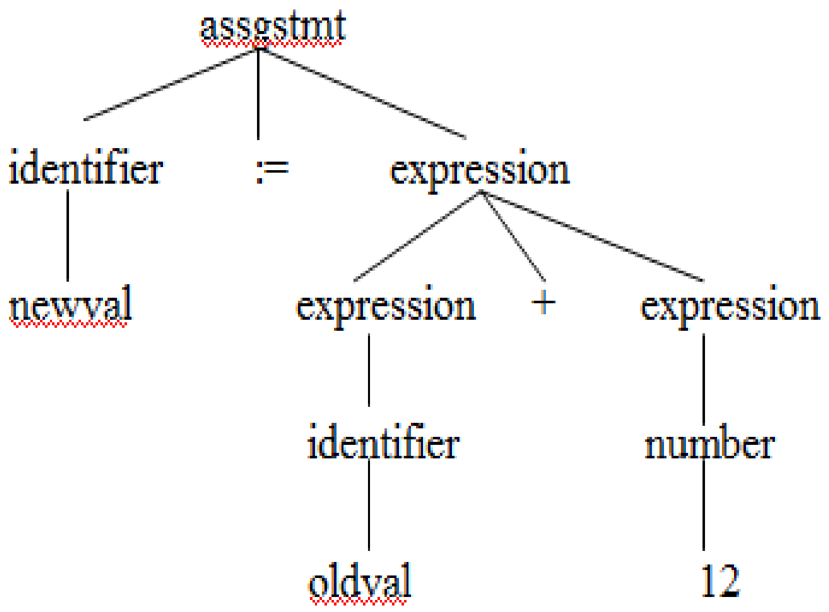
add operator

12 a number

- Puts information about identifiers into the symbol table.
- • Regular expressions are used to describe tokens (lexical constructs).

Syntax analyzer

- A **Syntax Analyzer** creates the syntactic structure (generally a parse tree) of the given program.
- A syntax analyzer is also called as a **parser**.
- A **parse tree** describes a syntactic structure.



- In a parse tree, all terminals are at leaves.
- All inner nodes are non-terminals in a context free grammar.

Syntax analyzer.

The syntax of a language is specified by a context free grammar (CFG).

- The rules in a CFG are mostly recursive.
- A syntax analyzer checks whether a given program satisfies the rules implied by a CFG or not.

If it satisfies, the syntax analyzer creates a parse tree for the given program.

Syntax analyzer

Ex:

We use BNF (Backus Naur Form) to specify a CFG

assgstmt -> identifier := expression

expression -> identifier

expression -> number

expression -> expression + expression

Semantic analyzer

- A semantic analyzer checks the source program for semantic errors and collects
- the type information for the code generation.
- Type-checking is an important part of semantic analyzer.
- Normally semantic information cannot be represented by a context-free language
- used in syntax analyzers.
- Context-free grammars used in the syntax analysis are integrated with attributes
- (semantic rules)
- the result is a syntax-directed translation,
- Attribute grammars

Semantic analyzer

Ex:

newval := oldval + 12

The type of the identifier *newval must match with type of the* expression *(oldval+12)*

Intermediate code generation

A compiler may produce an explicit intermediate codes representing the source program.

These intermediate codes are generally machine (architecture independent). But the level of intermediate codes is close to the level of machine codes.

Intermediate code generation

Ex:

newval := oldval * fact + 1

id1 := id2 * id3 + 1

MULT id2,id3,temp1 ***Intermediates Codes (Quadraples)***

ADD temp1,#1,temp2

MOV temp2,,id1

Code optimizer

- The code optimizer optimizes the code produced by the intermediate code generator in the terms of time and space.
- Ex:
- MULT id2,id3,temp1
- ADD temp1,#1,id1

Code generator

Produces the target language in a specific architecture.

The target program is normally is a relocatable object file containing the machine codes.

Ex:

(assume that we have an architecture with instructions whose at least one of its operands is a machine register)

Code generator

```
 MOVEid2,R1
MULT id3,R1
ADD #1,R1
MOVER1,id1
```

Compiler construction tools

- A number of tools have been developed variously called *compiler –compiler , compiler generator or translator writing system*
- The input for these systems may contain
- 1. a description of source language.
- 2. a description of what output to be
- generated.
- 3. a description of the target machine.

Compiler construction tools

The principal aids provided by compiler-compiler are

1. Scanner Generator
2. Parser generator
3. Facilities for code generation

Lexical Analyzer

- **The Role of the Lexical Analyzer**

  1st phase of compiler

  Reads i/p character & produce o/p sequence of tokens that the Parser uses for syntax analysis

  It can either work as a separate module or as sub module

Tokens , Patterns and Lexemes

- **Lexeme: Sequence of character in the source pm that is matched against the pattern for a token**

- **Pattern: The rule associated with each set of strings is called pattern.**

- **Lexeme is matched against pattern to generate token**

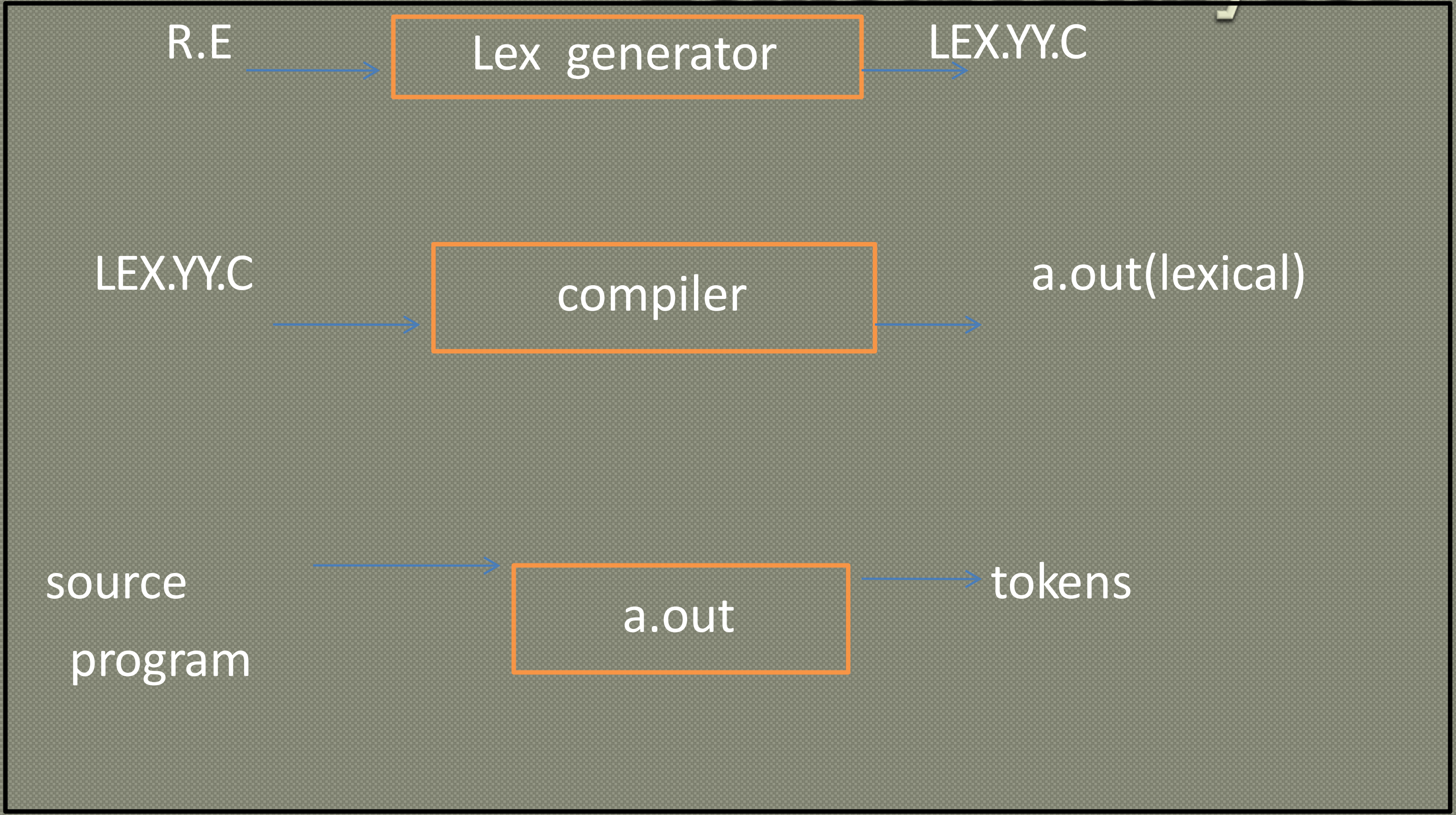
- **Token: Token is word, which describes the lexeme in source pgm. It is generated when lexeme is matched against pattern**

3 Methods of constructing Lexical Analyzer

.

- 1. Using Lexical Code Generator
- Such compilers/ tools are available that takes in Regular Expressions
- As i/p and generate code for that R.E.These tools can be used to generate Lexical Analyzer code from R.Es
- - Example of such Tool is LEX for Unix .

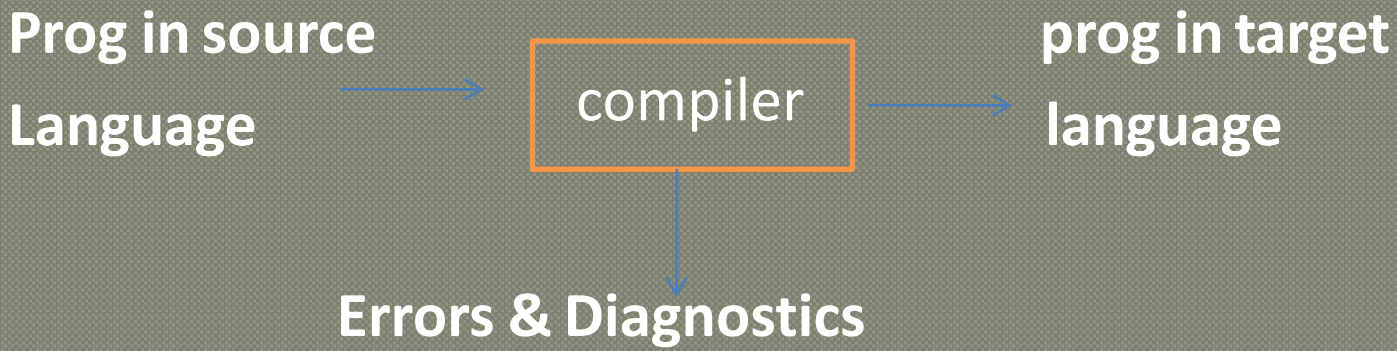
3 Methods of constructing Lexical Analyzer



Compiler and Translator

- Compiler is a form of translator that translate a program written in one language “
- The Source Language” to an equivalent program in a second language “ The Target
- language ” or “ The Object Language “

Compiler and Translator



Prog in source Language

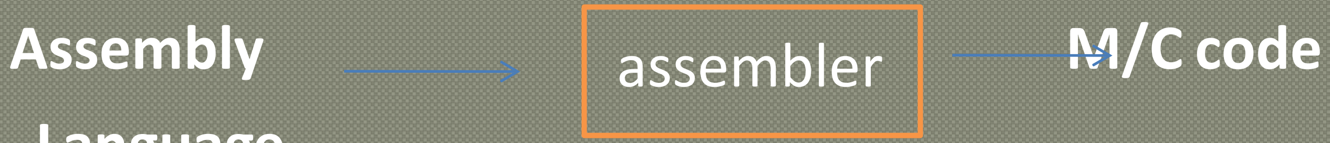
compiler

prog in target language

Errors & Diagnostics

Assemblers, Compilers and Interpreters are all specific translators

Assemblers



Assembly

assembler

M/C code

- Language
- (opcodes or mnemonics)
- <u>Interpreters</u>
- - Interpret the statements of the High level Language pgm as they are encountered .
- Produce o/p of statement as they are interpreted   .

Languages involved in Compiler

3 languages are involved in a compiler

- 1. Source language: Read by compiler
- 2. Target or Object language : translated by compiler /translator to another language
- 4. Host Language: Language in which compiler is written

Advantages of Compiler

- Conciseness which improves programmer productivity,
- semantic restriction
- Translate and analyze H.L.L.(source pgm) only once and then
- run the equivalent m/c code produce by compiler

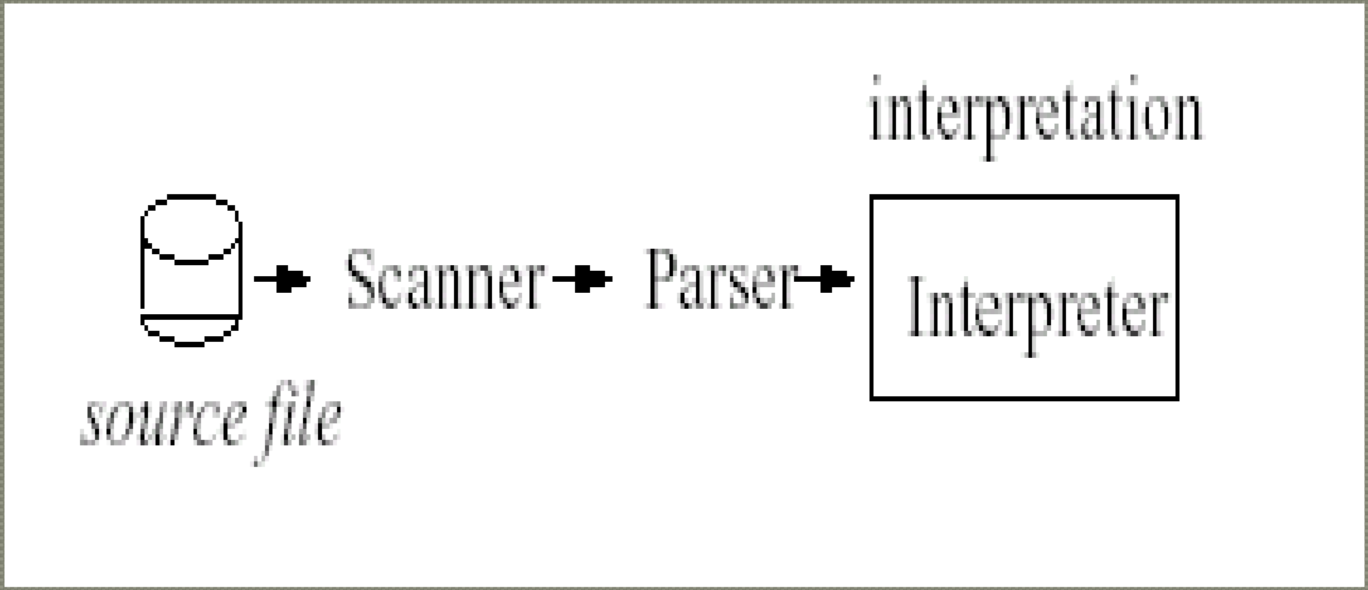
Disadvantage of Compiler

- Slower speed
- Size of compiler and compiled code
- Debugging involves all source code

Interpreter versus Compiler

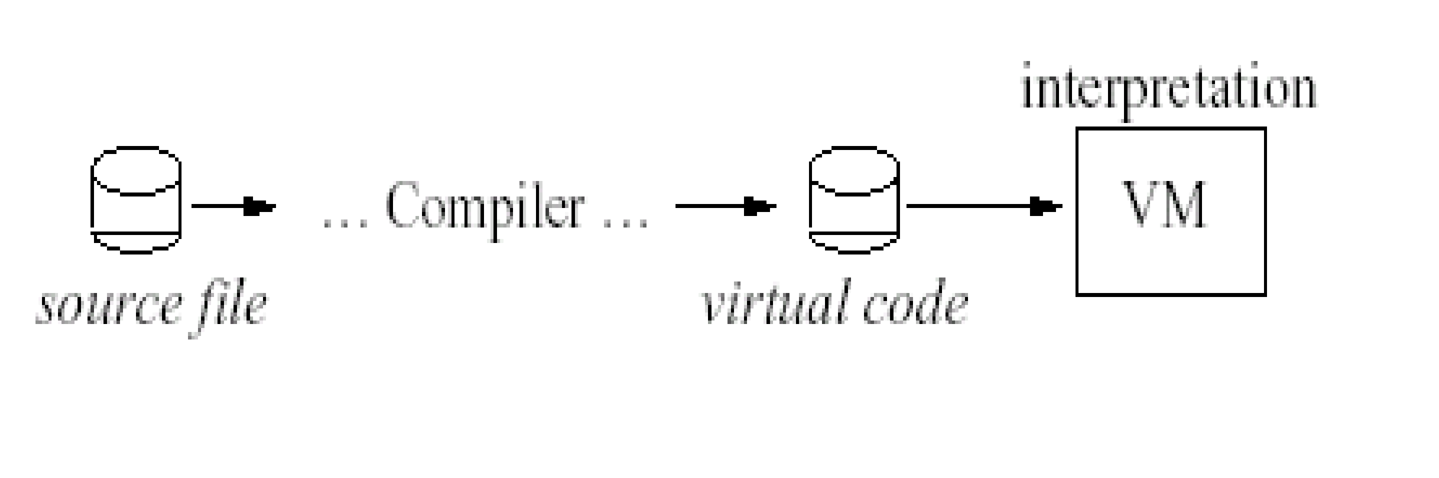
Compiler translates to machine code

Interpreter



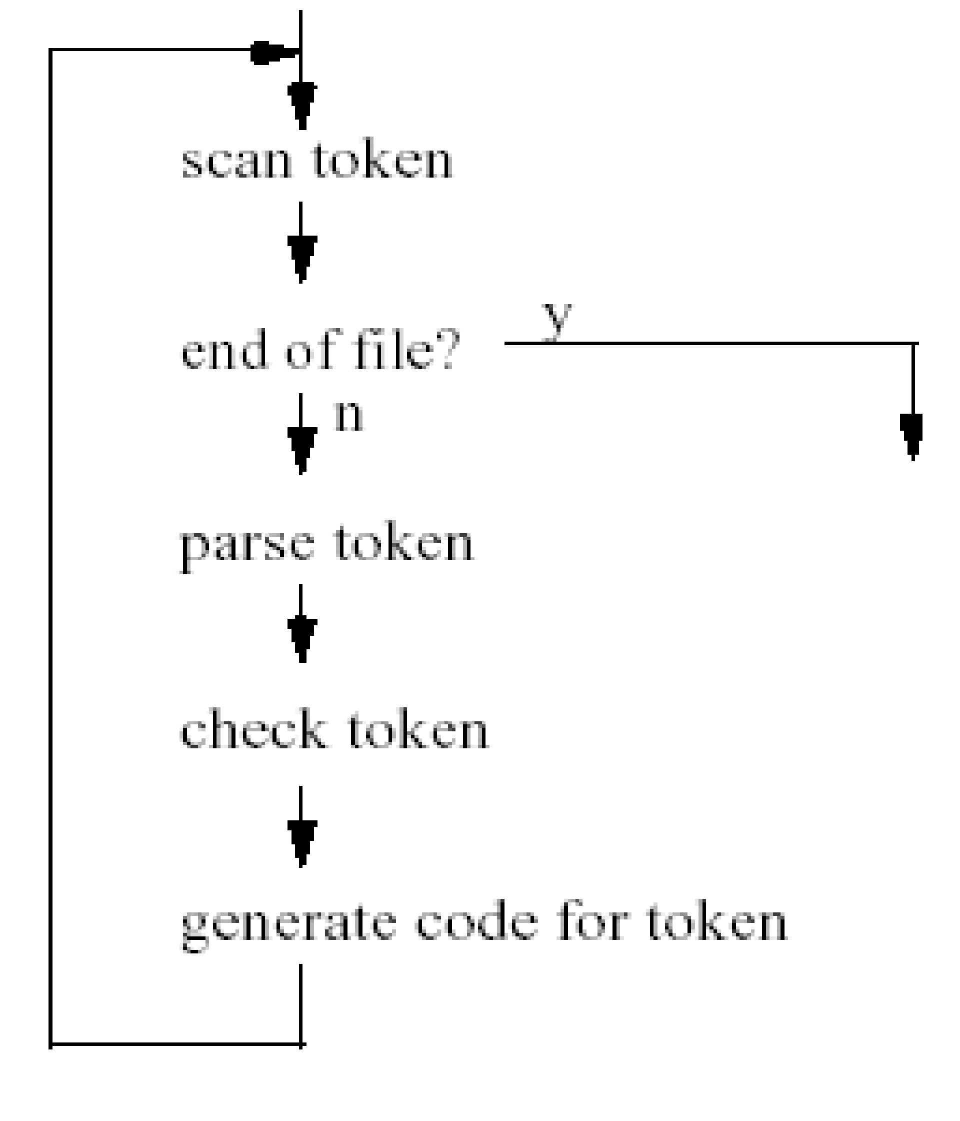
Variant: Interpretation of intermediate code

Compiler generates code for a "virtual machine" (VM)
VM is simulated by software

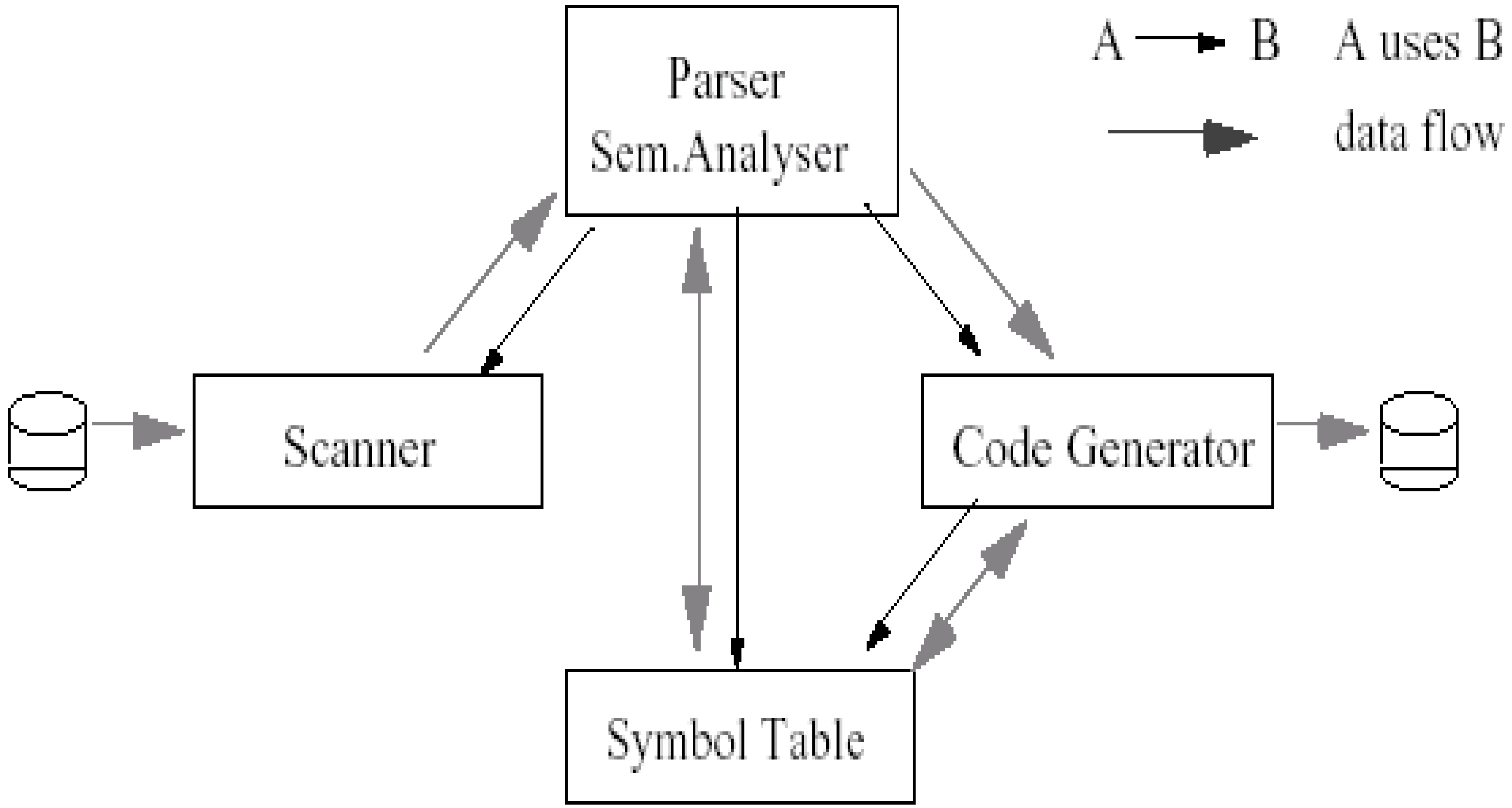


Single-Pass Compilers

Phases work in an interleaved way

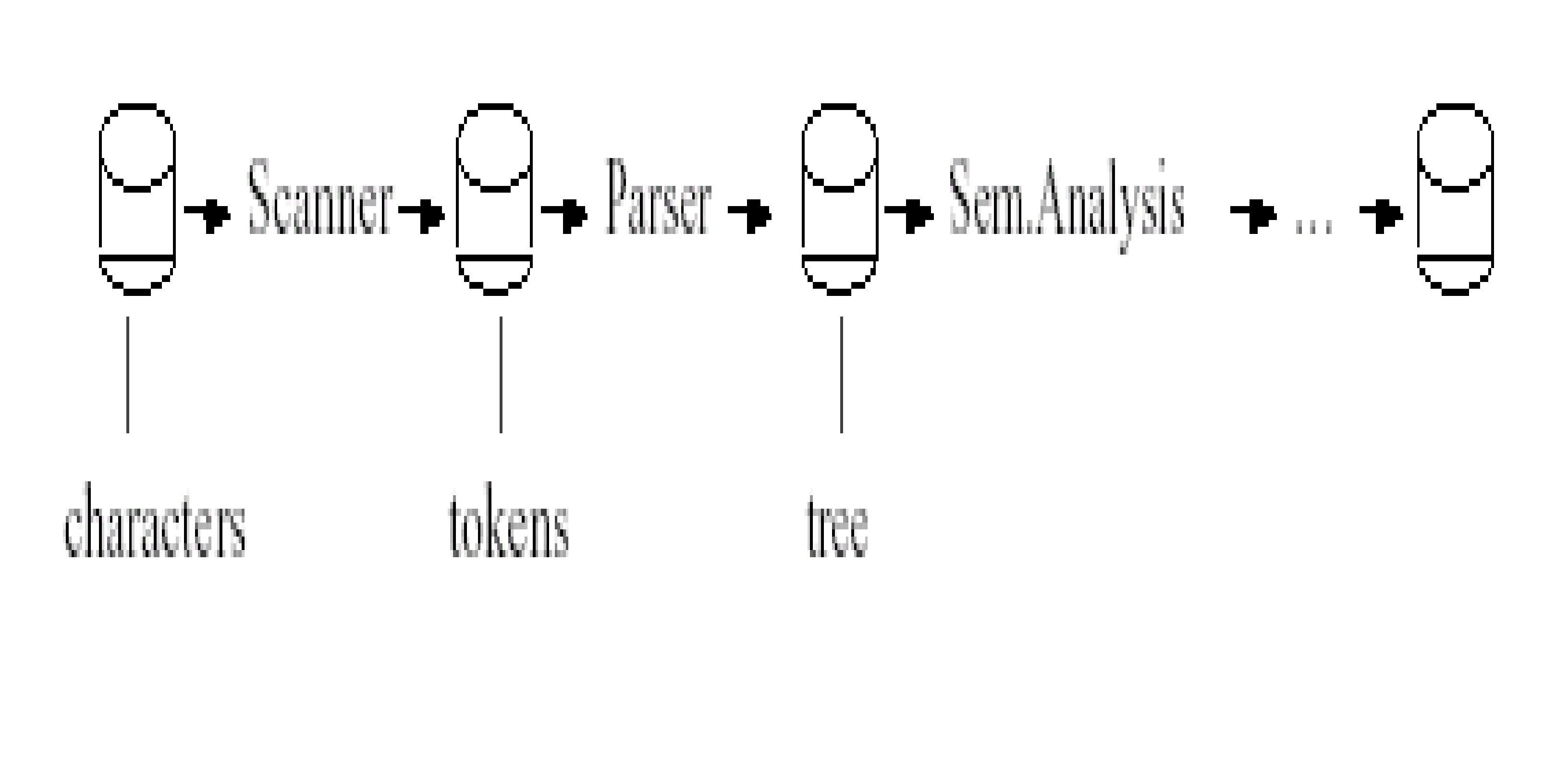


Static Structure of a (Single-Pass) Compiler



Multi-Pass Compilers

Phases are separate "Programs", which run sequentially



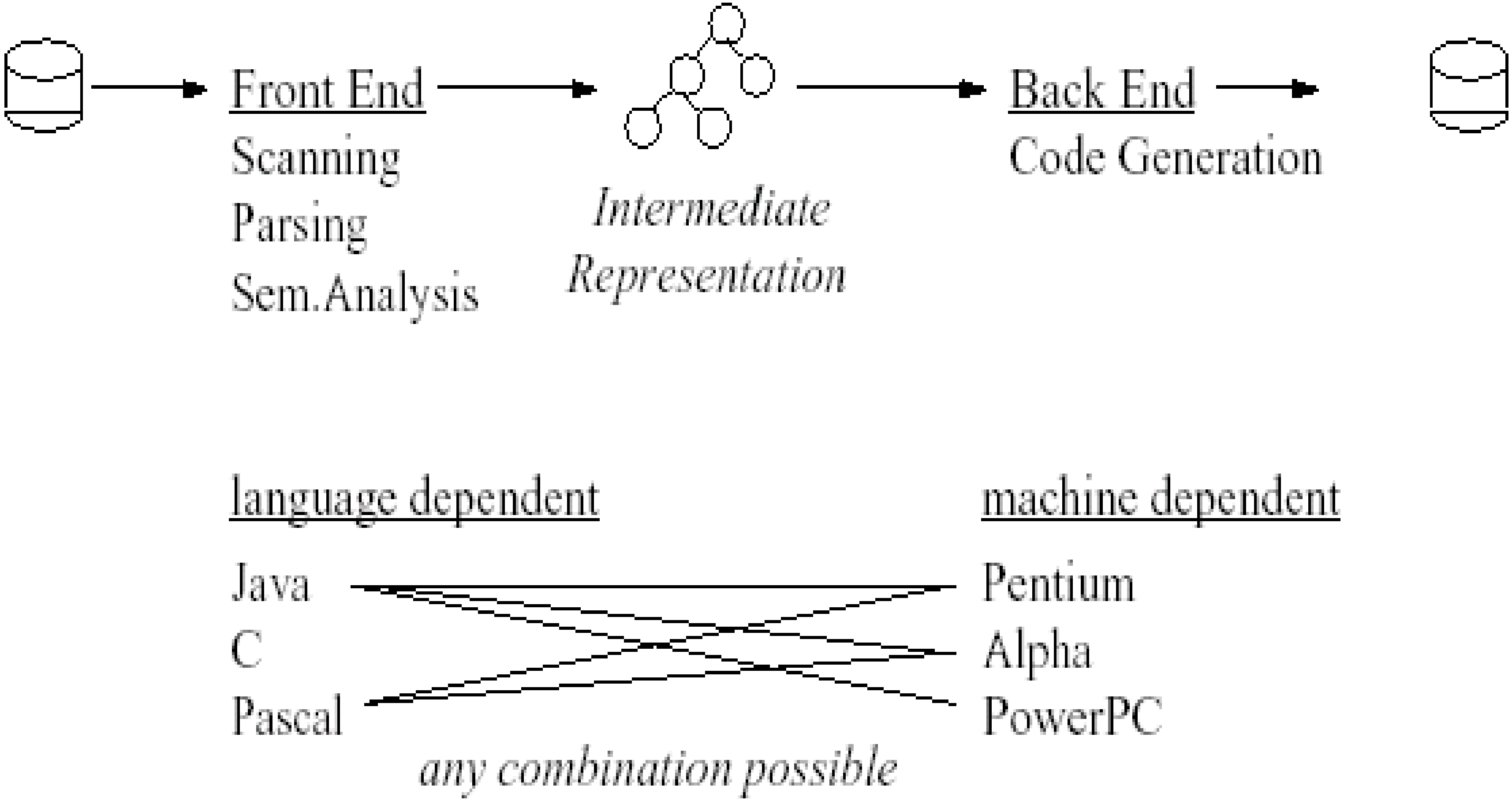
Why multi-pass?

If memory is scarce (irrelevant today)

If the language is complex

If portability is important

Often Two-Pass Compilers



Front End

Scanning

Parsing

Sem.Analysis

Intermediate Representation

Back End

Code Generation

language dependent

machine dependent

Java

C

Pascal

Pentium

Alpha

PowerPC

any combination possible

Loader/Linker Editor:

Performs 2 functions

- i. Loading : relocatable code is converted to absolute code

  i.e. placed at their specified position

  ii. Link-Editor :

  Make single pgm from several files of relocatable

  m/c code.

  The file may be o/p of different compilation .

  May be Library files or routine provided by system .

  Result is executable file

Preprocessor

Produce i/p to compiler. They may perform

i. Macro processing: shorthand for longer construct .

ii. File inclusion: separate module can be used by including their file e.g #include <iostream> .

iii. Rational preprocessors: give support for additional facilities which are not included in compiler itself .

iv. Language Extension: may include extra capabilities .

Major Types of Compiler

- **1. Self-resident Compiler: generates Target code for the same m/c   or   host .**
- **2. Cross Compilers: generates target code for m/c other then host .**

Phases and passes

- In logical terms a compiler is thought of as consisting of stages and phases
- Physically it is made up of passes
- The compiler has one pass for each time the source code, or a representation of
- it, is read
- Many compilers have just a single pass so that the complete compilation
- process is performed while the code is read once

Phases and passes

- The various phases described will therefore be executed in parallel
- Earlier compilers had a large number of passes, typically due to the limited
- memory space available
- Modern compilers are single pass since memory space is not usually a problem

Use of tools

- The 2 main types of tools used in compiler production are:
- 1. a lexical analyzer generator
- Takes as input the lexical structure of a language, which defines how its
- tokens are made up from characters
- Produces as output a lexical analyzer (a program in C for example) for the
- language
- Unix lexical analyzer Lex
- 2. a symbol analyzer generator

Use of tools

- Takes as input the syntactical definition of a language
- Produces as output a syntax analyzer (a program in C for example) for the
- language
- The most widely know is the Unix-based YACC (Yet Another
- Compiler-Compiler), used in conjunction with Lex.
- *Bison: public domain version*

Applications of compiler techniques

- Compiler technology is useful for a more general class of applications
- Many programs share the basic properties of compilers: they read textual input,
- organize it into a hierarchical structure and then process the structure
- An understanding how programming language compilers are designed and
- organized can make it easier to implement these compiler like applications as
- well
- More importantly, tools designed for compiler writing such as lexical analyzer

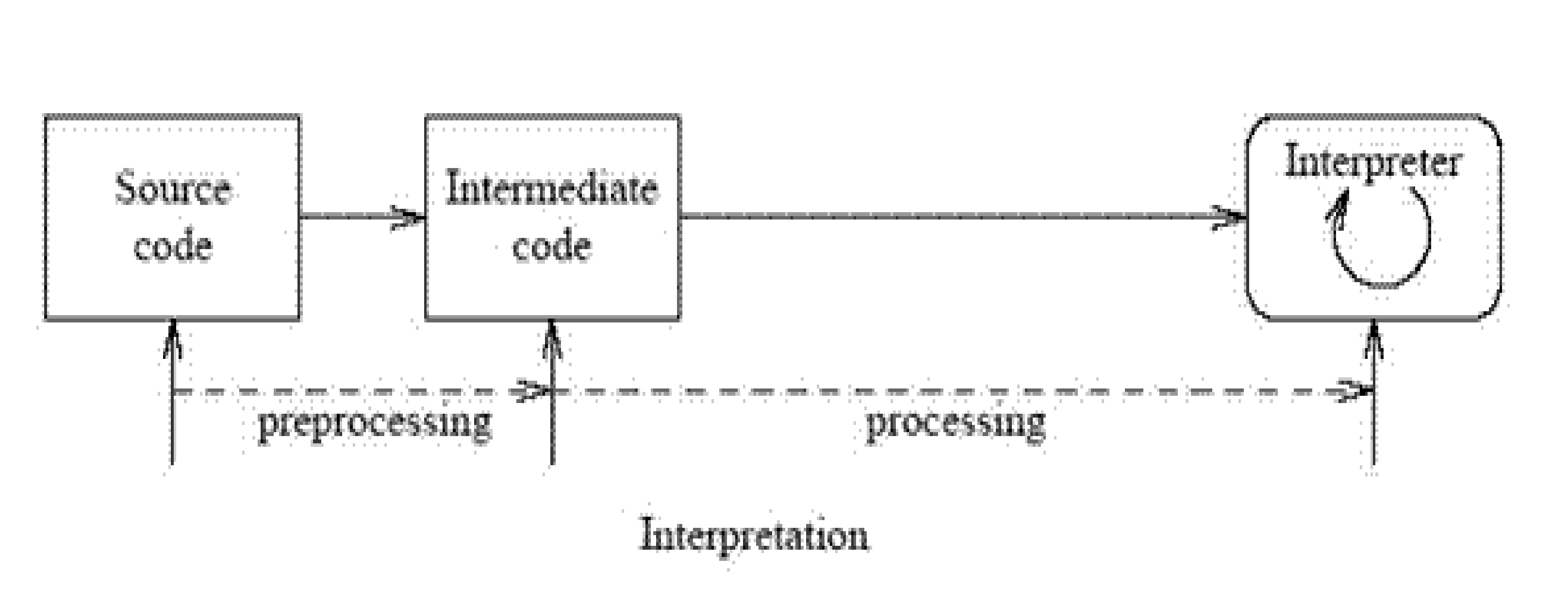
Applications of compiler techniqu

- generators and parser generators can make it vastly easier to implement such
- applications
- Thus, compiler techniques - An important knowledge for computer science
- engineers

- Examples:
- Document processing: Latex, HTML, XML
- User interfaces: interactive applications, file systems, databases
- Natural language treatment
- Automata Theory, La

Interpreters

- **Interpreters: Instead of producing a target program as a translation, an interpreter**
- **performs the operations implied by the source program**



Differences between compiler and Interpreter

no	COMPILER	INTERPRETER
1	It takes entire program as input	It takes single instruction as input
2	Intermediate object code is generated	No intermediate object code is generated
3	Conditional control statements are executes faster	Conditional control statements are executes slower
4	Memory requirement is more	Memory requirement is less
5	Program need not be compiled every time	Every time higher level program is converted into lower level program
6	Errors are displayed after entire program is checked	Errors are displayed for every instruction interpreted (if any)
7	Ex: C Compiler	Ex : Basic

Types of compilers

1.Incremental compiler :

It is a compiler it performs the recompilation of only modified source rather than compiling the whole source program.

Features:

1. It tracks the dependencies between output and source program.

2.It produces the same result as full recompilation.

3.It performs less task than recompilation.

4. The process of incremental compilation is effective for maintenance.

Types of compilers

2.Cross compiler:

Basically there exists 3 languages

1.source language i.e application program.

2.Target language in which machine code is return.

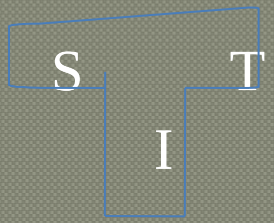
3.Implementation language in which a compiler is return.

All these 3 languages are different. In other words there may be a compiler which run on one machine and produces the target code for another machine. Such a compiler is called cross compiler.

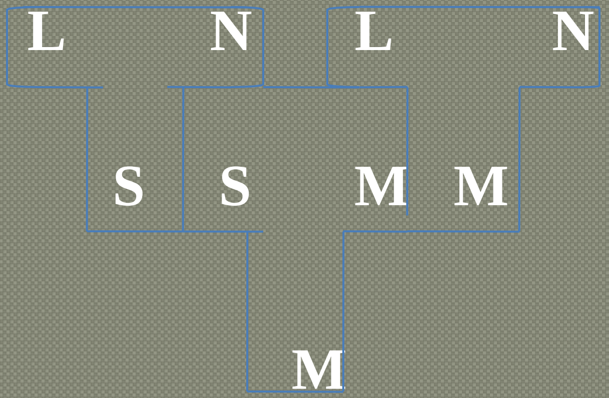
Types of compilers

To represent cross compiler T diagram is drawn as follows.

I



- **<u>CROSS COMPILER:</u>For source language L the target language N get generated which runs on machine M.**



Bootstrapping Compilers and T-diagrams

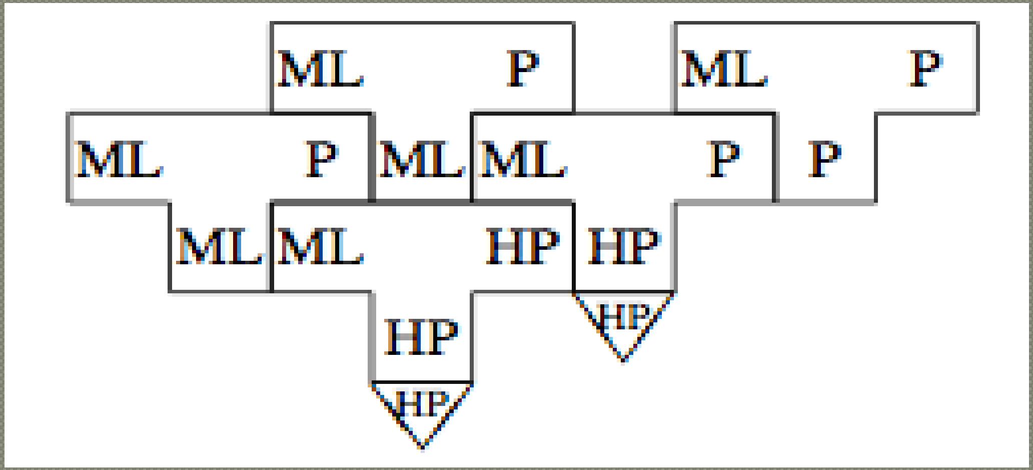
- The rules for T-diagrams are very simple. A compiler written in some language “C” (could be anything from machine code on up) that translates programs in language A to language B looks like this (these diagrams are from

Bootstrapping Compilers and T-diagrams

- **Now suppose you have a machine that can directly run HP machine code, and a compiler from ML to HP machine code, and you want to get a ML compiler running on different machine code P. You can start by writing an ML-to-P compiler in ML, and compile that to get an ML-to-P compiler in HP:**

Bootstrapping Compilers and T-diagrams

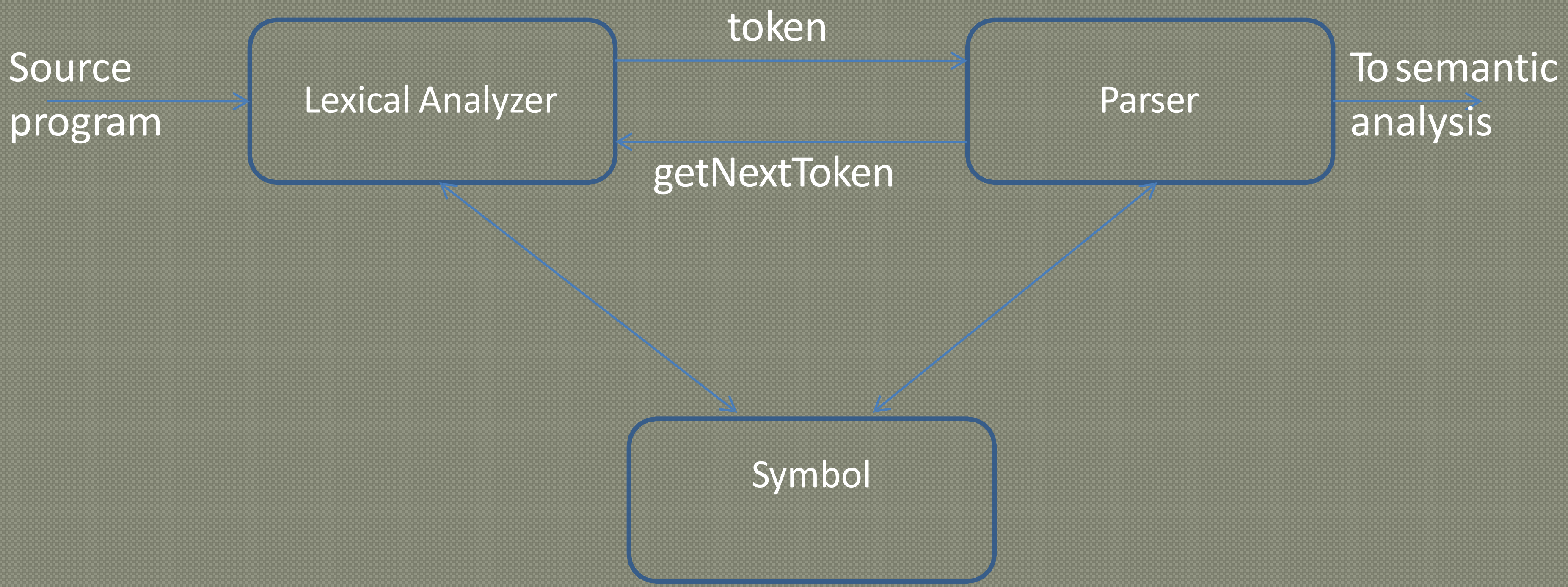
From there, feed the new ML-to-P compiler to itself, running on the HP machine, and you end up with an ML-to-P compiler that runs on a P machine!



Lexical Analysis

- lexical analyser or scanner is a program that groups sequences of characters into lexemes, and outputs (to the syntax analyser) a sequence of tokens. Here:
- (a) Tokens are symbolic names for the entities that make up the text of the program;
- e.g.
- If for the keyword if , and id
- for any identifier. These make up the output of
- the lexical analyser

The role of lexical analyzer



Source program

Lexical Analyzer

token

getNextToken

Parser

To semantic analysis

Symbol

Tokens, Patterns and Lexemes

- A token is a pair a token name and an optional token value
- A pattern is a description of the form that the lexemes of a token may take
- A lexeme is a sequence of characters in the source program that matches the pattern for a token

Example

Token	Informal description	Sample lexemes
if	Characters i, f	if
else	Characters e, l, s, e	else
comparison	< or > or <= or >= or == or !=	<=, !=
id	Letter followed by letter and digits	pi, score, D2
number	Any numeric constant	3.14159, 0, 6.02e23
Literal	Anything but “ sorrounded by “	“core dumped”

```
printf(“total = %d\n”, score);
```

Attributes for tokens

- E = M * C ** 2
  - <id, pointer to symbol table entry for E>
  - <assign-op>
  - <id, pointer to symbol table entry for M>
  - <mult-op>
  - <id, pointer to symbol table entry for C>
  - <exp-op>
  - <number, integer value 2>

Lexical errors

- Some errors are out of power of lexical analyzer to recognize:
  - fi (a == f(x)) …
- However it may be able to recognize errors like:
  - d = 2r
- Such errors are recognized when no pattern for tokens matches a character sequence

Specification of tokens

- In theory of compilation regular expressions are used to formalize the specification of tokens
- Regular expressions are means for specifying regular languages
- Example:
  - Letter_(letter_ | digit)*
- Each regular expression is a pattern specifying the form of strings

Regular expressions

- Ɛ is a regular expression, L(Ɛ) = {Ɛ}
- If a is a symbol in Σ then a is a regular expression, L(a) = {a}
- (r) | (s) is a regular expression denoting the language L(r) ∪ L(s)
- (r)(s) is a regular expression denoting the language L(r)L(s)
- (r)* is a regular expression denoting (L9r))*
- (r) is a regular expression denting L(r)

Regular definitions

d1 -> r1

d2 -> r2

…

dn -> rn

- Example:

  letter_ -> A | B | … | Z | a | b | … | Z | _

  digit -> 0 | 1 | … | 9

  id -> letter_ (letter_ | digit)*

Extensions

- One or more instances: (r)+
- Zero of one instances: r?
- Character classes: [abc]

- Example:
  - letter_ -> [A-Za-z_]
  - digit -> [0-9]
  - id -> letter_(letter|digit)*

Recognition of tokens

- **Starting point is the language grammar to understand the tokens:**

```
stmt -> if expr then stmt
        |  if expr then stmt else stmt
        | ε
expr -> term relop term
        |  term
term -> id
        |  number
```

Recognition of tokens (cont.)

- The next step is to formalize the patterns:

  digit -> [0-9]

  Digits -> digit+

  number -> digit(.digits)? (E[+-]? Digit)?

  letter -> [A-Za-z_]

  id -> letter (letter|digit)*

  If -> if

  Then -> then

  Else -> else

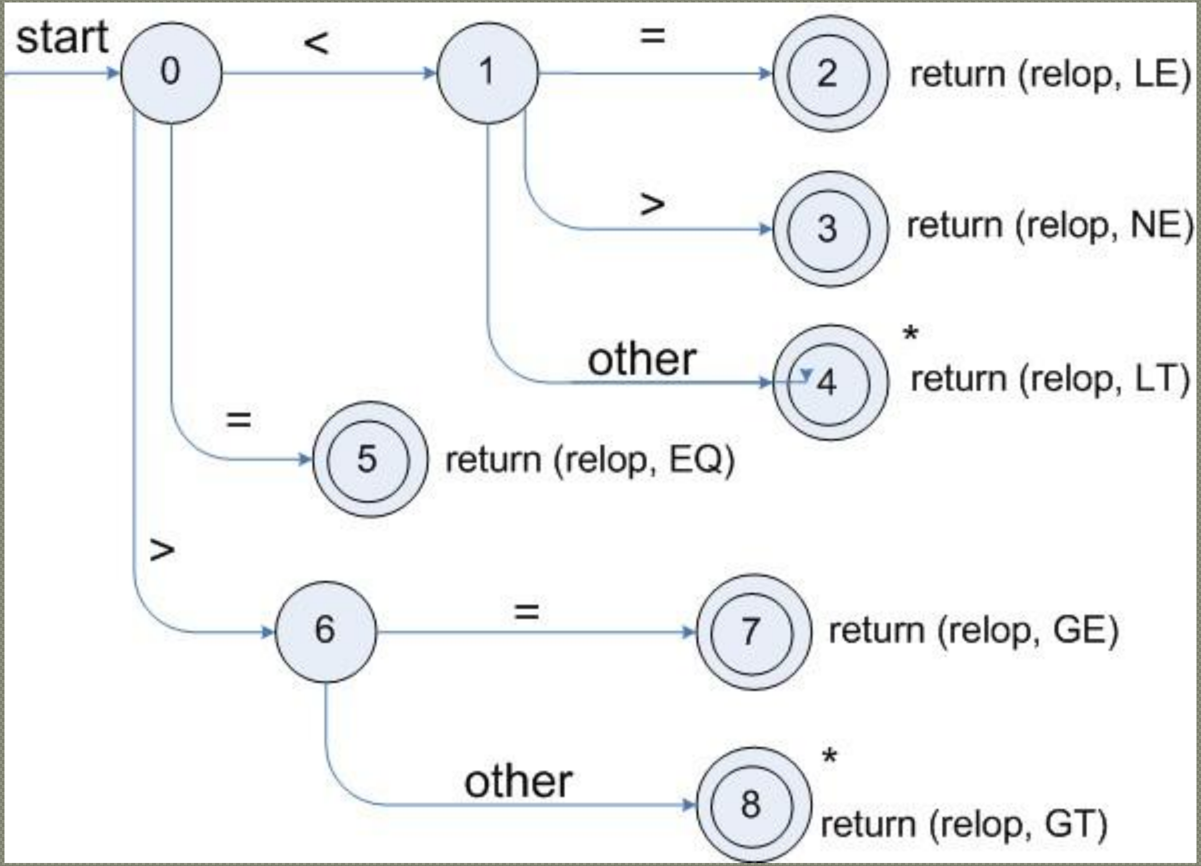
  Relop -> < | > | <= | >= | = | <>

- We also need to handle whitespaces:

  ws -> (blank | tab | newline)+

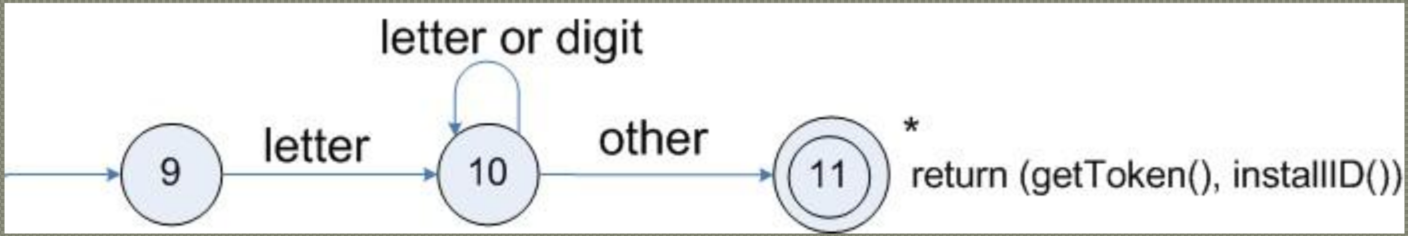
Transition diagrams

- **Transition diagram for relop**



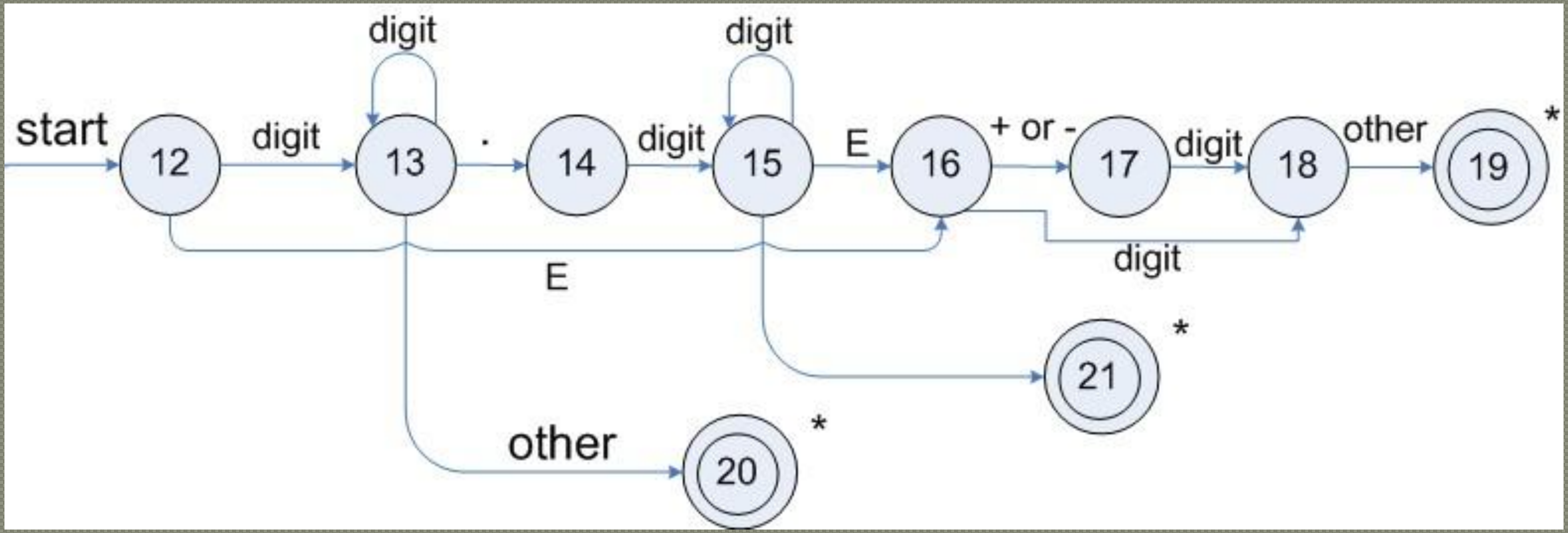
Transition diagrams (cont.)

- Transition diagram for reserved words and identifiers



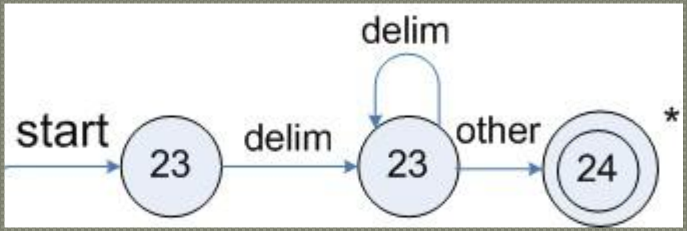
Transition diagrams (cont.)

- **Transition diagram for unsigned numbers**

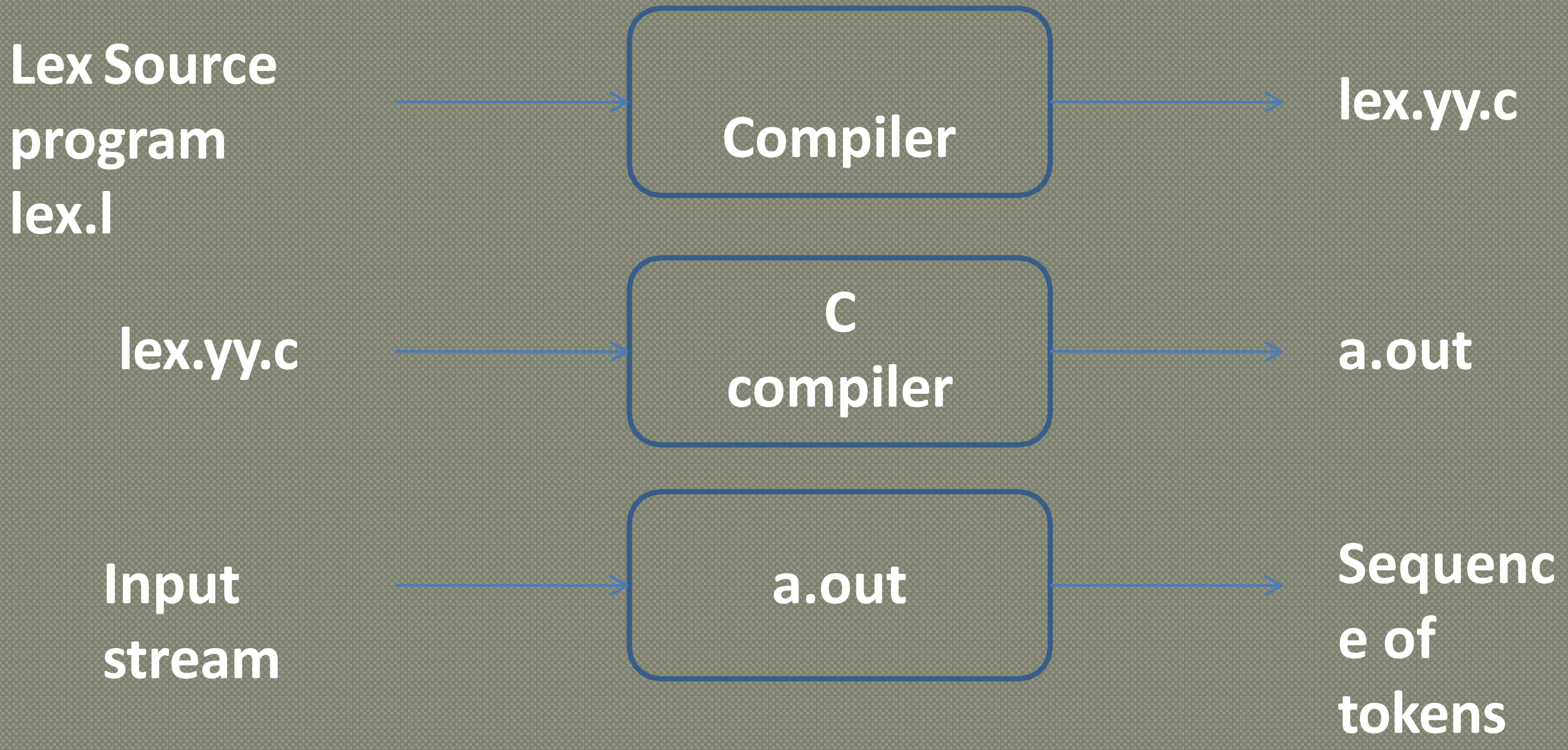


Transition diagrams (cont.)

- Transition diagram for whitespace



Lexical Analyzer Generator - Lex



Lex Source program lex.l → Compiler → lex.yy.c

lex.yy.c → C compiler → a.out

Input stream → a.out → Sequence of tokens

LEX Example

```
%{
    /* definitions of manifest constants
    LT, LE, EQ, NE, GT, GE,
    IF, THEN, ELSE, ID, NUMBER, RELOP */
%}

/* regular definitions
delim   [ \t\n]
ws            {delim}+
letter  [A-Za-z]
digit   [0-9]
id      {letter}({letter}|{digit})*
number        {digit}+(\.{digit}+)?(E[+-]?{digit}+)?
```

LEX Example

```
%%
{ws}     {/* no action and no return */}
if       {return(IF);}
then     {return(THEN);}
else     {return(ELSE);} {id}     {yylval = (int) installID();
   return(ID); }
{number}       {yylval = (int) installNum(); return(NUMBER);}
…
```

LEX Example

```
Int installID() {/* funtion to install the lexeme,whose
   first character is pointed to by yytext, and whose
   length is yyleng, into the symbol table and return a
   pointer thereto */
}

Int installNum() { /* similar to installID, but puts
   numerical constants into a separate table */
}
```

Finite Automata

- Regular expressions = specification
- Finite automata = implementation

- A finite automaton consists of
  - An input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions state $\rightarrow^{\text{input}}$ state

Finite Automata

- Transition

$$s_1 \rightarrow^{a} s_2$$

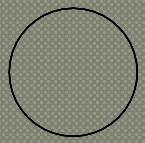
- Is read

In state s_1 on input “a” go to state s_2

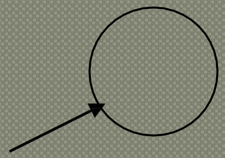
- If end of input
  - If in accepting state => accept, othewise => reject
- If no transition possible => reject

Finite Automata State Graphs

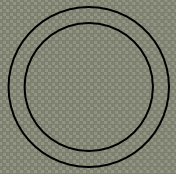
- A state



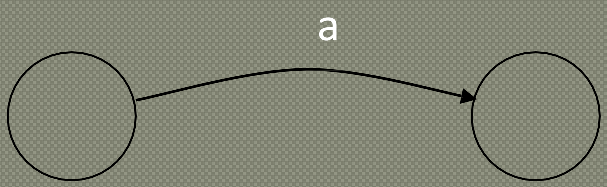
- The start state



- An accepting state

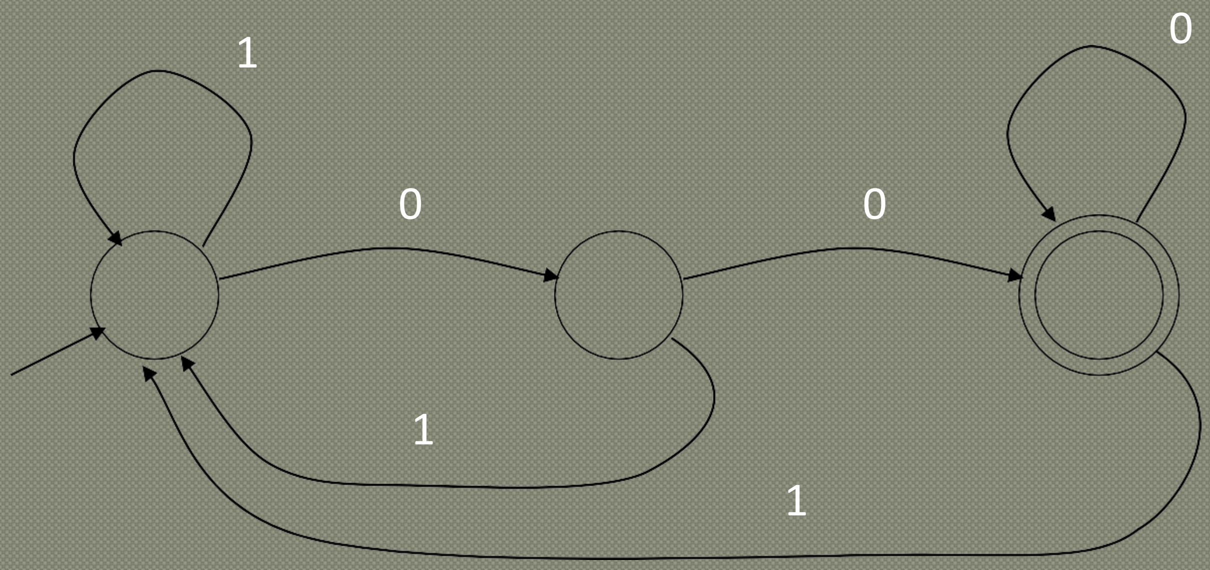


- A transition



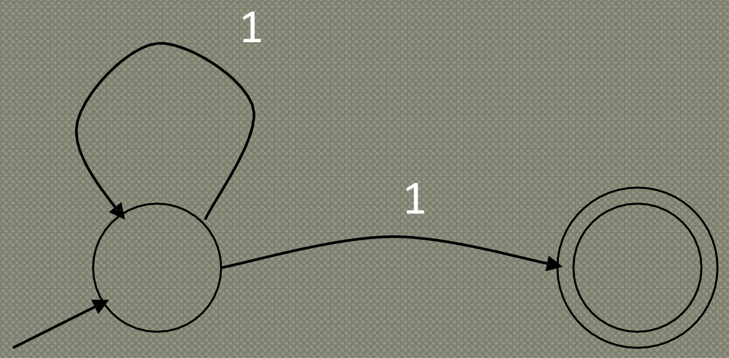
Example

- Alphabet {0,1}
- What language does this recognize?



And Another Example

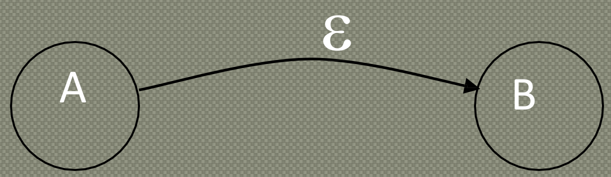
- Alphabet still { 0, 1 }



- The operation of the automaton is not completely defined by the input
  - On input “11” the automaton could be in either state

Epsilon Moves

- **Another kind of transition: ε-moves**



- **Machine can move from state A to state B without reading input**

Deterministic and Nondeterministic Automata

- **Deterministic Finite Automata (DFA)**
  - **One transition per input per state**
  - **No ε-moves**
- **Nondeterministic Finite Automata (NFA)**
  - **Can have multiple transitions for one input in a given state**
  - **Can have ε-moves**
- ***Finite* automata have *finite* memory**
  - **Need only to encode the current state**

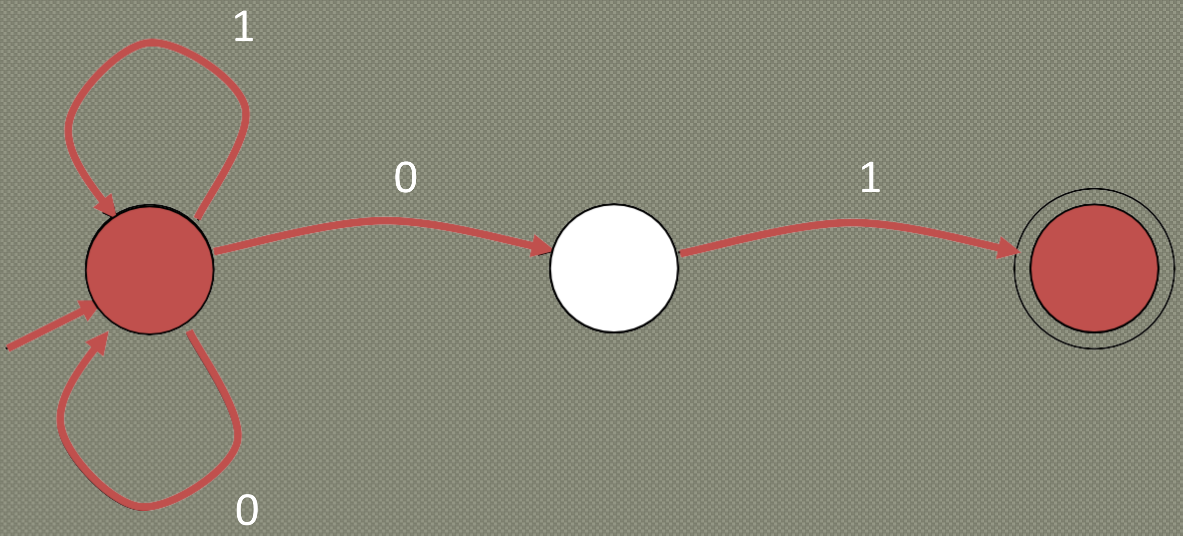
Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input

- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take .

Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 1

- Rule: NFA accepts if it <u>can</u> get in a final state

NFA vs. DFA (1)

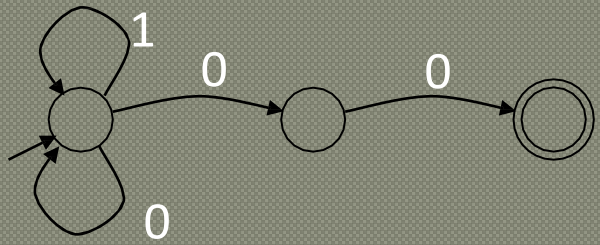
- **NFAs and DFAs recognize the same set of languages (regular languages)**

- **DFAs are easier to implement**
  - **There are no choices to consider**

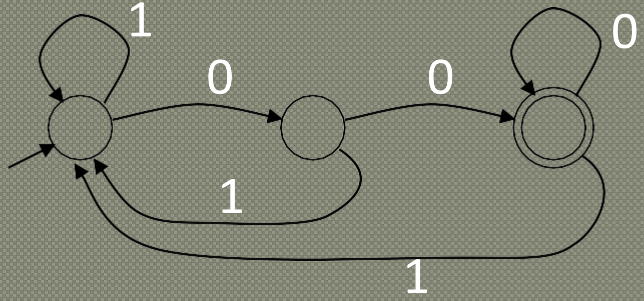
NFA vs. DFA (2)

- **For a given language the NFA can be simpler than the DFA**

NFA



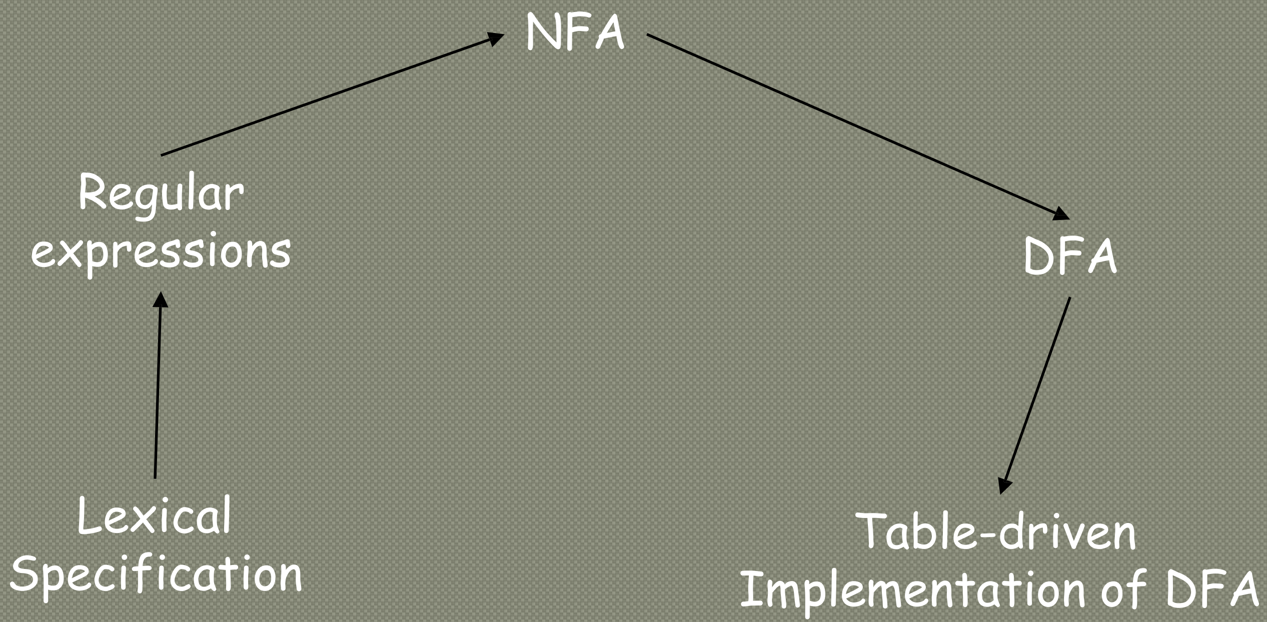
DFA



- **DFA can be exponentially larger than NFA**

Regular Expressions to Finite Automata

- High-level sketch



NFA

Regular expressions

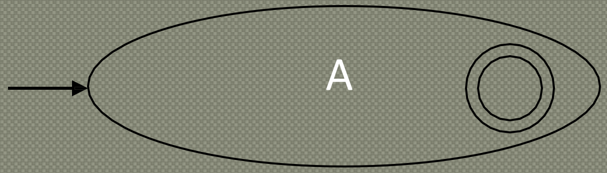
DFA

Lexical Specification

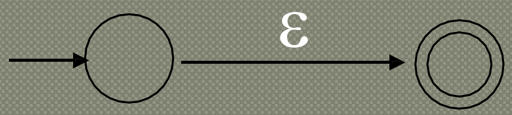
Table-driven Implementation of DFA

Expressions to NFA (1)

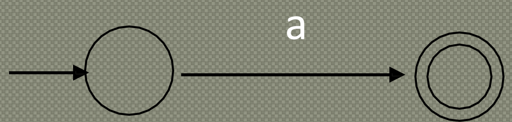
- For each kind of rexp, define an NFA
  - Notation: NFA for rexp A



- For ε

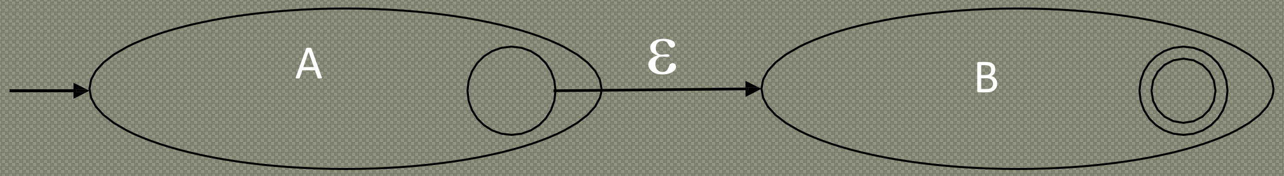


- For input a

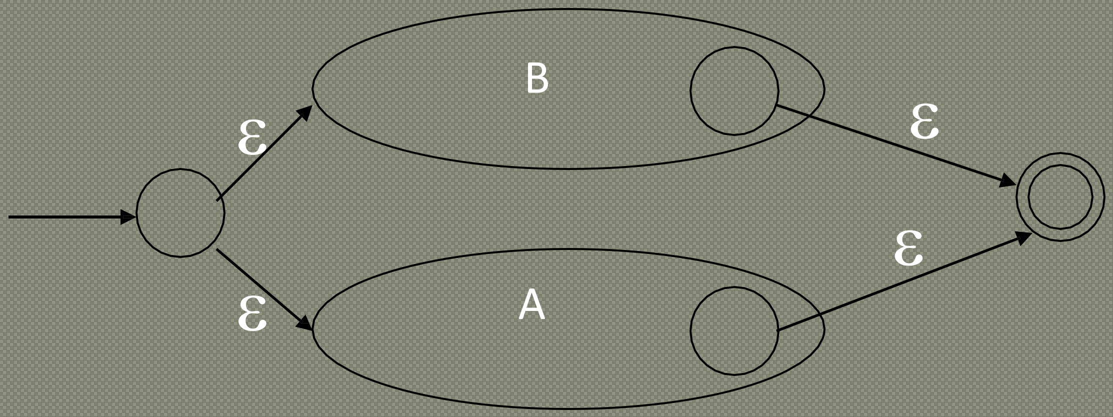


Expressions to NFA (2)

- For AB

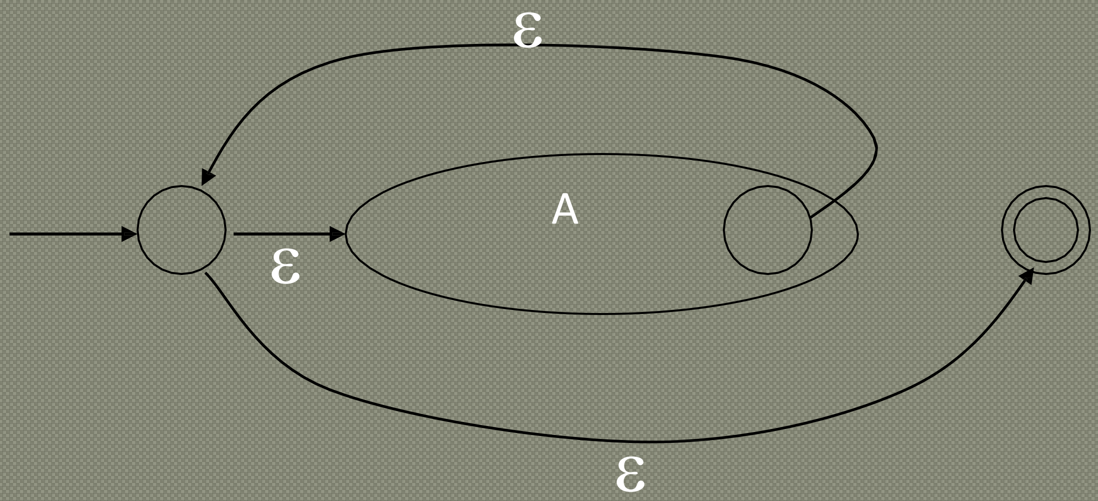


- For *A* | *B*



Expressions to NFA (3)

- For A*

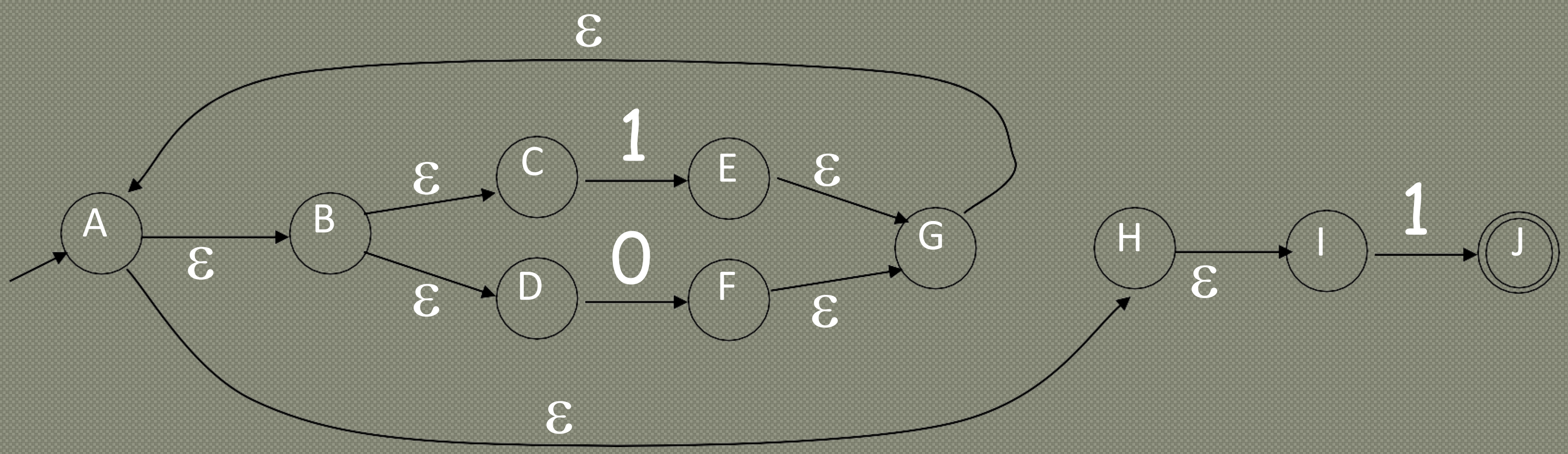


Example of RegExp -> NFA conversion

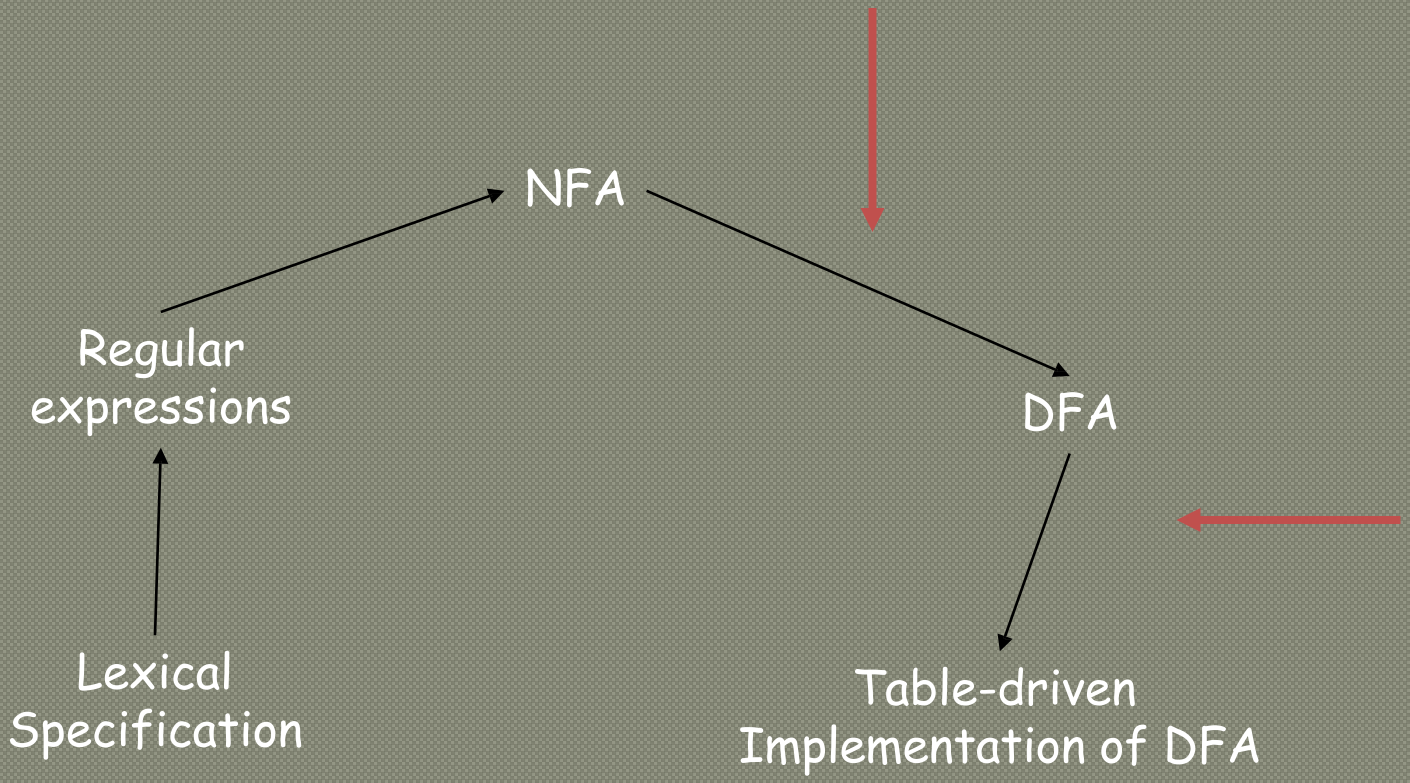
- Consider the regular expression

  (1 | 0)*1

- The NFA is



Next



NFA

Regular expressions

DFA

Lexical Specification

Table-driven Implementation of DFA

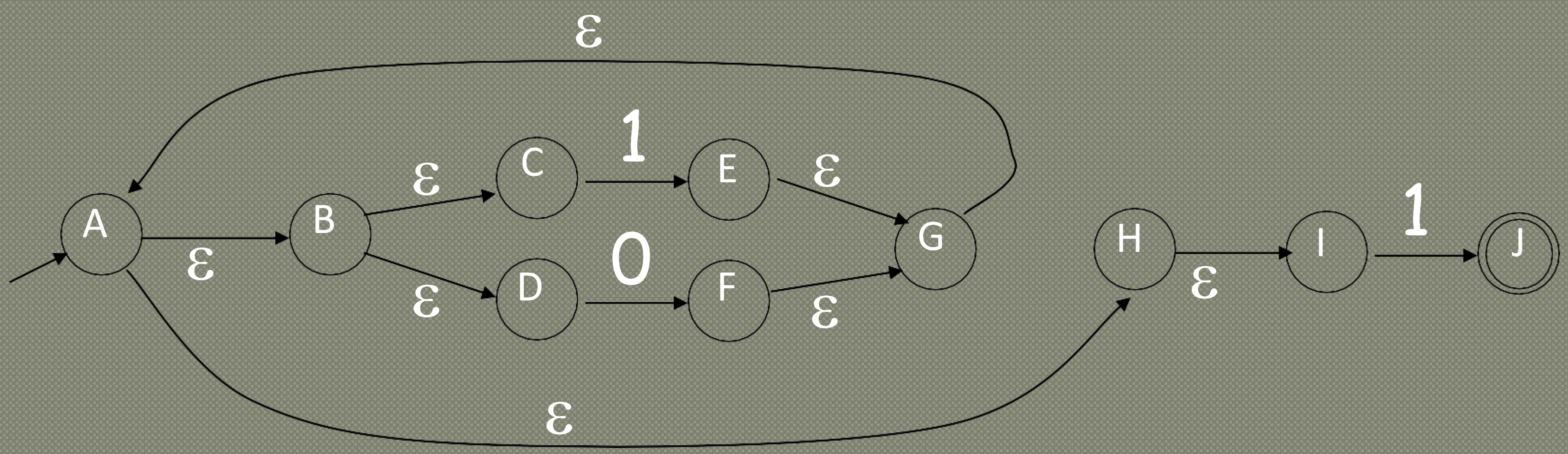
NFA to DFA. The Trick

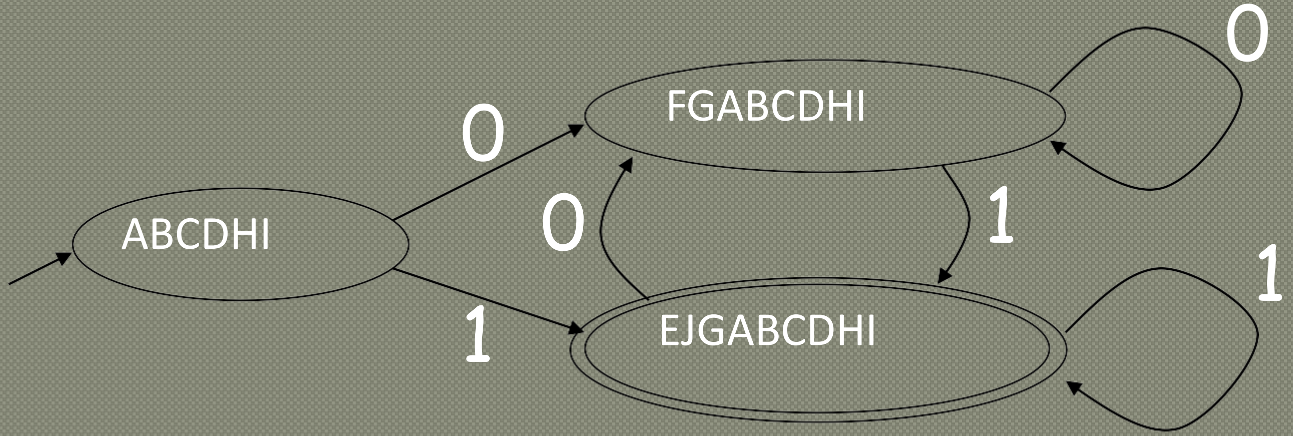
- Simulate the NFA
- Each state of resulting DFA

  = a non-empty subset of states of the NFA
- Start state

  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \rightarrow^{a} S'$ to DFA iff
  - S' is the set of NFA states reachable from the states in S after seeing the input a
    - considering ε-moves as well

NFA -> DFA Example





NFA to DFA. Remark

- An NFA may be in many states at any time

- How many different states ?

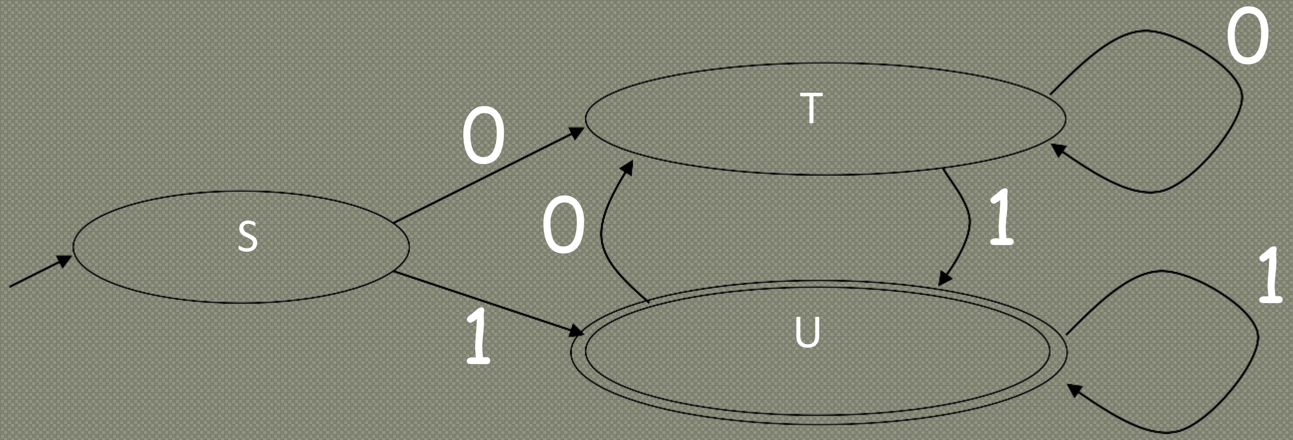
- If there are N states, the NFA must be in some subset of those N states

- How many non-empty subsets are there?
  - 2^N - 1 = finitely many, but exponentially many

Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is “states”
  - Other dimension is “input symbols”
  - For every transition $S_i \rightarrow^{a} S_k$ define T[i,a] = k
- DFA “execution”
  - If in state S_i and input a, read T[i,a] = k and skip to state S_k
  - Very efficient

Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- **NFA -> DFA conversion is at the heart of tools such as flex or jflex**

- **But, DFAs can be huge**

- **In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations**

UNIT-2

Bottom up parsing handle pruning LR Grammar Parsing, LALR Parsing, Parsing ambiguous grammars, YACC Programming specification.

Semantics: Syntax directed translation, S-attributed and L-attributed grammars, and Intermediate code-abstract, syntax tree, translation of simple statements and control flow statements.

Top-Down Parsing

- The parse tree is created top to bottom.
- Top-down parser
  - Recursive-Descent Parsing
    - Backtracking is needed (If a choice of a production rule does not work, we backtrack to try other alternatives.)
    - It is a general parsing technique, but not widely used.
    - Not efficient

Top-Down Parsing

- Predictive Parsing
  - no backtracking
  - efficient
  - needs a special form of grammars (LL(1) grammars).
  - Recursive Predictive Parsing is a special form of Recursive Descent parsing without backtracking.
  - Non-Recursive (Table Driven) Predictive Parser is also known as LL(1) parser.

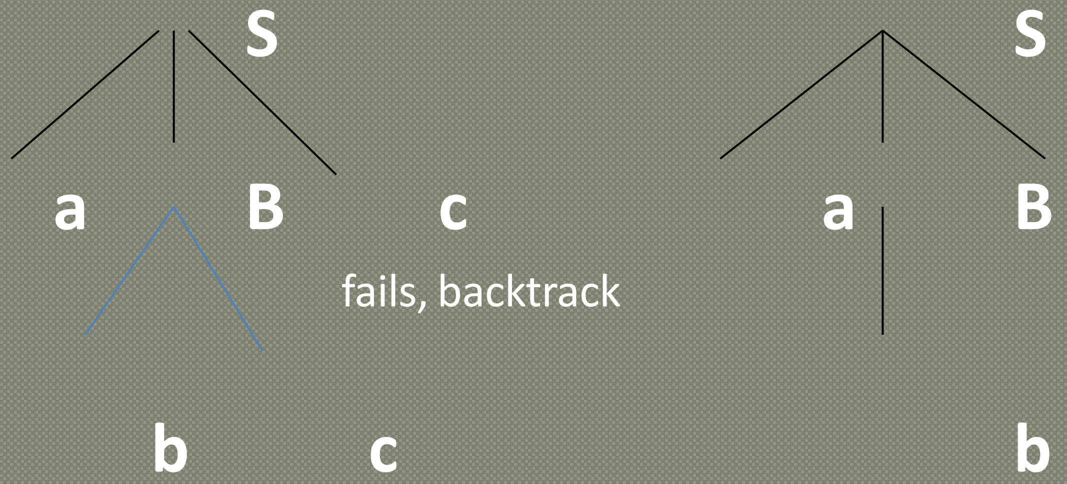
Recursive-Descent Parsing (uses Backtracking)

- **Backtracking is needed.**
- **It tries to find the left-most derivation.**

S → aBc
B → bc | b

input: abc

c



fails, backtrack

Predictive Parser

a grammar ➔ ➔ a grammar suitable
for predictive
eliminate left parsing (a LL(1)
grammar)
left recursion factor no %100 guarantee.

- When re-writing a non-terminal in a derivation step, a predictive parser can uniquely choose a production rule by just looking the current symbol in the input string.

$A \rightarrow \alpha_1 \mid \ldots \mid \alpha_n$ input: ... a ....

current token

Predictive Parser (example)

```
stmt → if ......        |
       while ......     |
       begin ......     |
       for .....
```

- When we are trying to write the non-terminal *stmt*, if the current token is `if` we have to choose first production rule.
- When we are trying to write the non-terminal *stmt*, we can uniquely choose the production rule by just looking the current token.
- We eliminate the left recursion in the grammar, and left factor it. But it may not be suitable for predictive parsing (not LL(1) grammar).

Unit-3

Context sensitive features-Chomsky hierarchy of languages and recognizers. Type checking, type conversions, equivalence of type expressions, overloading of functions and operations.

Bottom-Up Parsing

- **A bottom-up parser creates the parse tree of the given input starting from leaves towards the root.**
- **A bottom-up parser tries to find the right-most derivation of the given input in the reverse order.**

  $S \Rightarrow \ldots \Rightarrow \omega$ (the right-most derivation of ω)

  $\leftarrow$ (the bottom-up parser finds the right-most derivation in the reverse order)

- **Bottom-up parsing is also known as shift-reduce parsing because its two main actions are shift and reduce.**
  - **At each shift action, the current symbol in the input string is pushed to a stack.**

Bottom-Up Parsing

- At each reduction step, the symbols at the top of the stack (this symbol sequence is the right side of a production) will replaced by the non-terminal at the left side of that production.
- There are also two more actions: accept and error.

Parsing

- A shift-reduce parser tries to reduce the given input string into the starting symbol.

  a string $\xrightarrow[\text{reduced to}]{}$ the starting symbol

- At each reduction step, a substring of the input matching to the right side of a production rule is replaced by the non-terminal at the left side of that production rule.
- If the substring is chosen correctly, the right most derivation of that string is created in the reverse order.

Rightmost Derivation: $S \overset{*}{\underset{rm}{\Rightarrow}} \omega$

Shift-Reduce Parser finds: $\omega \underset{rm}{\Leftarrow} \ldots \underset{rm}{\Leftarrow} S$

Shift-Reduce Parsing -- Example

$S \rightarrow aABb$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

reduction

input string: `aaabb`

`aaAbb`

`aAbb` ⇓

`aABb`

`S`

$$S \underset{rm}{\Rightarrow} aABb \underset{rm}{\Rightarrow} aAbb \underset{rm}{\Rightarrow} aaAbb \underset{rm}{\Rightarrow} aaabb$$

Right Sentential Forms

- How do we know which substring to be replaced at each reduction step?

Handle

- **Informally, a handle of a string is a substring that matches the right side of a production rule.**
  - **But not every substring matches the right side of a production rule is handle**

- **A handle of a right sentential form γ ($\equiv \alpha\beta\omega$) is a production rule A $\rightarrow \beta$ and a position of γ where the string β may be found and replaced by A to produce the previous right-sentential form in a rightmost derivation of γ.**

$$S \overset{*}{\overset{rm}{\Rightarrow}} \alpha A \omega \overset{rm}{\Rightarrow} \alpha\beta\omega$$

- **If the grammar is unambiguous, then every right-sentential form of the grammar has exactly one handle.**
- **We will see that ω is a string of terminals.**

Handle Pruning

- A right-most derivation in reverse can be obtained by handle-pruning.

$$S=\gamma_0 \overset{rm}{\Rightarrow} \gamma_1 \overset{rm}{\Rightarrow} \gamma_2 \overset{rm}{\Rightarrow} \ldots \overset{rm}{\Rightarrow} \gamma_{n-1} \overset{rm}{\Rightarrow} \gamma_n = \omega$$

input string

- Start from γ_n, find a handle $A_n \rightarrow \beta_n$ in γ_n, and replace β_n in by A_n to get γ_{n-1}.
- Then find a handle $A_{n-1} \rightarrow \beta_{n-1}$ in γ_{n-1}, and replace β_{n-1} in by A_{n-1} to get γ_{n-2}.
- Repeat this, until we reach S.

A Shift-Reduce Parser

$E \rightarrow E+T \mid T$
$T \rightarrow T*F \mid F$
$F \rightarrow (E) \mid id$

Right-Most Derivation of `id+id*id`

$E \Rightarrow E+T \Rightarrow E+T*F \Rightarrow E+T*id \Rightarrow E+F*id$
$\Rightarrow E+id*id \Rightarrow T+id*id \Rightarrow F+id*id \Rightarrow id+id*id$

Right-Most Sentential Form	Reducing Production
$\underline{id}+id*id$	$F \rightarrow id$
$\underline{F}+id*id$	$T \rightarrow F$
$\underline{T}+id*id$	$E \rightarrow T$
$E+\underline{id}*id$	$F \rightarrow id$
$E+\underline{F}*id$	$T \rightarrow F$
$E+T*\underline{id}$	$F \rightarrow id$
$E+\underline{T*F}$	$T \rightarrow T*F$
$\underline{E+T}$	$E \rightarrow E+T$
E	

`Handles` are red and underlined in the right-sentential forms.

A Stack Implementation of A Shift-Reduce Parser

- There are four possible actions of a shift-parser action:

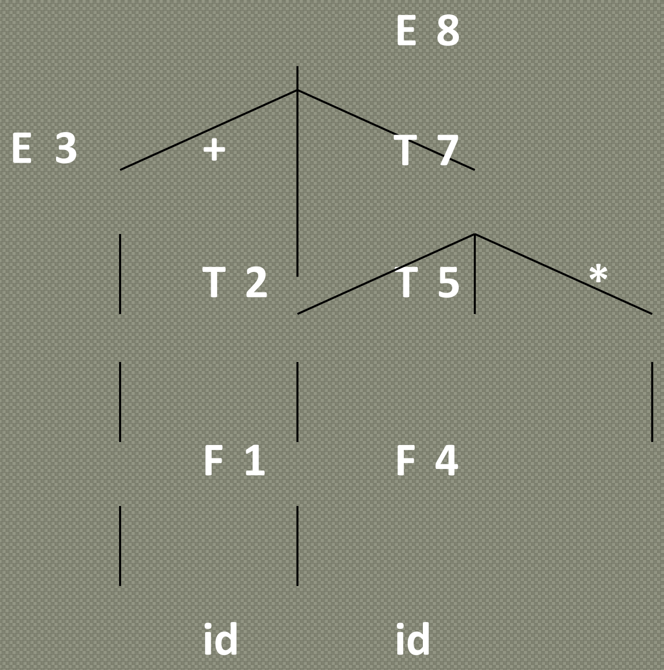
1. **Shift** : The next input symbol is shifted onto the top of the stack.
2. **Reduce**: Replace the handle on the top of the stack by the non-terminal.
3. **Accept**: Successful completion of parsing.
4. **Error**: Parser discovers a syntax error, and calls an error recovery routine.

- Initial stack just contains only the end-marker $.
- The end of the input string is marked by the end-marker $.

A Stack Implementation of A Shift-Reduce Parser

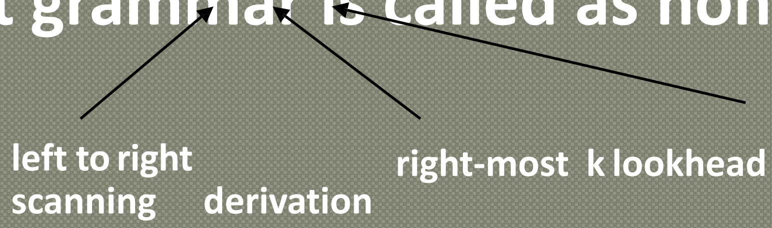
Stack	Input	Action
$	id+id*id$	shift
$id	+id*id$	reduce by F → id
$F	+id*id$	reduce by T → F
$T	+id*id$	reduce by E → T
$E	+id*id$	shift
$E+	id$	shift
id		
$E+id	*id$	reduce by F → id
*		
$E+F	*id$	reduce by T → F
F 6		
$E+T	*id$	shift
$E+T*	id$	shift
id		
$E+T*id	$	reduce by F → id
$E+T*F	$	reduce by T → T*F
$E+T	$	reduce by E → E+T
$E	$	accept

Parse Tree



Conflicts During Shift-Reduce Parsing

- There are context-free grammars for which shift-reduce parsers cannot be used.
- Stack contents and the next input symbol may not decide action:
  - shift/reduce conflict: Whether make a shift operation or a reduction.
  - reduce/reduce conflict: The parser cannot decide which of several reductions to make.
- If a shift-reduce parser cannot be used for a grammar, that grammar is called as non-LR(k) grammar.



left to right scanning

right-most derivation

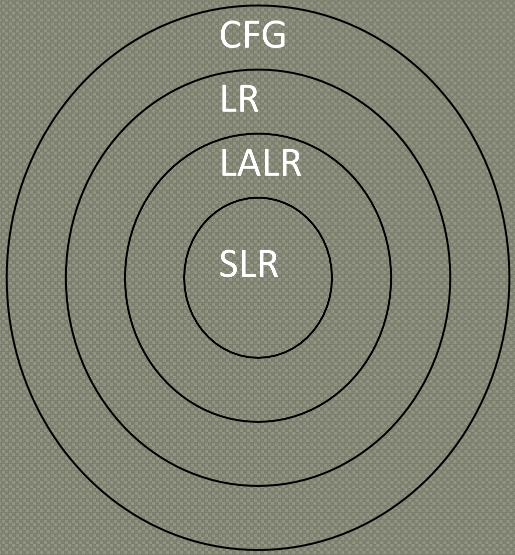
k lookhead

- An ambiguous grammar can never be a LR grammar.

Shift-Reduce Parsers

- There are two main categories of shift-reduce parsers

1. Operator-Precedence Parser
   – simple, but only a small class of grammars.



2. LR-Parsers
   – covers wide range of grammars.
     - SLR – simple LR parser
     - LR – most general LR parser
     - LALR – intermediate LR parser (lookhead LR parser)

Operator-Precedence Parser

- **Operator grammar**
  - **small, but an important class of grammars**
  - **we may have an efficient operator precedence parser (a shift-reduce parser) for an operator grammar.**
- **In an *operator grammar*, no production rule can have:**
  - **ε at the right side**
  - **two adjacent non-terminals at the right side.**

- **Ex:**

E→AB
A→a
B→b

not operator grammar

E→EOE
E→id
O→+|*|/

not operator grammar

E→E+E |
E*E |
E/E | id

operator grammar

Precedence Relations

- In operator-precedence parsing, we define three disjoint precedence relations between certain pairs of terminals.

a <· b	b has higher precedence than a
a =· b	b has same precedence as a
a ·> b	b has lower precedence than a

- The determination of correct precedence relations between terminals are based on the traditional notions of associativity and precedence of operators. (Unary minus causes a problem).

Using Operator-Precedence Relations

- The intention of the precedence relations is to find the handle of a right-sentential form,

  <· with marking the left end,

  =· appearing in the interior of the handle, and

  ·> marking the right hand.

- In our input string $\$a_1a_2...a_n\$$, we insert the precedence relation between the pairs of terminals (the precedence relation holds between the terminals in that pair).

Using Operator -Precedence Relations

E → E+E | E-E | E*E | E/E | E^E | (E) | -E | id

The partial operator-precedence table for this grammar

		id	*	$
id		·>	·>	·>
+	<·	·>	<·	·>
*	<·	·>	·>	·>
$	<·	<·	<·	

- **Then the input string id+id*id with the precedence relations inserted will be:**

$ <· id ·> + <· id ·> * <· id ·> $

To Find The Handles

1. **Scan the string from left end until the first ·> is encountered.**
2. **Then scan backwards (to the left) over any =· until a <· is encountered.**
3. **The handle contains everything to left of the first ·> and to the right of the <· is encountered.**

$ <· id ·> + <· id ·> * <· id ·> $	E → id	$ id + id * id $
$ <· + <· id ·> * <· id ·> $	E → id	$ E + id * id $
$ <· + <· * <· id ·> $	E → id	$ E + E * id $
$ <· + <· * ·> $	E → E*E	$ E + E * ·E $
$ <· + ·> $	E → E+E	$ E + E $
$ $		$ E $

Operator-Precedence Parsing Algorithm

- **The input string is w$, the initial stack is $ and a table holds precedence relations between certain terminals**

Algorithm:

```
set p to point to the first symbol of w$ ;
repeat forever
    if ( $ is on top of the stack and p points to $ ) then return
    else {
        let a be the topmost terminal symbol on the stack and let b be the symbol pointed to by p;
        if ( a <· b or a =· b ) then {            /* SHIFT */
```

Operator-Precedence Parsing Algorithm

```
push b onto the stack;
        advance p to the next input symbol;
    }else if  ( a ·> b )  then                 /* REDUCE */
         repeat  pop stack
         until  ( the top of stack terminal is related
 by <·  to the terminal most recently popped );
       else  error();
    }
```

Operator-Precedence Parsing Algorithm -- Example

stack	*input*	*action*	
$	id+id*id$	$ <· id shift	
$id	+id*id$	id ·> + reduce	E → id
$	+id*id$	shift	
$+	id*id$	shift	
$+id	*id$	id ·> * reduce	E → id
$+	*id$	shift	
$+*	id$	shift	
$+*id	$	id ·> $ reduce	E → id
$+*	$	* ·> $ reduce	E → E*E
$+	$	+ ·> $ reduce	E → E+E
$	$	accept	

How to Create Operator-Precedence Relations

- We use associativity and precedence relations among operators. 1.

If operator O_1 has higher precedence than operator O_2,

2. ➔ $O_1 \cdot> O_2$ and $O_2 <\cdot O_1$

If operator O_1 and operator O_2 have equal precedence,
they are left-associative ➔ $O_1 \cdot> O_2$ and $O_2 \cdot> O_1$
they are right-associative ➔ $O_1 <\cdot O_2$ and $O_2 <\cdot O_1$

3. For all operators O,

O <· id, id ·> O, O <· (, (<· O, O ·>),) ·> O, O ·> $, and $ <· O

4. Also, let

(=·)	$ <· (	id ·>)	) ·> $
(<· (	$ <· id	id ·> $	) ·>)
(<· id			

Operator-Precedence Relations

	+	-	*	/	^	id	(	)	$
+	·>	·>	<·	<·	<·	<·	<·	·>	·>
-	·>	·>	<·	<·	<·	<·	<·	·>	·>
*	·>	·>	·>	·>	<·	<·	<·	·>	·>
/	·>	·>	·>	·>	<·	<·	<·	·>	·>
^	·>	·>	·>	·>	<·	<·	<·	·>	·>
id	·>	·>	·>	·>	·>			·>	·>
(	<·	<·	<·	<·	<·	<·	<·	=·	
)	·>	·>	·>	·>	·>			·>	·>
$	<·	<·	<·	<·	<·	<·	<·		

Handling Unary Minus

- Operator-Precedence parsing cannot handle the unary minus when we have also the binary minus in our grammar.
- The best approach to solve this problem, let the lexical analyzer handle this problem.
  - The lexical analyzer will return two different operators for the unary minus and the binary minus.
  - The lexical analyzer will need a lookhead to distinguish the binary minus from the unary minus.
- Then, we make

  O <· unary-minus   for any operator

  unary-minus ·> O   if unary-minus has higher precedence than O

  unary-minus <· O   if unary-minus has lower (or equal) precedence than O

Precedence Functions

- **Compilers using operator precedence parsers do not need to store the table of precedence relations.**
- **The table can be encoded by two precedence functions f and g that map terminal symbols to integers.**
- **For symbols a and b.**

$f(a) < g(b)$ whenever $a <\cdot b$

$f(a) = g(b)$ whenever $a \doteq b$

$f(a) > g(b)$ whenever $a \cdot> b$

Disadvantages of Operator Precedence Parsing

- **Disadvantages**:
  - It cannot handle the unary minus (the lexical analyzer should handle the unary minus).
  - Small class of grammars.
  - Difficult to decide which language is recognized by the grammar.

- **Advantages:**
  - simple
  - powerful enough for expressions in programming languages

Error Recovery in Operator-Precedence Parsing

Error Cases:

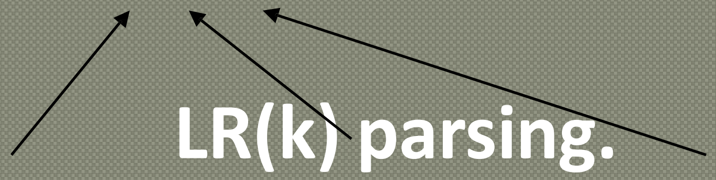
1. No relation holds between the terminal on the top of stack and the next input symbol.
2. A handle is found (reduction step), but there is no production with this handle as a right side

Error Recovery:

1. Each empty entry is filled with a pointer to an error routine.
2. Decides the popped handle "looks like" which right hand side. And tries to recover from that situation.

LR Parsers

- **The most powerful shift-reduce parsing (yet efficient) is:**



LR(k) parsing.

left to right right-most k lookhead
scanning derivation (k is omitted → it is 1)

LR Parsers

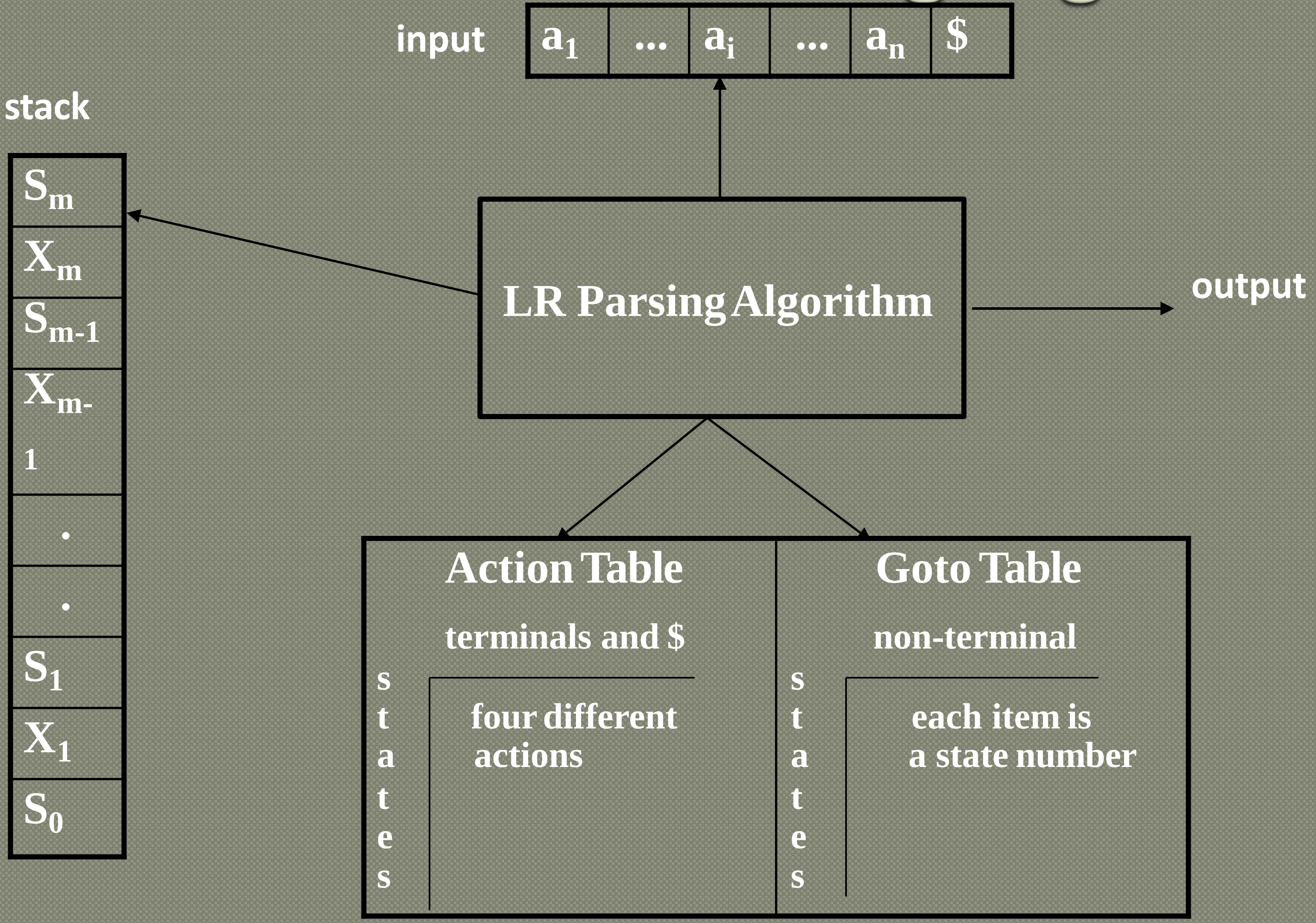
- **LR parsing is attractive because:**
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.

    LL(1)-Grammars $\subset$ LR(1)-Grammars
  - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

LR Parsers

- **LR-Parsers**
  - **covers wide range of grammars.**
  - **SLR – simple LR parser**
  - **CLR – most general LR parser**
  - **LALR – intermediate LR parser (look-head LR parser)**
  - **SLR, LR and LALR work same (they used the same algorithm), only their parsing tables are different.**

LR Parsing Algorithm



input

a_1	…	a_i	…	a_n	$

stack

S_m

X_m

S_{m-1}

X_{m-1}

.

.

S_1

X_1

S_0

LR Parsing Algorithm

output

Action Table

terminals and $

states

four different actions

Goto Table

non-terminal

states

each item is a state number

A Configuration of LR Parsing Algorithm

- **A configuration of a LR parsing is:**

  $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ \ a_i\ a_{i+1}\ \ldots\ a_n \$\ )$

  Stack **Rest of Input**

- **S_m and a_i decides the parser action by consulting the parsing action table. (*Initial Stack* contains just S_o)**

- **A configuration of a LR parsing represents the right sentential form:**

  $X_1 \ldots X_m\ a_i\ a_{i+1}\ \ldots\ a_n\ \$$

Actions of A LR-Parser

1. **shift s -- shifts the next input symbol and the state s onto the stack**

   $(S_o X_1 S_1 \ldots X_m S_m, a_i a_{i+1} \ldots a_n \$) \Rightarrow (S_o X_1 S_1 \ldots X_m S_m a_i s, a_{i+1} \ldots a_n \$)$

2. **reduce $A \rightarrow \beta$ (or `rn` where n is a production number)**
   - pop $2|\beta|$ (=r) items from the stack;
   - then push A and s where $s=goto[s_{m-r},A]$

   $(S_o X_1 S_1 \ldots X_m S_m, a_i a_{i+1} \ldots a_n \$) \Rightarrow (S_o X_1 S_1 \ldots X_{m-r} S_{m-r} A\ s, a_i \ldots a_n \$)$

   - Output is the reducing production reduce $A \rightarrow \beta$

3. **Accept – Parsing successfully completed**

4. **Error -- Parser detected an error (an empty entry in the action table)**

Reduce Action

- **pop 2|β| (=r) items from the stack; let us assume that $\beta = Y_1Y_2 \ldots Y_r$**
- **then push A and s where $s = goto[s_{m-r}, A]$**

$(S_o \ X_1 \ S_1 \ldots X_{m-r} \ S_{m-r} \ Y_1 \ S_{m-r} \ldots Y_r \ S_m, \ a_i \ a_{i+1} \ \ldots \ a_n \ \$ \ )$

➔ $(S_o \ X_1 \ S_1 \ldots X_{m-r} \ S_{m-r} \ A \ s, \ a_i \ldots a_n \ \$ \ )$

- **In fact, $Y_1Y_2 \ldots Y_r$ is a handle.**

$X_1 \ldots X_{m-r} \ A \ a_i \ldots a_n \ \$ \Rightarrow X_1 \ \ldots X_m \ Y_1 \ldots Y_r \ a_i \ a_{i+1} \ \ldots \ a_n \ \$$

(SLR) Parsing Tables for Expression Grammar

1)E → E+T

2) E → T

3) T → T*F

4)T → F 5)

F → (E)

6) F → id

	Action Table							Goto Table		
state	id	+	*	(	)	$		E	T	F
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

Actions of A (S)LR-Parser -- Example

stack	input	action	output
0	id*id+id$	shift 5	
0id5	*id+id$	reduce by F→id	F→id
0F3	*id+id$	reduce by T→F	T→F
0T2	*id+id$	shift 7	
0T2*7	id+id$	shift 5	
0T2*7id5	+id$	reduce by F→id	F→id
0T2*7F10	+id$	reduce by T→T*F	T→T*F
0T2	+id$	reduce by E→T	E→T

Actions of A (S)LR-Parser -- Example

0E1	+id$	shift 6
0E1+6	id$	shift 5
0E1+6id5	$	reduce by F→id
F→id		
0E1+6F3	$	reduce by T→F
T→F		
0E1+6T9	$	reduce by E→E+T
E→E+T		
0E1	$	accept

Constructing SLR Parsing Tables – LR(0) Item

- **An LR(0) item of a grammar G is a production of G a dot at the some position of the right side.**
- **Ex: A → aBb** **Possible LR(0) Items: A → .aBb**

  (four different possibility)

  A → a.Bb

  A → aB.b

  A → aBb.
- **Sets of LR(0) items will be the states of action and goto table of the SLR parser.**
- **A collection of sets of LR(0) items (the**

The Closure Operation

- **If *I* is a set of LR(0) items for a grammar G, then *closure(I)* is the set of LR(0) items constructed from I by the two rules:**

1. **Initially, every LR(0) item in I is added to closure(I).**
2. **If $A \rightarrow \alpha.B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow .\gamma$ will be in the closure(I).**
   We will apply this rule until no more new LR(0) items can be added to closure(I).

The Closure Operation -- Example

closure({E’ → .E})

E’ → E

E → E+T

kernel items

E → T

T → T*F

T → F

F → (E)

F → id

=

{ E’ → •E

E → •E+T

E → •T

T → •T*F

T → •F

F → •(E)

F → •id }

Goto Operation

- **If I is a set of LR(0) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:**
  - **If $A \rightarrow \alpha.X\beta$ in I**
    then every item in closure({$A \rightarrow \alpha X.\beta$}) will be in goto(I,X).

Example:

I ={ $E' \rightarrow .E$, $E \rightarrow .E+T$, $E \rightarrow .T$,
$T \rightarrow .T*F$, $T \rightarrow .F$,
$F \rightarrow .(E)$, $F \rightarrow .id$ }

goto(I,E) = { $E' \rightarrow E.$, $E \rightarrow E.+T$ }

goto(I,T) = { $E \rightarrow T.$, $T \rightarrow T.*F$ }

goto(I,F) = {$T \rightarrow F.$ }

goto(I,() = { $F \rightarrow (.E)$, $E \rightarrow .E+T$, $E \rightarrow .T$, $T \rightarrow$

Construction of The Canonical LR(0) Collection

- **To create the SLR parsing tables for a grammar G, we will create the canonical LR(0) collection of the grammar G'.**

- ***Algorithm*:**

```
C is { closure({S'→.S}) }
repeat the followings until no more set of LR(0) items can
    be added to C.
    for each I in C and each grammar symbol X
        if goto(I,X) is not empty and not in C
            add goto(I,X) to C
```

The Canonical LR(0) Collection -- Example

I_0: E’ → .E I_1: E’ → E. I_6: E → E+.T I_9: E → E+T.

E → .E+T E → E.+T T → .T*F T → T.*F

E → .T T → .F

T → .T*F I_2: E → T. F → .(E) I_{10}: T → T*F.

T → .F T → T.*F F → .id

F → .(E)

The Canonical LR(0) Collection -- Example

I_8: F → (E.)

T → .T*F

E → E.+T

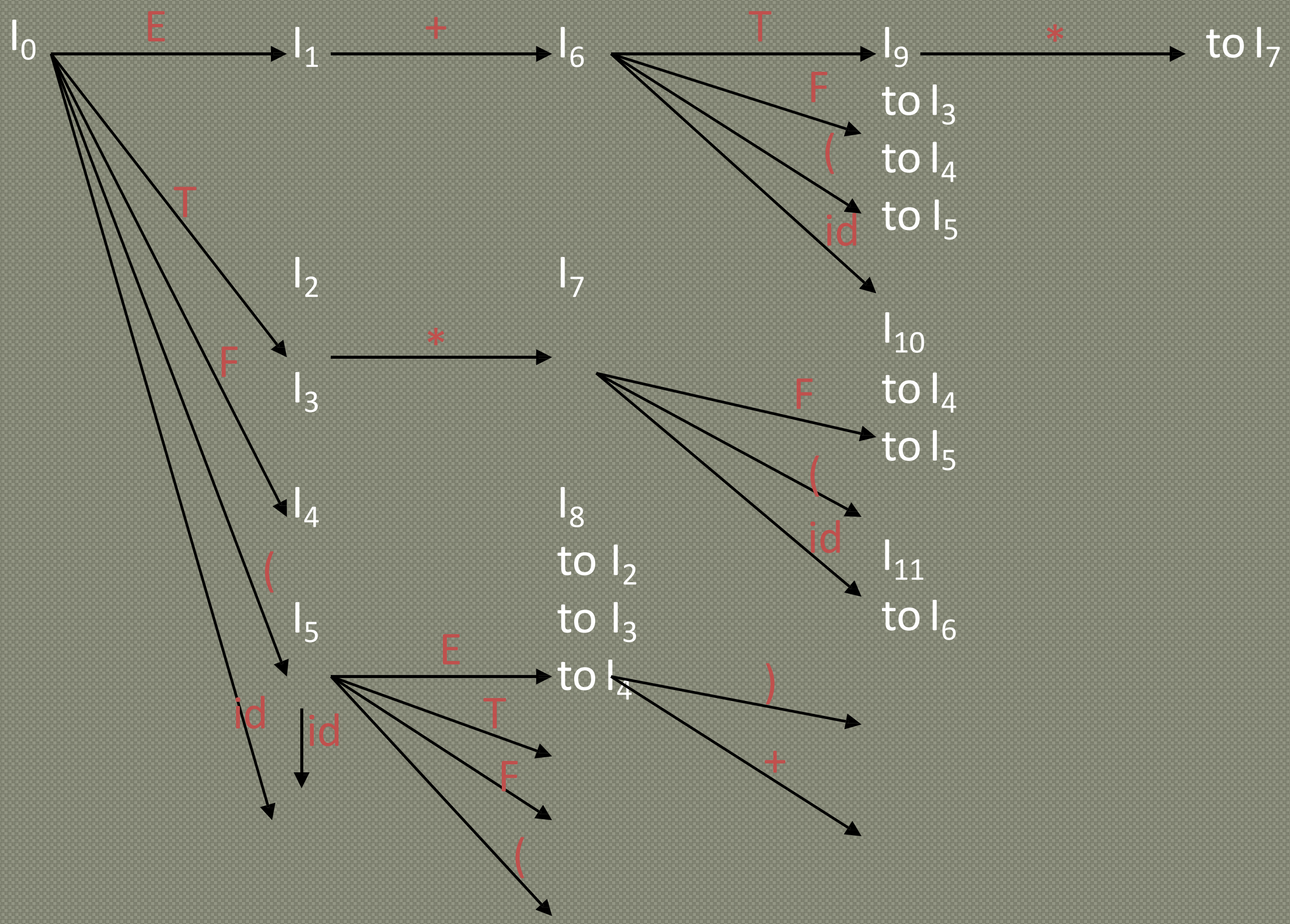
T → .F

F → .(E)

F → .id

I_5: F → id.

Transition Diagram (DFA) of Goto Function



Constructing SLR Parsing Table

(of an augumented grammar G')

1. Construct the canonical collection of sets of LR(0) items for G'. $C \leftarrow \{I_0,...,I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha.a\beta$ in I_i and goto(I_i,a)=I_j then action[i,a] is *shift j*.
   - If $A \rightarrow \alpha.$ is in I_i, then action[i,a] is *reduce* $A \rightarrow \alpha$ for all a in FOLLOW(A) where $A \neq S'$.
   - If $S' \rightarrow S.$ is in I_i, then action[i,$] is *accept*.
   - If any conflicting actions generated by these rules, the grammar is not SLR(1).

Constructing SLR Parsing Table

3. **Create the parsing goto table**
    - **for all non-terminals A, if goto(I_i,A)=I_j then goto[i,A]=j**

4. **All entries not defined by (2) and (3) are errors.**

5. **Initial state of the parser contains $S' \rightarrow .S$**

Parsing Tables of Expression Grammar

Action Table

Goto Table

state	id	+	*	(	)	$		E	T	F
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

SLR(1) Grammar

- **An LR parser using SLR(1) parsing tables for a grammar G is called as the SLR(1) parser for G.**
- **If a grammar G has an SLR(1) parsing table, it is called SLR(1) grammar (or SLR grammar in short).**
- **Every SLR grammar is unambiguous, but every unambiguous grammar is not a SLR grammar.**

shift/reduce and reduce/reduce conflicts

- **If a state does not know whether it will make a shift operation or reduction for a terminal, we say that there is a shift/reduce conflict.**

- **If a state does not know whether it will make a reduction operation using the production rule `i` or `j` for a terminal, we say that there is a reduce/reduce conflict.**

- **If the SLR parsing table of a grammar G has a**

Conflict Example

S → L=R

S → R

L→ *R

L → id

R → L

I_0: S' → .S

S → .L=R

S → .R

L → .*R

L → .id

R → .L

I_1: S' → S.

I_2: S → L.=R

R → L.

I_3: S → R.

I_4: L → *.R

R → .L

L→ .*R

L → .id

I_5: L → id.

I_6: S → L=.R

R → .L

L→ .*R

L → .id

I_7: L → *R.

I_8: R → L.

I_9: S → L=R.

Problem

FOLLOW(R)={=,$}

= shift 6

reduce by R → L

shift/reduce conflict

Conflict Example2

$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S$

$S \rightarrow .AaAb$

$S \rightarrow .BbBa$

$A \rightarrow .$

$B \rightarrow .$

Problem

FOLLOW(A)={a,b}

FOLLOW(B)={a,b}

a reduce by $A \rightarrow$

ε reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

b reduce by $A \rightarrow \varepsilon$

reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

Constructing Canonical LR(1) Parsing Tables

- **In SLR method, the state i makes a reduction by A→α when the current token is a:**
  - **if the A→α. in the I_i and a is FOLLOW(A)**
- **In some situations, βA cannot be followed by the terminal a in a right-sentential form when βα and the state i are on the top stack. This means that making reduction in this case is not correct.**

S → AaAb **S⇒AaAb⇒Aab⇒ab**

S⇒BbBa⇒Bba⇒ba

LR(1) Item

- **To avoid some of invalid reductions, the states need to carry more information.**
- **Extra information is put into a state by including a terminal symbol as a second component in an item.**

- **A LR(1) item is:**

$A \rightarrow \alpha.\beta$,a **where a is the look-head of the LR(1) item**

(a is a terminal or end-

LR(1) Item (cont.)

- **When β (in the LR(1) item $A \rightarrow \alpha.\beta,a$) is not empty, the look-head does not have any affect.**
- **When β is empty ($A \rightarrow \alpha.,a$), we do the reduction by $A\rightarrow\alpha$ only if the next input symbol is a (not for any terminal in FOLLOW(A)).**

- **A state will contain $A \rightarrow \alpha.,a_1$ where $\{a_1,\ldots,a_n\} \subseteq$ FOLLOW(A)**

...

$$A \rightarrow \alpha.,a_n$$

Canonical Collection of Sets of LR(1) Items

- The construction of the canonical collection of the sets of LR(1) items are similar to the construction of the canonical collection of the sets of LR(0) items, except that *closure* and *goto* operations work a little bit different.

closure(I) is: (where I is a set of LR(1) items)

- every LR(1) item in I is in closure(I)
- if $A \rightarrow \alpha.B\beta,a$ in closure(I) and $B \rightarrow \gamma$ is a production rule of G: then $B \rightarrow .\gamma,b$ will be in the closure(I) for each

goto operation

- If I is a set of LR(1) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \rightarrow \alpha.X\beta,a$ in I
    then every item in closure({$A \rightarrow \alpha X.\beta,a$}) will be in goto(I,X).

Construction of The Canonical LR(1) Collection

- ***Algorithm***:

```
C is { closure({S'→.S,$}) }
repeat the followings until no more set of LR(1) items can be added to C.
    for each I in C and each grammar symbol X
        if goto(I,X) is not empty and not in C
            add goto(I,X) to C
```

- **goto function is a DFA on the sets in C.**

A Short Notation for The Sets of LR(1) Items

- **A set of LR(1) items containing the following items**

  $$A \rightarrow \alpha\bullet\beta, a_1$$

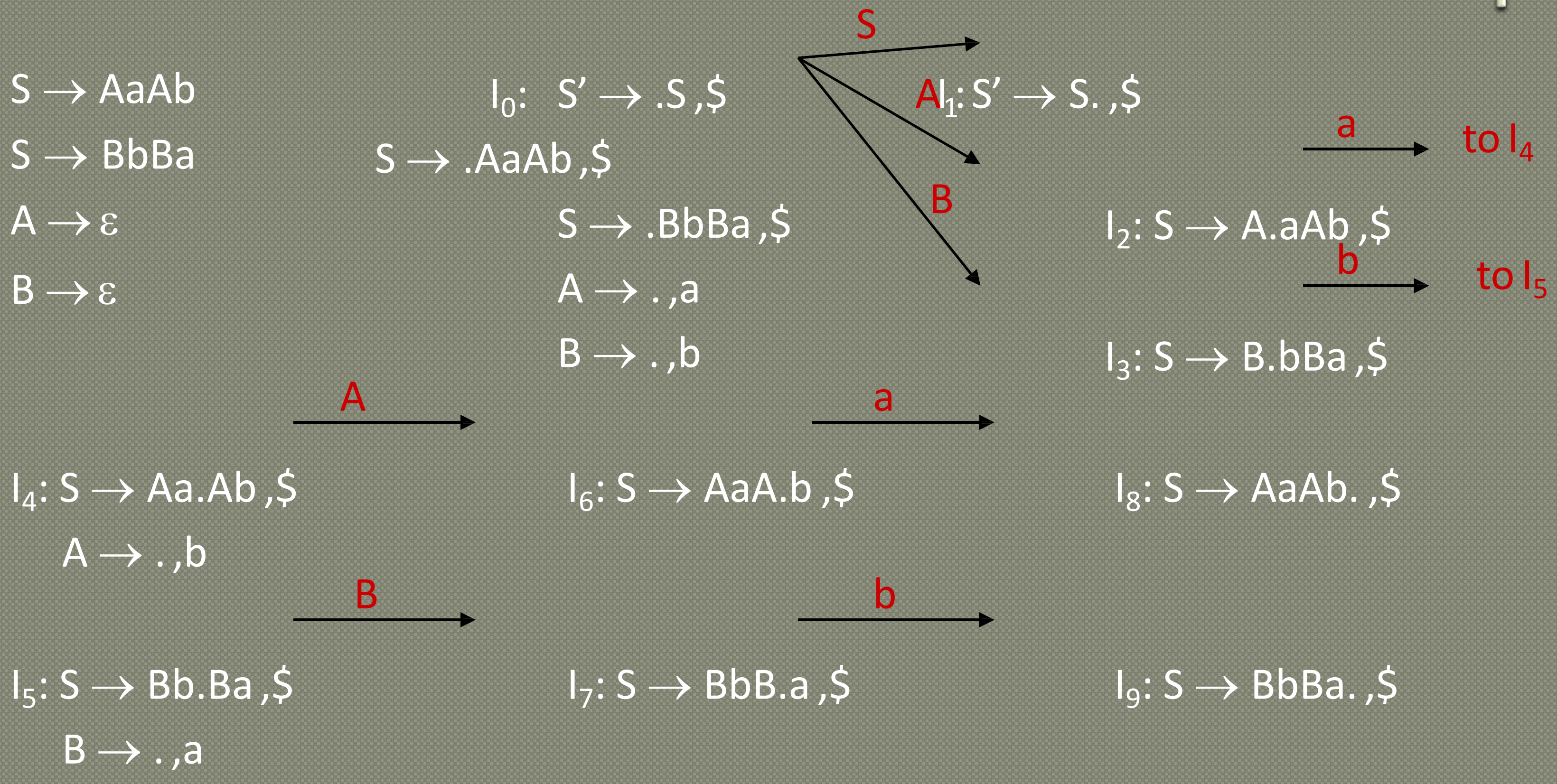
  ...

  $$A \rightarrow \alpha\bullet\beta, a_n$$

can be written as

$$A \rightarrow \alpha\bullet\beta, a_1/a_2/\ldots/a_n$$

Canonical LR(1) Collection -- Example



$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

$I_0:\ S' \rightarrow .S\,,\$$

$S \rightarrow .AaAb\,,\$$

$S \rightarrow .BbBa\,,\$$

$A \rightarrow .\,,a$

$B \rightarrow .\,,b$

$I_1:\ S' \rightarrow S.\,,\$$

$I_2:\ S \rightarrow A.aAb\,,\$$

$I_3:\ S \rightarrow B.bBa\,,\$$

$I_4:\ S \rightarrow Aa.Ab\,,\$$

$A \rightarrow .\,,b$

$I_5:\ S \rightarrow Bb.Ba\,,\$$

$B \rightarrow .\,,a$

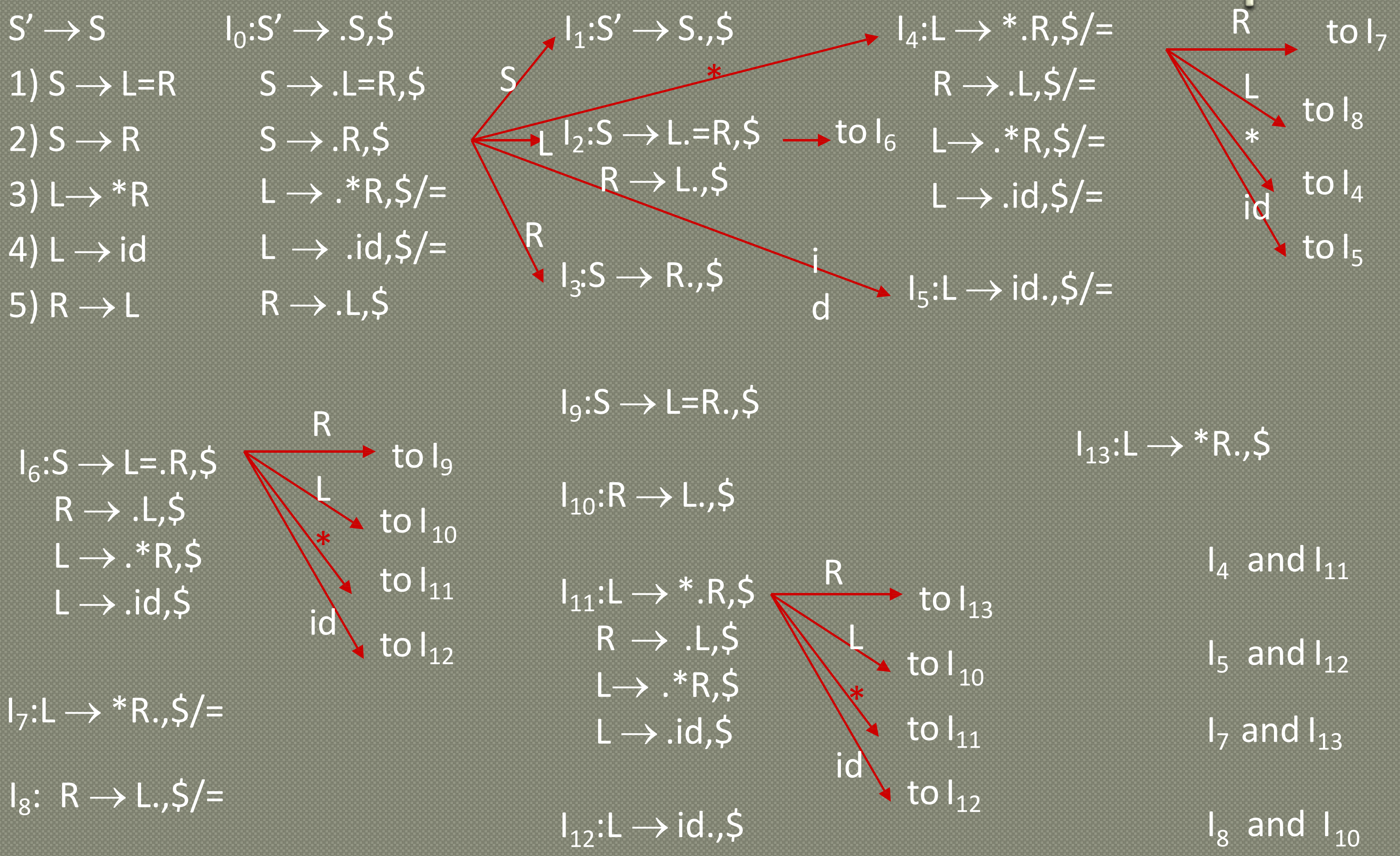
$I_6:\ S \rightarrow AaA.b\,,\$$

$I_7:\ S \rightarrow BbB.a\,,\$$

$I_8:\ S \rightarrow AaAb.\,,\$$

$I_9:\ S \rightarrow BbBa.\,,\$$

Canonical LR(1) Collection – Example2



$S' \to S$

1) $S \to L{=}R$

2) $S \to R$

3) $L \to {*}R$

4) $L \to id$

5) $R \to L$

I_0: $S' \to .S,\$$
$S \to .L{=}R,\$$
$S \to .R,\$$
$L \to .{*}R,\$/{=}$
$L \to .id,\$/{=}$
$R \to .L,\$$

I_1: $S' \to S.,\$$

I_2: $S \to L.{=}R,\$$ → to I_6
$R \to L.,\$$

I_3: $S \to R.,\$$

I_4: $L \to {*}.R,\$/{=}$
$R \to .L,\$/{=}$
$L \to .{*}R,\$/{=}$
$L \to .id,\$/{=}$

to I_7
to I_8
to I_4
to I_5

I_5: $L \to id.,\$/{=}$

I_6: $S \to L{=}.R,\$$
$R \to .L,\$$
$L \to .{*}R,\$$
$L \to .id,\$$

to I_9
to I_{10}
to I_{11}
to I_{12}

I_7: $L \to {*}R.,\$/{=}$

I_8: $R \to L.,\$/{=}$

I_9: $S \to L{=}R.,\$$

I_{10}: $R \to L.,\$$

I_{11}: $L \to {*}.R,\$$
$R \to .L,\$$
$L \to .{*}R,\$$
$L \to .id,\$$

to I_{13}
to I_{10}
to I_{11}
to I_{12}

I_{12}: $L \to id.,\$$

I_{13}: $L \to {*}R.,\$$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

Construction of LR(1) Parsing Tables

1. **Construct the canonical collection of sets of LR(1) items for G'.** $C \leftarrow \{I_0, \ldots, I_n\}$

2. **Create the parsing action table as follows**
   - **If a is a terminal, $A \rightarrow \alpha.a\beta, b$ in I_i and $goto(I_i, a) = I_j$ then action[i,a] is *shift j*.**
   - **If $A \rightarrow \alpha., a$ is in I_i, then action[i,a] is *reduce $A \rightarrow \alpha$* where $A \neq S'$.**
   - **If $S' \rightarrow S., \$$ is in I_i, then action[i,\$] is *accept*.**
   - **If any conflicting actions generated by these rules, the grammar is not LR(1).**

Construction of LR(1) Parsing Tables

3. **Create the parsing goto table**
   - **for all non-terminals A, if goto(I_i,A)=I_j then goto[i,A]=j**

4. **All entries not defined by (2) and (3) are errors.**

5. **Initial state of the parser contains S'→.S,$**

LR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			
10				r5			
11	s12	s11				10	13
12				r4			
13				r3			

no shift/reduce or
no reduce/reduce conflict

⇓

so, it is a LR(1) grammar

LALR Parsing Tables

- **LALR** stands for **LookAhead LR.**

- LALR parsers are often used in practice because LALR parsing tables are smaller than LR(1) parsing tables.
- The number of states in SLR and LALR parsing tables for a grammar G are equal.
- But LALR parsers recognize more grammars than SLR parsers.
- ***yacc*** creates a LALR parser for the given

Creating LALR Parsing Tables

Canonical LR(1) Parser → LALR Parser

shrink # of states

- This shrink process may introduce a **reduce/reduce** conflict in the resulting LALR parser (so the grammar is NOT LALR)
- But, this shrink process does not produce a **shift/reduce** conflict.

The Core of A Set of LR(1) Items

- The core of a set of LR(1) items is the set of its first component.

Ex: $S \rightarrow L.{=}R,\$$ ➔ $S \rightarrow L.{=}R$ Core

$R \rightarrow L.,\$$ $R \rightarrow L.$

- We will find the states (sets of LR(1) items) in a canonical LR(1) parser with same cores. Then we will merge them as a single state.

$I_1 : L \rightarrow id.,=$

A new state:

$I \ \ : L \rightarrow id.,=$

The Core of A Set of LR(1) Items

- **We will do this for all states of a canonical LR(1) parser to get the states of the LALR parser.**
- **In fact, the number of the states of the LALR parser for a grammar will be equal to the number of states of the SLR parser for that grammar**.

Creation of LALR Parsing Tables

- Create the canonical LR(1) collection of the sets of LR(1) items for the given grammar.
- Find each core; find all sets having that same core; replace those sets having same cores with a single set which is their union.

  $C=\{I_0,...,I_n\}$ ➔ $C'=\{J_1,...,J_m\}$ where $m \leq n$
- Create the parsing tables (action and goto tables) same as the construction of the parsing tables of LR(1) parser.
  - Note that: If $J=I_1 \cup ... \cup I_k$ since $I_1,...,I_k$ have same cores

    ➔

Shift/Reduce Conflict

- **We say that we cannot introduce a shift/reduce conflict during the shrink process for the creation of the states of a LALR parser.**
- **Assume that we can introduce a shift/reduce conflict. In this case, a state of LALR parser must have:**

$$A \to \alpha\bullet,a \quad \text{and} \quad B \to \beta\bullet a\gamma,b$$

- **This means that a state of the canonical LR(1) parser must have:**

$$A \to \alpha\bullet,a \quad \text{and} \quad B \to \beta\bullet a\gamma,c$$

Reduce/Reduce Conflict

- **But, we may introduce a reduce/reduce conflict during the shrink process for the creation of the states of a LALR parser.**

$I_1 : A \rightarrow \alpha\bullet, a$ $\qquad$ $I_2 : A \rightarrow \alpha\bullet, b$

$B \rightarrow \beta\bullet, b$ $\qquad$ $B \rightarrow \beta\bullet, c$

$$\Downarrow$$

$I_{12} : A \rightarrow \alpha\bullet, a/b$ ➔ **reduce/reduce conflict**

$B \rightarrow \beta\bullet, b/c$

Canonical LALR(1) Collection – Example2

$S' \rightarrow S$

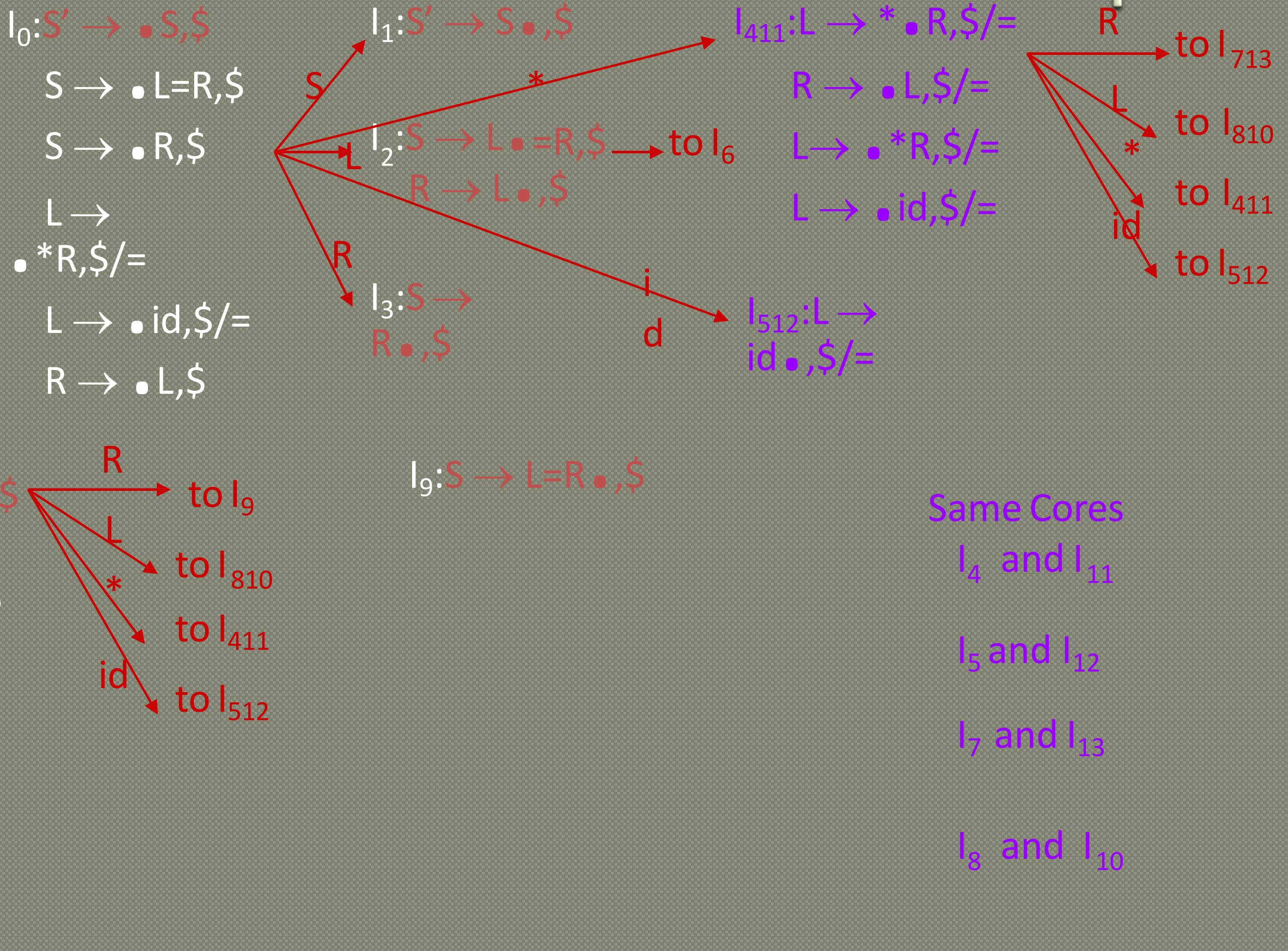
1) $S \rightarrow L{=}R$

2) $S \rightarrow R$

3) $L \rightarrow {*}R$

4) $L \rightarrow id$

5) $R \rightarrow L$



I_0: $S' \rightarrow \bullet S,\$$
$S \rightarrow \bullet L{=}R,\$$
$S \rightarrow \bullet R,\$$
$L \rightarrow \bullet {*}R,\$/{=}$
$L \rightarrow \bullet id,\$/{=}$
$R \rightarrow \bullet L,\$$

I_1: $S' \rightarrow S \bullet,\$$

I_2: $S \rightarrow L \bullet {=}R,\$$ → to I_6
$R \rightarrow L \bullet,\$$

I_3: $S \rightarrow R \bullet,\$$

I_{411}: $L \rightarrow {*} \bullet R,\$/{=}$
$R \rightarrow \bullet L,\$/{=}$
$L \rightarrow \bullet {*}R,\$/{=}$
$L \rightarrow \bullet id,\$/{=}$

to I_{713}
to I_{810}
to I_{411}
to I_{512}

I_{512}: $L \rightarrow id \bullet,\$/{=}$

I_6: $S \rightarrow L{=} \bullet R,\$$
$R \rightarrow \bullet L,\$$
$L \rightarrow \bullet {*}R,\$$
$L \rightarrow \bullet id,\$$

to I_9
to I_{810}
to I_{411}
to I_{512}

I_9: $S \rightarrow L{=}R \bullet,\$$

I_{713}: $L \rightarrow {*}R \bullet,\$/{=}$

I_{810}: $R \rightarrow L \bullet,\$/{=}$

Same Cores
I_4 and I_{11}
I_5 and I_{12}
I_7 and I_{13}
I_8 and I_{10}

LALR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			

no shift/reduce or
no reduce/reduce conflict

⇓

so, it is a LALR(1) rammar
g

Using Ambiguous Grammars

- All grammars used in the construction of LR-parsing tables must be un-ambiguous.
- Can we create LR-parsing tables for ambiguous grammars ?
  - Yes, but they will have conflicts.
  - We can resolve these conflicts in favor of one of them to disambiguate the grammar.
  - At the end, we will have again an unambiguous grammar.
- Why we want to use an ambiguous grammar?
  - Some of the ambiguous grammars are much natural, and a corresponding unambiguous grammar can be very complex.
  - Usage of an ambiguous grammar may eliminate unnecessary reductions.
- Ex.

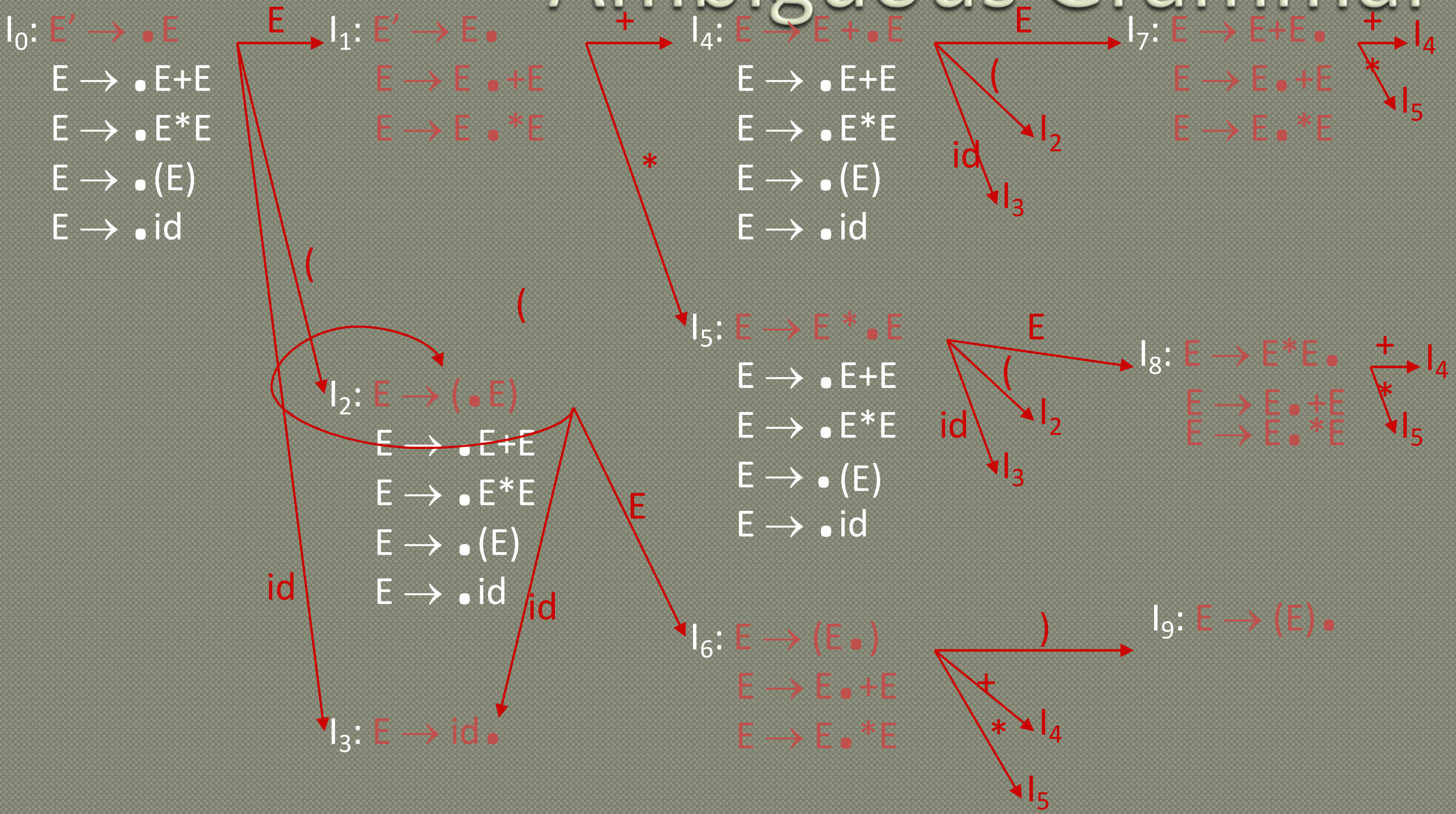
$E \rightarrow E+E \mid E*E \mid (E) \mid id$ ➔

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

Sets of LR(0) Items for Ambiguous Grammar



I_0: E′ → •E
E → •E+E
E → •E*E
E → •(E)
E → •id

I_1: E′ → E•
E → E•+E
E → E•*E

I_2: E → (•E)
E → •E+E
E → •E*E
E → •(E)
E → •id

I_3: E → id•

I_4: E → E+•E
E → •E+E
E → •E*E
E → •(E)
E → •id

I_5: E → E*•E
E → •E+E
E → •E*E
E → •(E)
E → •id

I_6: E → (E•)
E → E•+E
E → E•*E

I_7: E → E+E•
E → E•+E
E → E•*E

I_8: E → E*E•
E → E•+E
E → E•*E

I_9: E → (E)•

SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { \$, + , * ,) }

State I_7 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{+} I_4 \xrightarrow{E} I_7$$

when current token is +
shift ➔ + is right-associative
reduce ➔ + is left-associative

when current token is *
shift ➔ * has higher precedence than +
reduce ➔ + has higher precedence than *

SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, + , * ,) }

State I_8 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{*} I_5 \xrightarrow{E} I_7$$

when current token is *

shift → * is right-associative

reduce → * is left-associative

when current token is +

shift → + has higher precedence than *

reduce → * has higher precedence than +

SLR-Parsing Tables for Ambiguous Grammar

	Action							Goto
	id	**+**	*****	**(**	**)**	**$**		**E**
0	s3			s2				1
1		s4	s5			acc		
2	s3			s2				6
3		r4	r4		r4	r4		
4	s3			s2				7
5	s3			s2				8
6		s4	s5		s9			
7		r1	s5		r1	r1		
8		r2	r2		r2	r2		
9		r3	r3		r3	r3		

Error Recovery in LR Parsing

- An LR parser will detect an error when it consults the parsing action table and finds an error entry. All empty entries in the action table are error entries.
- Errors are never detected by consulting the goto table.
- An LR parser will announce error as soon as there is no valid continuation for the scanned portion of the input.
- A canonical LR parser (LR(1) parser) will never

Panic Mode Error Recovery in LR Parsing

- **Scan down the stack until a state s with a goto on a particular nonterminal A is found. (Get rid of everything from the stack before this state s).**
- **Discard zero or more input symbols until a symbol a is found that can legitimately follow A.**
  - **The symbol a is simply in FOLLOW(A), but this may not work for all situations.**
- **The parser stacks the nonterminal A and the state goto[s,A], and it resumes the normal**

Phrase-Level Error Recovery in LR Parsing

- **Each empty entry in the action table is marked with a specific error routine.**
- **An error routine reflects the error that the user most likely will make in that case.**
- **An error routine inserts the symbols into the stack or the input (or it deletes the symbols from the stack and the input, or it can do both insertion and deletion).**
  - **missing operand**
  - **unbalanced right parenthesis**

Recursive Predictive Parsing

- **Each non-terminal corresponds to a procedure.**

Ex: $A \rightarrow aBb$ (This is only the production rule for A)

```
proc A {
    - match the current token with a, and move to the next token;
    - call 'B';
    - match the current token with b, and move to the next token;
}
```

Recursive Predictive Parsing (cont.)

$A \rightarrow aBb \;\mid\; bAB$

```
proc A {
    case of the current token {
        'a':    - match the current token with a, and move to the next token;
                - call 'B';
                - match the current token with b, and move to the next token;
        'b':    - match the current token with b, and move to the next token;
                - call 'A';
                - call 'B';
    }
}
```

Recursive Predictive Parsing (cont.)

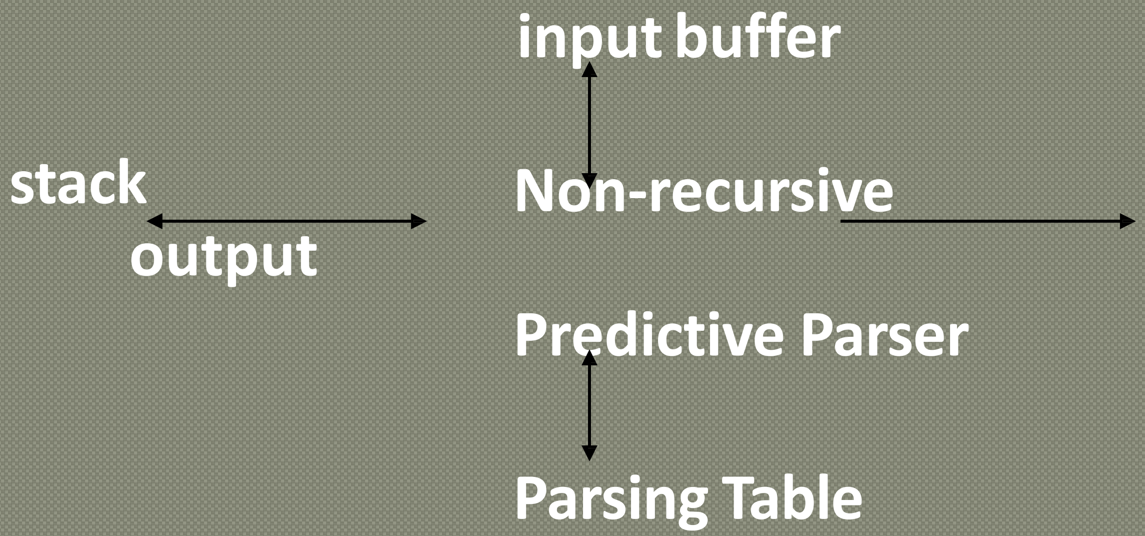
- **When to apply ε-productions.**

  A → aA | bB | ε

- **If all other productions fail, we should apply an ε-production. For example, if the current token is not a or b, we may apply the ε-production.**
- **Most correct choice: We should apply an ε-production for a non-terminal A when the current token is in the follow set of A (which terminals can follow A in the sentential forms).**

Non-Recursive Predictive Parsing -- LL(1) Parser

- Non-Recursive predictive parsing is a table-driven parser.
- It is a top-down parser.
- It is also known as LL(1) Parser.



input buffer

stack

Non-recursive

output

Predictive Parser

Parsing Table

LL(1) Parser

input buffer

- our string to be parsed. We will assume that its end is marked with a special symbol $.

output

- a production rule representing a step of the derivation sequence (left-most derivation) of the string in the input buffer.

stack

- contains the grammar symbols
- at the bottom of the stack, there is a special end marker symbol $.
- initially the stack contains only the symbol $ and the starting symbol S. $S ← initial stack
- when the stack is emptied (ie. only $ left in the stack), the parsing is completed.

LL(1) Parser

parsing table

- a two-dimensional array M[A,a]
- each row is a non-terminal symbol
- each column is a terminal symbol or the special symbol $
- each entry holds a production rule.

LL(1) Parser – Parser Actions

- **The symbol at the top of the stack (say X) and the current symbol in the input string (say a) determine the parser action.**
- **There are four possible parser actions.**

1. **If X and a are $ ➔ parser halts (successful completion)**

2. **If X and a are the same terminal symbol (different from $)**
   ➔ parser pops X from the stack, and moves the next symbol in the input buffer.

LL(1) Parser – Parser Actions

3. If X is a non-terminal

   ➔ parser looks at the parsing table entry M[X,a]. If M[X,a] holds a production rule $X \rightarrow Y_1 Y_2 ... Y_k$, it pops X from the stack and pushes $Y_k, Y_{k-1}, ..., Y_1$ into the stack. The parser also outputs the production rule $X \rightarrow Y_1 Y_2 ... Y_k$ to represent a step of the derivation.

4. none of the above ➔ error
   - all empty entries in the parsing table are errors.
   - If X is a terminal symbol different from a, this is also an error case.

LL(1) Parser – Example1

LL(1)

S → aBa

Parsing

B → bB | ε

Table

	a	b	$
S	S → aBa		
B	B → ε	B → bB	

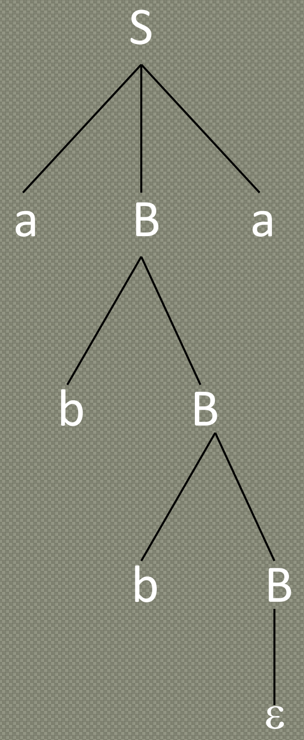
LL(1) Parser – Example

stack	input	output
$S	abba$	S → aBa
$aBa	abba$	
$aB	bba$	B → bB
$aBb	bba$	
$aB	ba$	B → bB
$aBb	ba$	
$aB	a$	B → ε
$a	a$	
$	$	accept, successful

LL(1) Parser – Example1 (cont.)

Outputs: S → aBa   B → bB   B → bB   B → ε

Derivation(left-most):   S⇒aBa⇒abBa⇒abbBa⇒abba



parse tree

LL(1) Parser – Example2

E → TE′
E′ → +TE′ | ε
T → FT′
T′ → *FT′ | ε
F → (E) | id

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → id			F → (E)		

LL(1) Parser – Example2

stack	input	output
$\$E$	$id+id\$$	$E \to TE'$
$\$E'T$	$id+id\$$	$T \to FT'$
$\$E'T'F$	$id+id\$$	$F \to id$
$\$ E'T'id$	$id+id\$$	
$\$ E'T'$	$+id\$$	$T' \to \varepsilon$
$\$ E'$	$+id\$$	$E' \to +TE'$
$\$ E'T+$	$+id\$$	
$\$ E'T$	$id\$$	$T \to FT'$
$\$ E'T'F$	$id\$$	$F \to id$
$\$ E'T'id$	$id\$$	
$\$ E'T'$	$\$$	$T' \to \varepsilon$
$\$ E'$	$\$$	$E' \to \varepsilon$
$\$$	$\$$	accept

Constructing LL(1) Parsing Tables

- **Two functions are used in the construction of LL(1) parsing tables:**
  - **FIRST FOLLOW**

- **FIRST(α) is a set of the terminal symbols which occur as first symbols in strings derived from α where α is any string of grammar symbols.** *

- **if α derives to ε, then* ε is also in FIRST(α) .**

- **FOLLOW(A) is the set of the terminals which**

Compute FIRST for Any String X

- If X is a terminal symbol ➔ FIRST(X)={X}
- If X is a non-terminal symboland $X \rightarrow \varepsilon$ is a production rule ➔ ε is in FIRST(X).
- If X is a non-terminal symbol and $X \rightarrow Y_1Y_2..Y_n$ is a production rule ➔ if a terminal a in FIRST(Y_i) and ε is in all FIRST(Y_j) for j=1,...,i-1 then a is in FIRST(X).
  ➔ if ε is in all FIRST(Y_j) for j=1,...,n then ε is in FIRST(X).
- If X is ε ➔ FIRST(X)={ε}
- If X is $Y_1Y_2..Y_n$

FIRST Example

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \varepsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \varepsilon$

$F \rightarrow (E) \mid id$

FIRST(F) = { (,id}
{ (,id}

FIRST(T′) = {*, ε}
{+}

FIRST(T) = { (,id}

FIRST(E′) = {+, ε}

FIRST(TE′) =

FIRST(+TE′) =

FIRST(ε) = {ε}

FIRST(FT′) =

Compute FOLLOW (for non-terminals)

- **If S is the start symbol ➔ $ is in FOLLOW(S)**

- **if A $\rightarrow$ αBβ is a production rule**
  ➔ everything in FIRST(β) is FOLLOW(B) except ε

- **If (A $\rightarrow$ αB is a production rule) or**
  (A $\rightarrow$ αBβ is a production rule and ε is in FIRST(β)) ➔ everything in FOLLOW(A) is in FOLLOW(B).

FOLLOW Example

$E \rightarrow TE'$

$E' \rightarrow +TE' \;\;|\;\; \varepsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \;\;|\;\; \varepsilon$

$F \rightarrow (E) \;\;|\;\; id$

FOLLOW(E) = { $,) }

FOLLOW(E’) = { $,) }

FOLLOW(T) = { +,), $ }

FOLLOW(T’) = { +,), $ }

Constructing LL(1) Parsing Table -- Algorithm

- for each production rule $A \rightarrow \alpha$ of a grammar G
  - for each terminal a in FIRST(α)
    ➔ add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α)
    ➔ for each terminal a in FOLLOW(A) add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α) and \$ in FOLLOW(A)
    ➔ add $A \rightarrow \alpha$ to M[A,\$]

- All other undefined entries of the parsing table are error entries.

Constructing LL(1) Parsing Table -- Example

$E \rightarrow TE'$ FIRST(TE')={(,id} ➔ $E \rightarrow TE'$ into M[E,(] and M[E,id]

$E' \rightarrow +TE'$ FIRST($+TE'$)={+} ➔ $E' \rightarrow +TE'$ into M[E',+]

$E' \rightarrow \varepsilon$ FIRST(ε)={ε} ➔ none
but since ε in FIRST(ε)
and FOLLOW(E')={\$,)} ➔ $E' \rightarrow \varepsilon$ into M[E',\$] and M[E',)]

$T \rightarrow FT'$ FIRST(FT')={(,id} ➔ $T \rightarrow FT'$ into M[T,(] and M[T,id]

$T' \rightarrow *FT'$ FIRST($*FT'$)={*} ➔ $T' \rightarrow *FT'$ into M[T',*]

$T' \rightarrow \varepsilon$ FIRST(ε)={ε} ➔ none
but since ε in FIRST(ε)
and FOLLOW(T')={\$,),+} ➔ $T' \rightarrow \varepsilon$ into M[T',\$], M[T',)] and
M[T',+]

$F \rightarrow (E)$ FIRST((E))={(} ➔ $F \rightarrow (E)$ into M[F,(]

$F \rightarrow id$ FIRST(id)={id} ➔ $F \rightarrow id$ into M[F,id]

LL(1) Grammars

- A grammar whose parsing table has no multiply-defined entries is said to be LL(1) grammar.

one input symbol used as a look-head symbol do determine parser action

LL(1) left most derivation

input scanned from left to right

A Grammar which is not LL(1)

S → i C t S E | a

E → e S | ε

C → b

FOLLOW(S) = { $,e }

FOLLOW(E) = { $,e }

FOLLOW(C) = { t }

FIRST(iCtSE) = {i}

FIRST(a) = {a}

FIRST(eS) = {e}

FIRST(ε) = {ε}

FIRST(b) = {b}

	a	b	e	i	t	$
S	S → a			S → iCtSE		
E			E → e S E → ε			E → ε
C		C → b				

A Grammar which is not LL(1) (cont.)

- What do we have to do it if the resulting parsing table contains multiply defined entries?
  - If we didn’t eliminate left recursion, eliminate the left recursion in the grammar.
  - If the grammar is not left factored, we have to left factor the grammar.
  - If its (new grammar’s) parsing table still contains multiply defined entries, that grammar is ambiguous or it is inherently not a LL(1) grammar.

A Grammar which is not LL(1) (cont.)

- **A left recursive grammar cannot be a LL(1) grammar.**
  - $A \to A\alpha \mid \beta$

    ➔ any terminal that appears in FIRST(β) also appears FIRST($A\alpha$) because $A\alpha \Rightarrow \beta\alpha$.

    ➔ If β is ε, any terminal that appears in FIRST(α) also appears in FIRST($A\alpha$) and FOLLOW(A).
- **A grammar is not left factored, it cannot be a LL(1) grammar**
  - $A \to \alpha\beta_1 \mid \alpha\beta_2$

    ➔any terminal that appears in FIRST($\alpha\beta_1$) also appears in FIRST($\alpha\beta_2$).

Properties of LL(1) Grammars

- **A grammar G is LL(1) if and only if the following conditions hold for two distinctive production rules $A \rightarrow \alpha$ and $A \rightarrow \beta$**

1. **Both α and β cannot derive strings starting with same terminals.**

2. **At most one of α and β can derive to ε.**

3. **If β can derive to ε, then α cannot derive to any string starting with a terminal in FOLLOW(A).**

Error Recovery in Predictive Parsing

- **An error may occur in the predictive parsing (LL(1) parsing)**
  - **if the terminal symbol on the top of stack does not match with the current input symbol.**
  - **if the top of stack is a non-terminal A, the current input symbol is a, and the parsing table entry M[A,a] is empty.**
- **What should the parser do in an error case?**
  - **The parser should be able to give an error message (as much as possible meaningful error message).**
  - **It should be recover from that error case, and it should be able to continue the parsing with the rest of the input.**

Error Recovery Techniques

- Panic-Mode Error Recovery
  - Skipping the input symbols until a synchronizing token is found.
- Phrase-Level Error Recovery
  - Each empty entry in the parsing table is filled with a pointer to a specific error routine to take care that error case.
- Error-Productions
  - If we have a good idea of the common errors that might be encountered, we can augment the grammar with productions that generate erroneous constructs.
  - When an error production is used by the parser, we can generate appropriate error diagnostics.
  - Since it is almost impossible to know all the errors that can be made by the programmers, this method is not practical.

Error Recovery Techniques

- Global-Correction
  - Ideally, we would like a compiler to make as few change as possible in processing incorrect inputs.
  - We have to globally analyze the input to find the error.
  - This is an expensive method, and it is not in practice.

Panic-Mode Error Recovery in LL(1) Parsing

- In panic-mode error recovery, we skip all the input symbols until a synchronizing token is found.
- What is the synchronizing token?
  - All the terminal-symbols in the follow set of a non-terminal can be used as a synchronizing token set for that non-terminal.
- So, a simple panic-mode error recovery for the LL(1) parsing:
  - All the empty entries are marked as *synch* to indicate that the parser will skip all the input symbols until a symbol in the follow set of the non-terminal A which on the top of the stack. Then the parser will pop that non-terminal A from the

Panic-Mode Error Recovery - Example

S → AbS | e | ε
A → a | cAd

FOLLOW(S)={$}
FOLLOW(A)={b,d}

	a	b	c	d	e	$
S	S → AbS	*sync*	S → AbS	*sync*	S → e	S → ε
A	A → a	*sync*	A → cAd	*sync*	*sync*	*sync*

stack	input	output
$S	aab$	S → AbS
$SbA	aab$	A → a
$Sba	aab$	
$Sbab$	Error: missing b, inserted	
$S	ab$	S → AbS
$SbA	ab$	A → a
$Sba	ab$	
Sbb		
$S	$	S → ε
$	$	accept

stack	input	output
$S	ceadb$	S → AbS
$SbA	ceadb$	A → cAd
$SbdAc	ceadb$	
$SbdA	eadb$	Error:unexpected e (illegal A)
	(Remove all input tokens until first b or d, pop A)	
$Sbd	db$	
$Sb	b$	
$S	$	S → ε
$	$	accept

Phrase-Level Error Recovery

- **Each empty entry in the parsing table is filled with a pointer to a special error routine which will take care that error case.**
- **These error routines may:**
  - **change, insert, or delete input symbols.**
  - **issue appropriate error messages**
  - **pop items from the stack.**
- **We should be careful when we design these error routines, because we may put the parser into an infinite loop.**

Syntax Analyzer

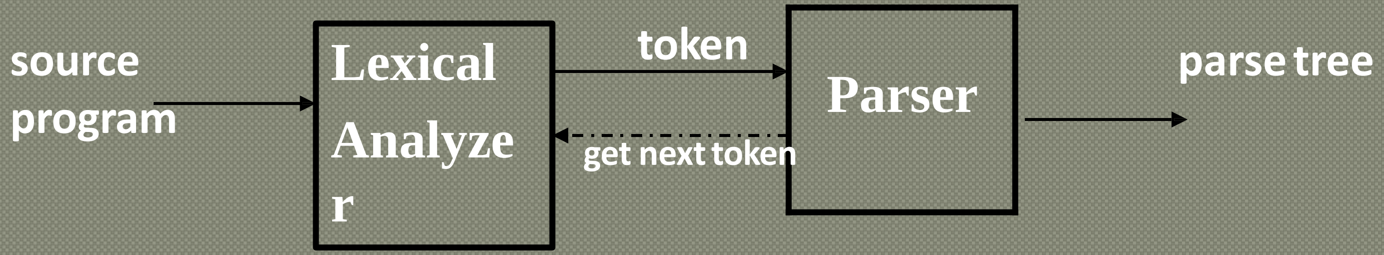
- ***Syntax Analyzer* creates the syntactic structure of the given source program.**
- **This syntactic structure is mostly a *parse tree*.**
- **Syntax Analyzer is also known as *parser*.**
- **The syntax of a programming is described by a *context-free grammar (CFG)*. We will use BNF (Backus-Naur Form) notation in the description of CFGs.**
- **The syntax analyzer (parser) checks whether a given source program satisfies the rules**

Syntax Analyzer

- **A context-free grammar**
  - **gives a precise syntactic specification of a programming language.**
  - **the design of the grammar is an initial phase of the design of a compiler.**
  - **a grammar can be directly converted into a parser by some tools.**

Parser

- Parser works on a stream of tokens.
- The smallest item is a token.



source program

Lexical Analyzer

token

get next token

Parser

parse tree

Parsers (cont.)

- **We categorize the parsers into two groups:**

1. **Top-Down Parser**
   - **the parse tree is created top to bottom, starting from the root.**
2. **Bottom-Up Parser**
   - **the parse is created bottom to top; starting from the leaves**

- **Both top-down and bottom-up parsers scan the input from left to right (one symbol at a**

Context-Free Grammars

- **Inherently recursive structures of a programming language are defined by a context-free grammar.**

- **In a context-free grammar, we have:**
  - **A finite set of terminals (in our case, this will be the set of tokens)**
  - **A finite set of non-terminals (syntactic-variables)**
  - **A finite set of productions rules in the following form**
    - **$A \rightarrow \alpha$ where A is a non-terminal and α is a string of terminals and non-terminals (including the empty string)**

Context-Free Grammars

- **Example:**

  $E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid -E$

  $E \rightarrow (E)$

  $E \rightarrow id$

Derivations

$E \Rightarrow E+E$

- E+E derives from E
  - we can replace E by E+E
  - to able to do this, we have to have a production rule E→E+E in our grammar.

$E \Rightarrow E+E \Rightarrow id+E \Rightarrow id+id$

*

+

- A sequence of replacements of non-terminal symbols is called a derivation of id+id from E.

Derivations

- In general a derivation step is

$\alpha A\beta \Rightarrow \alpha\gamma\beta$ if there is a production rule $A \rightarrow \gamma$ in our grammar

where α and β are arbitrary strings of terminal and non-terminal symbols

$\alpha_1 \Rightarrow \alpha_2 \Rightarrow \ldots \Rightarrow \alpha_n$ (α_n derives from α_1 or α_1 derives α_n)

$\Rightarrow$: derives in one step

CFG - Terminology

- L(G) is *the language of G* (the language generated by G) which is a set of sentences.
- *A sentence of L(G)* is a string of terminal symbols of G.
- If S is the start symb$_{+}$ol of G then

  ω is a sentence of L(G) iff S $\Rightarrow$ ω where ω is a string of terminals of G.

ω If G is a context-free grammar, L(G) is a *context-free language*.

- Two grammars are *equivalent* if they produce the same language.
- S $\Rightarrow$ α - If α contains non-terminals, it is called as a *se***ntential* form of G.

  - If α does not contain non-terminals, it is called as a *sentence* of G.

Derivation Example

$E \Rightarrow -E \Rightarrow -(E) \Rightarrow -(E+E) \Rightarrow -(id+E) \Rightarrow -(id+id)$

OR

$E \Rightarrow -E \Rightarrow -(E) \Rightarrow -(E+E) \Rightarrow -(E+id) \Rightarrow -(id+id)$

- **At each derivation step, we can choose any of the non-terminal in the sentential form of G for the replacement.**
- **If we always choose the left-most non-terminal in each derivation step, this derivation is called as left-most derivation.**
- **If we always choose the right-most non-terminal in each derivation step, this derivation is called as right-most derivation.**

Left-Most and Right-Most Derivations

Left-Most Derivation

$$E \overset{lm}{\Rightarrow} -E \overset{lm}{\Rightarrow} -(E) \overset{lm}{\Rightarrow} -(E+E) \overset{lm}{\Rightarrow} -(id+E) \overset{lm}{\Rightarrow} -(id+id)$$

Right-Most Derivation

$$E \underset{rm}{\Rightarrow} -E \underset{rm}{\Rightarrow} -(E) \underset{rm}{\Rightarrow} -(E+E) \underset{rm}{\Rightarrow} -(E+id) \underset{rm}{\Rightarrow} -(id+id)$$

- **We will see that the top-down parsers try to find the left-most derivation of the given source program.**

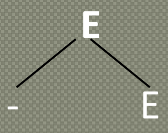
- **We will see that the bottom-up parsers try to find the right-most derivation of the given source program in the reverse order.**

Parse Tree

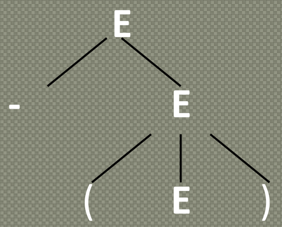
- Inner nodes of a parse tree are non-terminal symbols.
- The leaves of a parse tree are terminal symbols.

- A parse tree can be seen as a graphical representation of a derivation.

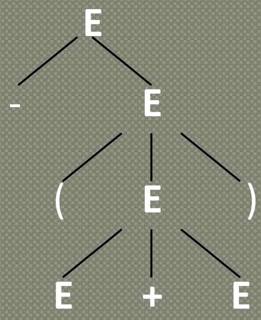
$E \Rightarrow -E$



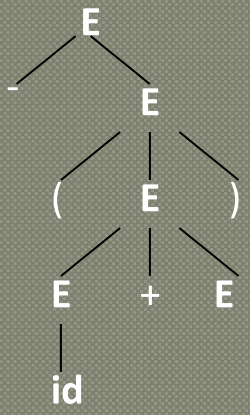
$\Rightarrow -(E)$



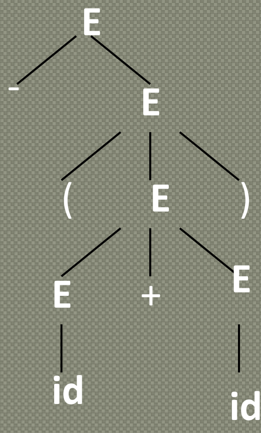
$\Rightarrow -(E+E)$



$\Rightarrow -(id+E)$



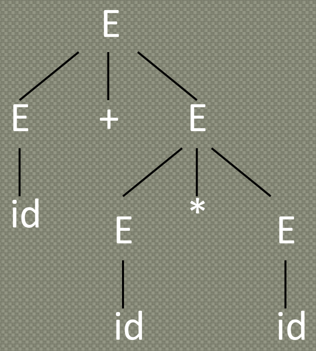
$\Rightarrow -(id+id)$



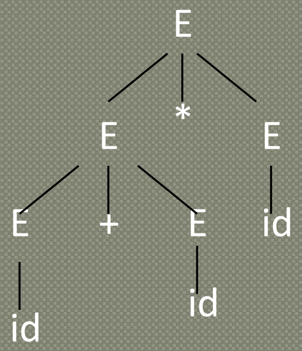
Ambiguity

- A grammar produces more than one parse tree for a sentence is called as an *ambiguous* grammar.

$$E \Rightarrow E+E \Rightarrow id+E \Rightarrow id+E*E \Rightarrow id+id*E \Rightarrow id+id*id$$



$$E \Rightarrow E*E \Rightarrow E+E*E \Rightarrow id+E*E \Rightarrow id+id*E \Rightarrow id+id*id$$



Ambiguity (cont.)

- **For the most parsers, the grammar must be unambiguous.**

- **unambiguous grammar**

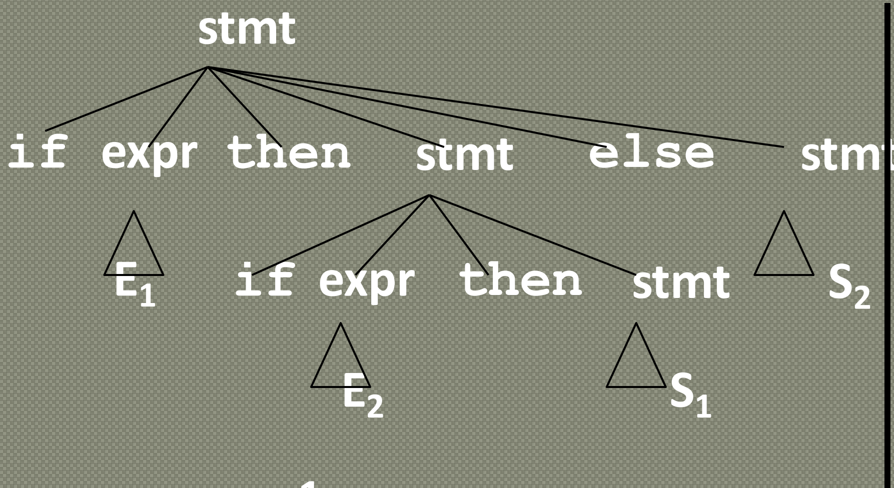
  ➔ unique selection of the parse tree for a sentence

- **We should eliminate the ambiguity in the grammar during the design phase of the compiler.**
- **An unambiguous grammar should be written to eliminate the ambiguity.**
- **We have to prefer one of the parse trees of a sentence (generated by an ambiguous grammar) to disambiguate that grammar to restrict to this choice.**

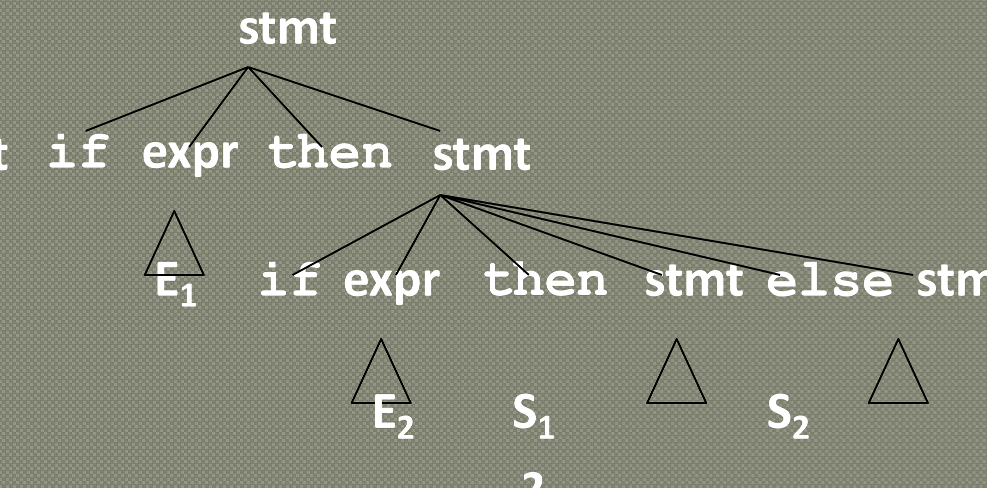
Ambiguity (cont.)

stmt → if expr then stmt |
if expr then stmt else stmt | otherstmts

if E_1 then if E_2 then S_1 else S_2



1



2

Ambiguity

- We prefer the second parse tree (else matches with closest if).
- So, we have to disambiguate our grammar to reflect this choice.

- The unambiguous grammar will be:

```
stmt → matchedstmt | unmatchedstmt

matchedstmt → if expr then matchedstmt else matchedstmt
                                      | otherstmts

unmatchedstmt → if expr then stmt   |
                if expr then matchedstmt else unmatchedstmt
```

Ambiguity – Operator Precedence

- **Ambiguous grammars (because of ambiguous operators) can be disambiguated according to the precedence and associativity rules.**

E → E+E | E*E | E^E | id | (E)

⇓ disambiguate the grammar

precedence:	**^**	**(right to left)**
	*****	**(left to right)**
	+	**(left to right)**

E → E+T | T
T → T*F | F
F → G^F | G
G → id | (E)

Left Recursion

- A grammar is *left recursive* if it has a non-terminal A such that there is a derivation.

  $A \overset{+}{\Rightarrow} A\alpha$ for some string α

- Top-down parsing techniques cannot handle left-recursive grammars.
- So, we have to convert our left-recursive grammar into an equivalent grammar which is not left-recursive.
- The left-recursion may appear in a single step of the derivation (*immediate left-recursion*), or may appear in more than one step of the derivation.

Immediate Left-Recursion

$A \rightarrow A\,\alpha \mid \beta$ where β does not start with A

$\Downarrow$ eliminate immediate left recursion

$A \rightarrow \beta\, A'$

$A' \rightarrow \alpha\, A' \mid \varepsilon$ an equivalent grammar

In general,

$A \rightarrow A\,\alpha_1 \mid \ldots \mid A\,\alpha_m \mid \beta_1 \mid \ldots \mid \beta_n$ where $\beta_1 \ldots \beta_n$ do not start with A

$\Downarrow$ eliminate immediate left recursion

$A \rightarrow \beta_1 A' \mid \ldots \mid \beta_n A'$

$A' \rightarrow \alpha_1 A' \mid \ldots \mid \alpha_m A' \mid \varepsilon$ an equivalent grammar

Immediate Left-Recursion -- Example

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow id \mid (E)$

$\Downarrow$ eliminate immediate left recursion

$E \rightarrow T E'$

$E' \rightarrow +T E' \mid \varepsilon$

$T \rightarrow F T'$

$T' \rightarrow *F T' \mid \varepsilon$

$F \rightarrow id \mid (E)$

Left-Recursion -- Problem

- A grammar cannot be immediately left-recursive, but it still can be left-recursive.
- By just eliminating the immediate left-recursion, we may not get a grammar which is not left-recursive.

$S \rightarrow Aa \mid b$

$A \rightarrow Sc \mid d$ This grammar is not immediately left-recursive, but it is still left-recursive.

$\underline{S} \Rightarrow Aa \Rightarrow \underline{S}ca$ or

$\underline{A} \Rightarrow Sc \Rightarrow \underline{A}ac$ causes to a left-recursion

- So, we have to eliminate all left-recursions from our grammar

Eliminate Left-Recursion -- Algorithm

```
- Arrange non-terminals in some order:  $A_1$ ... $A_n$
- for  i  from  1  to  n  do  {
     - for  j from 1 to i-1 do  {
          replace each production
                    $A_i \rightarrow A_j \gamma$
                        by
                    $A_i \rightarrow \alpha_1 \gamma \mid \ldots \mid \alpha_k \gamma$
                    where $A_j \rightarrow \alpha_1 \mid \ldots \mid \alpha_k$
      }
     - eliminate immediate left-recursions among $A_i$
     productions
}
```

Eliminate Left-Recursion -- Example

$S \rightarrow Aa \mid b$
$A \rightarrow Ac \mid Sd \mid f$

-Order of non-terminals: S, A

for S:

- we do not enter the inner loop.
- there is no immediate left recursion in S.

for A:

- Replace $A \rightarrow Sd$ with $A \rightarrow Aad \mid bd$
  So, we will have $A \rightarrow Ac \mid Aad \mid bd \mid f$
- Eliminate the immediate left-recursion in A

$A \rightarrow bdA' \mid fA'$

$A' \rightarrow cA' \mid adA' \mid \varepsilon$

Eliminate Left-Recursion -- Example

So, the resulting equivalent grammar which is not left-recursive is:

$S \rightarrow Aa \mid b$

$A \rightarrow bdA' \mid fA'$

$A' \rightarrow cA' \mid adA' \mid \varepsilon$

Eliminate Left-Recursion – Example2

$S \rightarrow Aa \mid b$
$A \rightarrow Ac \mid Sd \mid f$

- **Order of non-terminals: A, S**

for A:

- **we do not enter the inner loop.**
- **Eliminate the immediate left-recursion in A**

$$A \rightarrow SdA' \mid fA'$$
$$A' \rightarrow cA' \mid \varepsilon$$

Eliminate Left-Recursion – Example2

for S:

- Replace $S \rightarrow Aa$ with $S \rightarrow SdA'a \mid fA'a$

  So, we will have $S \rightarrow SdA'a \mid fA'a \mid b$
- Eliminate the immediate left-recursion in S

  $S \rightarrow fA'aS' \mid bS'$

  $S' \rightarrow dA'aS' \mid \varepsilon$

So, the resulting equivalent grammar which is not left-recursive is:

$S \rightarrow fA'aS' \mid bS'$

$S' \rightarrow dA'aS' \mid \varepsilon$

$A \rightarrow SdA' \mid fA'$

$A' \rightarrow cA' \mid \varepsilon$

Left-Factoring

- **A predictive parser (a top-down parser without backtracking) insists that the grammar must be *left-factored*.**

  grammar ➔ a new equivalent grammar suitable for predictive parsing

```
stmt → if expr then stmt else stmt  |
       if expr then stmt
```

- **when we see `if`, we cannot now which production rule to choose to re-write *stmt* in the derivation.**

Left-Factoring (cont.)

- In general,

  $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$ where α is non-empty and the first symbols

  of β_1 and β_2 (if they have one)are different.

- when processing α we cannot know whether expand A to $\alpha\beta_1$ or A to $\alpha\beta_2$

- But, if we re-write the grammar as follows $A \rightarrow \alpha A'$

  $A' \rightarrow \beta_1 \mid \beta_2$ so, we can immediately expand A to $\alpha A'$

Left-Factoring -- Algorithm

- **For each non-terminal A with two or more alternatives (production rules) with a common non-empty prefix, let say**

  $$A \rightarrow \alpha\beta_1 \mid \ldots \mid \alpha\beta_n \mid \gamma_1 \mid \ldots \mid \gamma_m$$

  convert it into

  $$A \rightarrow \alpha A' \mid \gamma_1 \mid \ldots \mid \gamma_m$$
  $$A' \rightarrow \beta_1 \mid \ldots \mid \beta_n$$

Left-Factoring – Example1

$$A \rightarrow \underline{a}bB \mid \underline{a}B \mid cdg \mid cdeB \mid cdfB$$

$$\Downarrow$$

$$A \rightarrow aA' \mid \underline{cd}g \mid \underline{cd}eB \mid \underline{cd}fB$$

$$A' \rightarrow bB \mid B$$

$$\Downarrow$$

$$A \rightarrow aA' \mid cdA''$$

$$A' \rightarrow bB \mid B$$

$$A'' \rightarrow g \mid eB \mid fB$$

Left-Factoring – Example2

$$A \rightarrow ad \mid a \mid ab \mid abc \mid b$$

$$\Downarrow$$

$$A \rightarrow aA' \mid b$$

$$A' \rightarrow d \mid \varepsilon \mid b \mid bc$$

$$\Downarrow$$

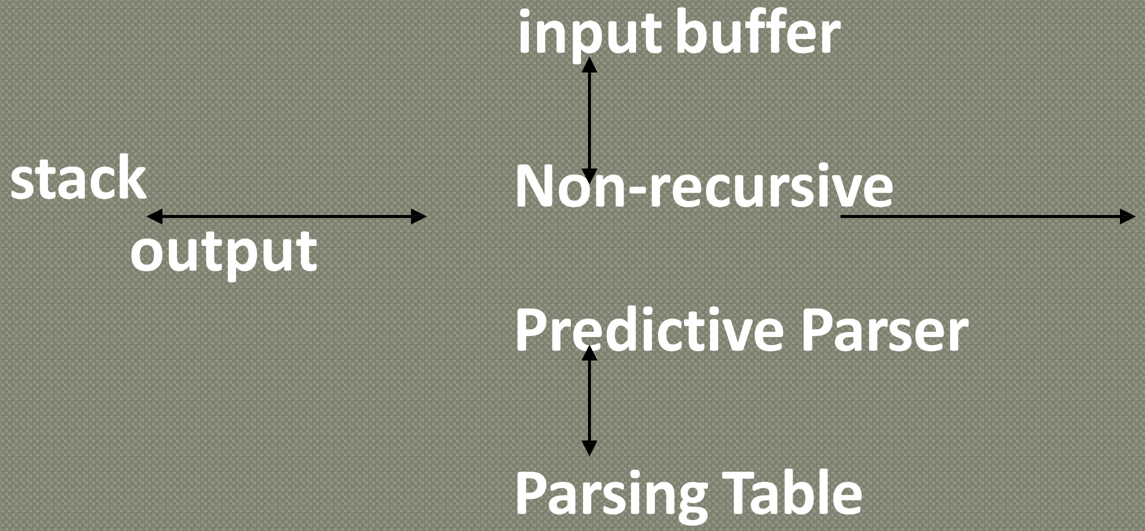
$$A \rightarrow aA' \mid b$$

$$A' \rightarrow d \mid \varepsilon \mid bA''$$

$$A'' \rightarrow \varepsilon \mid c$$

Non-Recursive Predictive Parsing -- LL(1) Parser

- Non-Recursive predictive parsing is a table-driven parser.
- It is a top-down parser.
- It is also known as LL(1) Parser.



input buffer

stack

Non-recursive

output

Predictive Parser

Parsing Table

LL(1) Parser

input buffer

- our string to be parsed. We will assume that its end is marked with a special symbol $.

output

- a production rule representing a step of the derivation sequence (left-most derivation) of the string in the input buffer.

stack

- contains the grammar symbols
- at the bottom of the stack, there is a special end marker symbol $.
- initially the stack contains only the symbol $ and the starting symbol S. $S ← initial stack

LL(1) Parser

- when the stack is emptied (ie. only $ left in the stack), the parsing is completed.

parsing table

- a two-dimensional array M[A,a]
- each row is a non-terminal symbol
- each column is a terminal symbol or the special symbol $
- each entry holds a production rule.

LL(1) Parser – Parser Actions

- **The symbol at the top of the stack (say X) and the current symbol in the input string (say a) determine the parser action.**
- **There are four possible parser actions.**

1. **If X and a are $ ➔ parser halts (successful completion)**

2. **If X and a are the same terminal symbol (different from $)**

   ➔ parser pops X from the stack, and moves the next symbol in the input buffer.

LL(1) Parser – Parser Actions

3. **If X is a non-terminal**

   ➔ parser looks at the parsing table entry M[X,a]. If M[X,a] holds a production rule $X \rightarrow Y_1 Y_2 ... Y_k$, it pops X from the stack and pushes $Y_k, Y_{k-1}, ..., Y_1$ into the stack. The parser also outputs the production rule $X \rightarrow Y_1 Y_2 ... Y_k$ to represent a step of the derivation.

4. **none of the above ➔ error**
   - **all empty entries in the parsing table are errors.**
   - **If X is a terminal symbol different from a, this is also an error case.**

LL(1) Parser – Example1

$S \rightarrow aBa$

Parsing

$B \rightarrow bB \quad | \; \varepsilon$

	a	b	$
S	$S \rightarrow aBa$		
B	$B \rightarrow \varepsilon$	$B \rightarrow bB$	

LL(1)

Table

<u>stack</u>	<u>input</u>	<u>output</u>
$S	abba$	$S \rightarrow aBa$
$aBa	abba$	$B \rightarrow bB$
$aB	bba$	
$aBb	bba$	$B \rightarrow bB$
$aB	ba$	
$aBb	ba$	

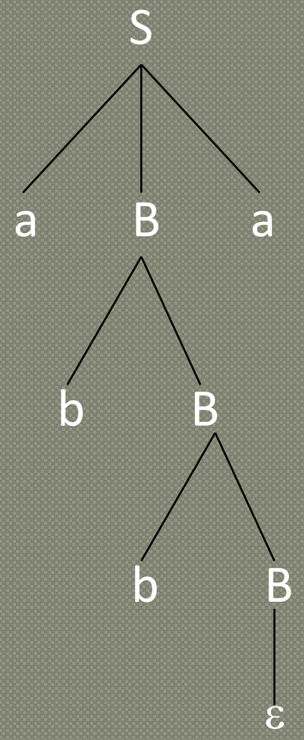
LL(1) Parser – Example1

$aB	a$	B → ε
$a	a$	
$	$	**accept, successful completion**

LL(1) Parser – Example1 (cont.)

Outputs: S → aBa   B → bB   B → bB   B → ε

Derivation(left-most):   S⇒aBa⇒abBa⇒abbBa⇒abba



parse tree

LL(1) Parser – Example2

E → TE′
E′ → +TE′ | ε
T → FT′
T′ → *FT′ | ε
F → (E) | id

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε

LL(1) Parser – Example2

stack	input	output
$\$E$	$id+id\$$	$E \rightarrow TE'$
$\$E'T$	$id+id\$$	$T \rightarrow FT'$
$\$E'T'F$	$id+id\$$	$F \rightarrow id$
$\$ E'T'id$	$id+id\$$	
$\$ E'T'$	$+id\$$	$T' \rightarrow \varepsilon$
$\$ E'$	$+id\$$	$E' \rightarrow +TE'$
$\$ E'T+$	$+id\$$	
$\$ E'T$	$id\$$	$T \rightarrow FT'$

LL(1) Parser – Example2

$ E′ T′ F	id$	F → id
$ E′ T′id	id$	
$ E′ T′	$	T′ → ε
$ E′	$	E′ → ε
$	$	accept

Constructing LL(1) Parsing Tables

- **Two functions are used in the construction of LL(1) parsing tables:**
  - **FIRST FOLLOW**

- **FIRST(α) is a set of the terminal symbols which occur as first symbols in strings derived from α where α is any string of grammar symbols.**
- **if α derives to ε, then ε is also in FIRST(α) .**

- **FOLLOW(A) is the set of the terminals which occur immediately after (follow) the *non-terminal A* in the strings derived from the starting symbol.**
  - **a terminal a is in FOLLOW(A) if $S \Rightarrow \alpha A a \beta$**
  - **\$ is in FOLLOW(A) if $S \Rightarrow \alpha A$**

$*$

$*$

Compute FIRST for Any String X

- If X is a terminal symbol ➔ FIRST(X)={X}
- If X is a non-terminal symbol and $X \rightarrow \varepsilon$ is a production rule ➔ ε is in FIRST(X).
- If X is a non-terminal symbol and $X \rightarrow Y_1Y_2..Y_n$ is a production rule ➔ if a terminal a in FIRST(Y_i) and ε is in all FIRST(Y_j) for j=1,...,i-1 then a is in FIRST(X).
  ➔ if ε is in all FIRST(Y_j) for j=1,...,n
  then ε is in FIRST(X).
- If X is ε ➔ FIRST(X)={ε}
- If X is $Y_1Y_2..Y_n$
  ➔ if a terminal a in FIRST(Y_i) and ε is in all FIRST(Y_j) for j=1,...,i-1 then a is in FIRST(X).
  ➔ if ε is in all FIRST(Y_j) for j=1,...,n
  then ε is in FIRST(X).

FIRST Example

$E \rightarrow TE'$
$E' \rightarrow +TE' \mid \varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \mid \varepsilon$
$F \rightarrow (E) \mid id$

FIRST(F) = { (,id}
FIRST(T′) = {*, ε}
FIRST(T) = { (,id}
FIRST(E′) = {+, ε}
FIRST(E) = { (,id}

FIRST(TE′) = { (,id}
FIRST(+TE′) = {+}
FIRST(ε) = {ε}
FIRST(FT′) = { (,id}
FIRST(*FT′) = {*}
FIRST(ε) = {ε}
FIRST((E)) = { (}
FIRST(id) = {id}

Compute FOLLOW (for non-terminals)

- If S is the start symbol ➔ $ is in FOLLOW(S)

- if $A \rightarrow \alpha B \beta$ is a production rule
  ➔ everything in FIRST(β) is FOLLOW(B) except ε

- If ($A \rightarrow \alpha B$ is a production rule) or
  ($A \rightarrow \alpha B \beta$ is a production rule and ε is in FIRST(β))
  ➔ everything in FOLLOW(A) is in FOLLOW(B).

We apply these rules until nothing more can be added to any follow set.

FOLLOW Example

$E \rightarrow TE'$
$E' \rightarrow +TE' \;|\; \varepsilon$
$T \rightarrow FT'$
$T' \rightarrow *FT' \;|\; \varepsilon$
$F \rightarrow (E) \;|\; id$

FOLLOW(E) = { $,) }

FOLLOW(E′) = { $,) }

FOLLOW(T) = { +,), $ }

FOLLOW(T′) = { +,), $ }

FOLLOW(F) = {+, *,), $ }

Constructing LL(1) Parsing Table -- Algorithm

- **for each production rule $A \rightarrow \alpha$ of a grammar G**
  - for each terminal a in FIRST(α)
    ➔ add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α) ➔
    for each terminal a in FOLLOW(A) add $A \rightarrow \alpha$ to M[A,a]
  - If ε in FIRST(α) and \$ in FOLLOW(A) ➔
    add $A \rightarrow \alpha$ to M[A,\$]

- **All other undefined entries of the parsing table are error entries.**

Constructing LL(1) Parsing Table - - Example

$E \rightarrow TE'$ FIRST(TE')={(,id} ➔ $E \rightarrow TE'$ into M[E,(] and M[E,id]

$E' \rightarrow +TE'$ FIRST($+TE'$)={+} ➔ $E' \rightarrow +TE'$ into M[E′,+]

$E' \rightarrow \varepsilon$ FIRST(ε)={ε} ➔ none
but since ε in FIRST(ε)
and FOLLOW(E')={\$,)} ➔ $E' \rightarrow \varepsilon$ into M[E′,\$] and M[E′,)]

$T \rightarrow FT'$ FIRST(FT')={(,id} ➔ $T \rightarrow FT'$ into M[T,(] and M[T,id]

$T' \rightarrow *FT'$ FIRST($*FT'$)={*} ➔ $T' \rightarrow *FT'$ into M[T′,*]

Constructing LL(1) Parsing Table -- Example

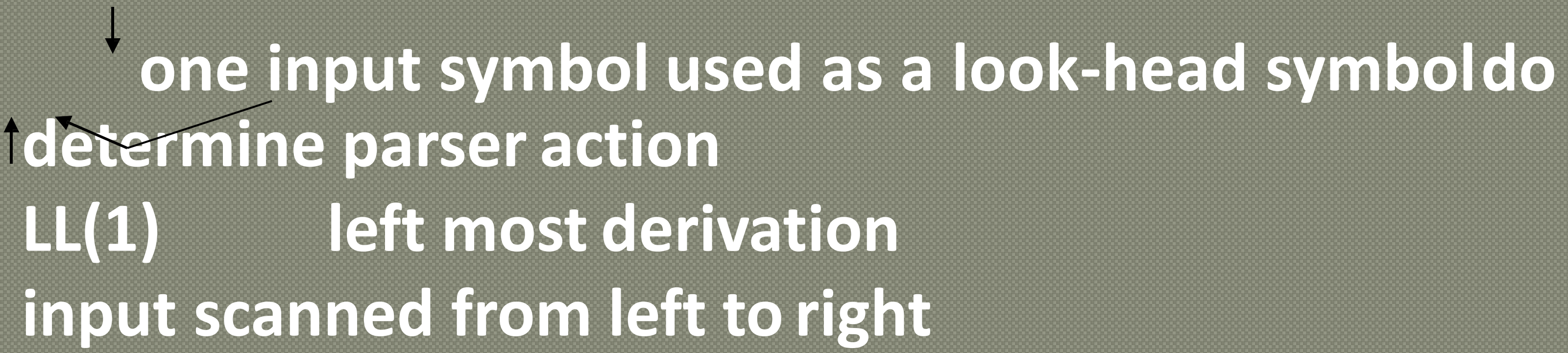
$T' \rightarrow \varepsilon$ FIRST(ε)={ε} ➔ none
but since ε in FIRST(ε)
and FOLLOW(T')={\$,),+} ➔ $T' \rightarrow \varepsilon$
into M[T',\$], M[T',)] and M[T',+]

$F \rightarrow$ (E) FIRST((E))={(} ➔ $F \rightarrow$
(E) into M[F,(]

$F \rightarrow$ id FIRST(id)={id} ➔ $F \rightarrow$ id
into M[F,id]

LL(1) Grammars

- A grammar whose parsing table has no multiply-defined entries is said to be LL(1) grammar.



one input symbol used as a look-head symbol do determine parser action

LL(1) left most derivation

input scanned from left to right

- The parsing table of a grammar may contain more than one production rule. In this case, we say that it is not a LL(1) grammar.

A Grammar which is not LL(1)

S → i C t S E | a
E → e S | ε
C → b

FOLLOW(S) = { $,e }
FOLLOW(E) = { $,e }
FOLLOW(C) = { t }

FIRST(iCtSE) = {i}
FIRST(a) = {a}
FIRST(eS) = {e}
FIRST(ε) = {ε}
FIRST(b) = {b}

	a	b	e	i	t	$
S	S → a			S → iCtSE		
E			E → e S E → ε			E → ε
C		C → b				

two production rules for M[E,e]

Problem ➔ ambiguity

A Grammar which is not LL(1) (cont.)

- **What do we have to do it if the resulting parsing table contains multiply defined entries?**
  - **If we didn’t eliminate left recursion, eliminate the left recursion in the grammar.**
  - **If the grammar is not left factored, we have to left factor the grammar.**
  - **If its (new grammar’s) parsing table still contains multiply defined entries, that grammar is ambiguous or it is inherently not a LL(1) grammar.**

A Grammar which is not LL(1) (cont.)

- **A left recursive grammar cannot be a LL(1) grammar.**
  - $A \rightarrow A\alpha \mid \beta$
    - ➔ any terminal that appears in FIRST(β) also appears FIRST($A\alpha$) because $A\alpha \Rightarrow \beta\alpha$.
    - ➔ If β is ε, any terminal that appears in FIRST(α) also appears in FIRST($A\alpha$) and FOLLOW(A).
- **A grammar is not left factored, it cannot be a LL(1) grammar**
  - $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2$
    - ➔any terminal that appears in FIRST($\alpha\beta_1$) also appears in FIRST($\alpha\beta_2$).
- **An ambiguous grammar cannot be a LL(1) grammar.**

Properties of LL(1) Grammars

- **A grammar G is LL(1) if and only if the following conditions hold for two distinctive production rules $A \rightarrow \alpha$ and $A \rightarrow \beta$**

1. **Both α and β cannot derive strings starting with same terminals.**

2. **At most one of α and β can derive to ε.**

3. **If β can derive to ε, then α cannot derive to any string starting with a terminal in FOLLOW(A).**

Error Recovery in Predictive Parsing

- **An error may occur in the predictive parsing (LL(1) parsing)**
  - **if the terminal symbol on the top of stack does not match with the current input symbol.**
  - **if the top of stack is a non-terminal A, the current input symbol is a, and the parsing table entry M[A,a] is empty.**
- **What should the parser do in an error case?**
  - **The parser should be able to give an error message (as much as possible meaningful error message).**
  - **It should be recover from that error case, and it should be able to continue the parsing with the rest of the input.**

Error Recovery Techniques

- **Panic-Mode Error Recovery**
  - Skipping the input symbols until a synchronizing token is found.
- **Phrase-Level Error Recovery**
  - Each empty entry in the parsing table is filled with a pointer to a specific error routine to take care that error case.
- **Error-Productions**
  - If we have a good idea of the common errors that might be encountered, we can augment the grammar with productions that generate erroneous constructs.
  - When an error production is used by the parser, we can generate appropriate error diagnostics.

Error Recovery Techniques

- **Global-Correction**
  - **Ideally, we would like a compiler to make as few change as possible in processing incorrect inputs.**
  - **We have to globally analyze the input to find the error.**
  - **This is an expensive method, and it is not in practice.**

Recovery in LL(1) Parsing

- **In panic-mode error recovery, we skip all the input symbols until a synchronizing token is found.**
- **What is the synchronizing token?**
  - **All the terminal-symbols in the follow set of a non-terminal can be used as a synchronizing token set for that non-terminal.**
- **So, a simple panic-mode error recovery for the LL(1) parsing:**
  - **All the empty entries are marked as *synch* to indicate that the parser will skip all the input symbols until a symbol in the follow set of the non-terminal A which on the top of the stack. Then the parser will pop that non-terminal A from the stack. The parsing continues from that state.**
  - **To handle unmatched terminal symbols, the parser pops that unmatched terminal symbol from the stack and it issues an error message saying that that unmatched terminal is inserted.**

Panic-Mode Error Recovery - Example

S → AbS | e | ε
A → a | cAd

FOLLOW(S)={$}
FOLLOW(A)={b,d}

	a	b	c	d	e	$
S	S → AbS	*sync*	S → AbS	*sync*	S → e	S → ε
A	A → a	*sync*	A → cAd	*sync*	*sync*	*sync*

stack **input** **out put**
output

$S aab$ S → AbS
S → AbS

$SbA aab$ A → a
A → cAd

$Sba aab$

$Sb ab$

stack **input**

$S ceadb$

$SbA ceadb$

$SbdAc ceadb$

$SbdAeadb$

Panic-Mode Error Recovery - Example

$S ab$ S → AbS (Remove all input tokens until first b or d, pop A)

$SbA ab$ A → a $Sbd db$

$Sba ab$ $Sb b$

$Sb b$ $S $ S → ε

$S $ S → ε $ $
accept

$ $ accept

Error : unexpected e (illegal A)

Phrase-Level Error Recovery

- Each empty entry in the parsing table is filled with a pointer to a special error routine which will take care that error case.
- These error routines may:
  - change, insert, or delete input symbols.
  - issue appropriate error messages
  - pop items from the stack.
- We should be careful when we design these error routines, because we may put the parser into an infinite loop.

UNIT-4

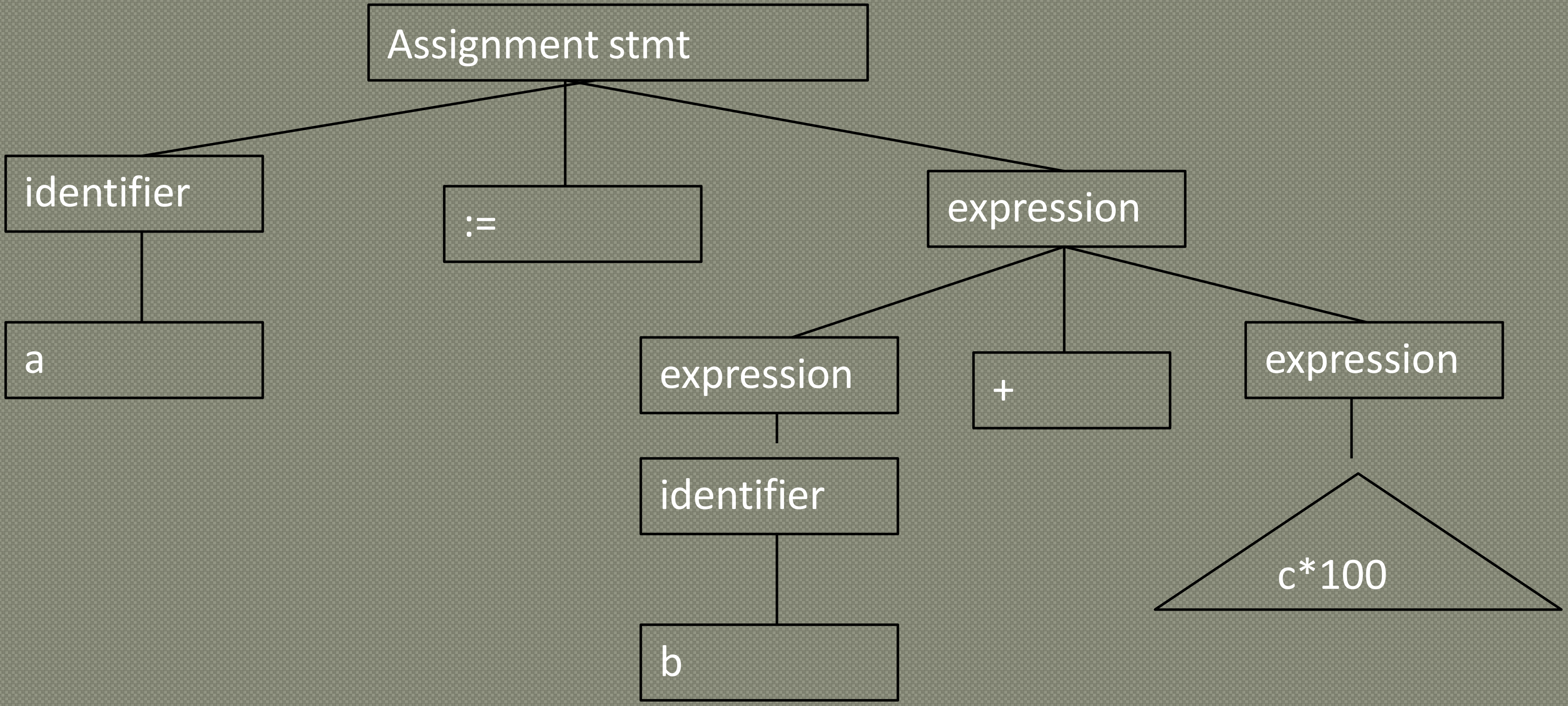
Run time storage: Storage organization, storage allocation strategies scope access to now local names, parameters, language facilities for dynamics storage allocation.

Code optimization: Principal sources of optimization of basic blocks, peephole optimization, flow graphs, data flow analysis of flow graphs

Syntax Analysis Example

a := b + c* 100

- The seven tokens are grouped into a parse tree

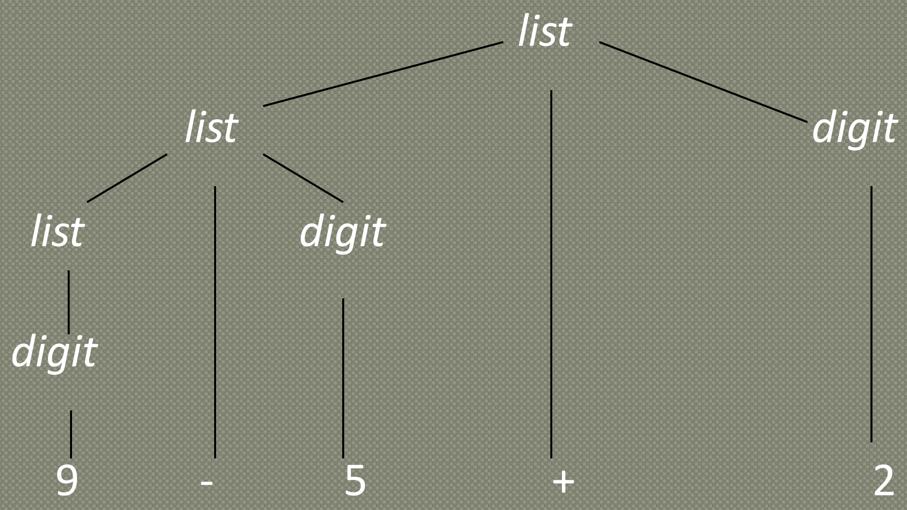


Example of Parse Tree

Given the grammar:

list → *list* + *digit* (2.2
)
list → *list* - *digit*
list → *digit* (2.3) (2.4
digit → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9)
(2.5
)

What is the parse tree for 9-5+2?

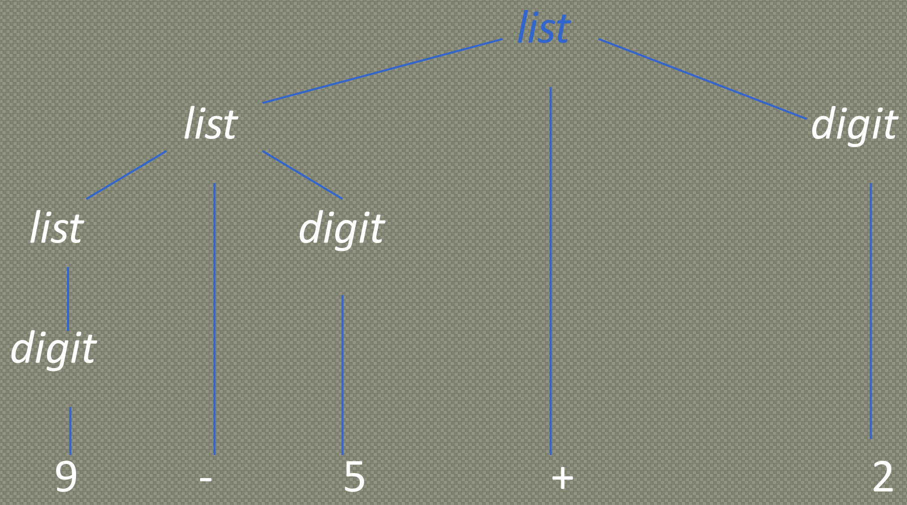


Abstract Syntax Tree (AST)

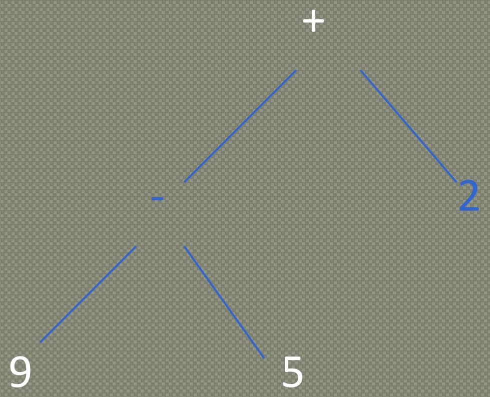
The AST is a condensed/simplified/abstract form of the *parse tree* in which:

1. Operators are directly associated with interior nodes (non-terminals)
2. Chains of single productions are collapsed.
3. Terminals that have no attributes are ignored, i.e., the corresponding leaf nodes are discarded.

Abstract and Concrete Trees



Parse or concrete tree



Abstract syntax tree

Advantages of the AST Representation

- **Convenient representation for semantic analysis and intermediate-language (IL) generation**
- **Useful for building other programming language tools e.t., a syntax-directed editor**

Syntax Directed Translation (SDT)

Syntax-directed translation is a method of translating a string into a sequence of actions by attaching on such action to each rule of a grammar.

A syntax-directed translation is defined by augmenting the CFG: a translation rule is defined for each production. A translation rule defines the translation of the left-hand side non terminal.

Translation Schemes

A. Syntax-Directed Definitions:

- give high-level specifications for translations
- hide many implementation details such as order of evaluation of semantic actions.
- We associate a production rule with a set of semantic actions, and we do not say when they will be evaluated.

B. Translation Schemes:

- Indicate the order of evaluation of semantic actions associated with a production rule.
- In other words, translation schemes give a little bit information about implementation details.

Example Syntax-Directed Definition

```
term ::= ID
{ term.place := ID.place ; term.code = “” }

$term_1$ ::= $term_2$ * ID
       {$term_1$.place := newtemp( );
        $term_1$.code := $term_2$.code || ID.code ||
                      gen($term_1$.place ‘:=‘ $term_2$.place ‘*’ ID.place}

expr ::= term
       { expr.place := term.place ; expr.code := term.code }

$expr_1$ ::= $expr_2$ + term
       { $expr_1$.place := newtemp( )
          expr1.code := $expr_2$.code || term.code ||
                      gen($expr_1$.place ‘:=‘ $expr_2$.place ‘+’ term.place }
```

YACC – Yet Another Compiler-Compiler

- A bottom-up parser generator
- It provides semantic stack manipulation and supports specification of semantic routines.
- Developed by Steve Johnson and others at AT&T Bell Lab.
- Can use scanner generated by Lex or hand-coded scanner in C
- Used by many compilers and tools, including production compilers.

Syntax-Directed Translation

- Grammar symbols are associated with attributes to associate information with the programming language constructs that they represent.
- Values of these attributes are evaluated by the semantic rules associated with the production rules.
- Evaluation of these semantic rules:
  - may generate intermediate codes
  - may put information into the symbol table
  - may perform type checking
  - may issue error messages
  - may perform some other activities
  - in fact, they may perform almost any activities.
- An attribute may hold almost any thing.
  - a string, a number, a memory location, a complex record.

Translation Schemes

- **When we associate semantic rules with productions, we use two notations:**
  - **Syntax-Directed Definitions**
  - **Translation Schemes**
- **Syntax-Directed Definitions:**
  - **give high-level specifications for translations**
  - **hide many implementation details such as order of evaluation of semantic actions.**
  - **We associate a production rule with a set of semantic actions, and we do not say when they will be evaluated.**

Syntax-Directed Definitions and Translation Schemes

- **Translation Schemes:**
  - **indicate the order of evaluation of semantic actions associated with a production rule.**
  - **In other words, translation schemes give a little bit information about implementation details.**

Syntax-Directed Definitions

- **A syntax-directed definition is a generalization of a context-free grammar in which:**
  - **Each grammar symbol is associated with a set of attributes.**
  - **This set of attributes for a grammar symbol is partitioned into two subsets called synthesized and inherited attributes of that grammar symbol.**
  - **Each production rule is associated with a set of semantic rules.**
- ***Semantic rules* set up dependencies between attributes which can be represented by a *dependency graph*.**
- **This *dependency graph* determines the evaluation order of these semantic rules.**
- **Evaluation of a semantic rule defines the value of an attribute. But a semantic rule may also have some side effects such as**

Annotated Parse Tree

- **A parse tree showing the values of attributes at each node is called an annotated parse tree.**
- **The process of computing the attributes values at the nodes is called annotating (or decorating) of the parse tree.**
- **Of course, the order of these computations depends on the dependency graph induced by the semantic rules.**

Syntax-Directed Definition

- In a syntax-directed definition, each production $A\rightarrow\alpha$ is associated with a set of semantic rules of the form:

    $b=f(c_1,c_2,\ldots,c_n)$ where f is a function,

    and b can be one of the followings:

    ➔ b is a synthesized attribute of A and $c_1,c_2,\ldots,c_n$ are attributes of the grammar symbols in the production ($A\rightarrow\alpha$).

    OR

    ➔ b is an inherited attribute one of the grammar symbols in α (on the right side of the production), and $c_1,c_2,\ldots,c_n$ are attributes of the grammar symbols in the production ($A\rightarrow\alpha$).

Attribute Grammar

- **So, a semantic rule *$b=f(c_1,c_2,\ldots,c_n)$* indicates that the attribute b *depends* o*n* attributes *$c_1,c_2,\ldots,c_n$*.**
- **In a syntax-directed definition, a semantic rule may just evaluate a value of an attribute or it may have some side effects such as printing values.**

- **An attribute grammar is a syntax-directed definition in which the functions in the semantic rules cannot have side effects (they can only evaluate values of attributes).**

Syntax-Directed Definition -- Example

Production

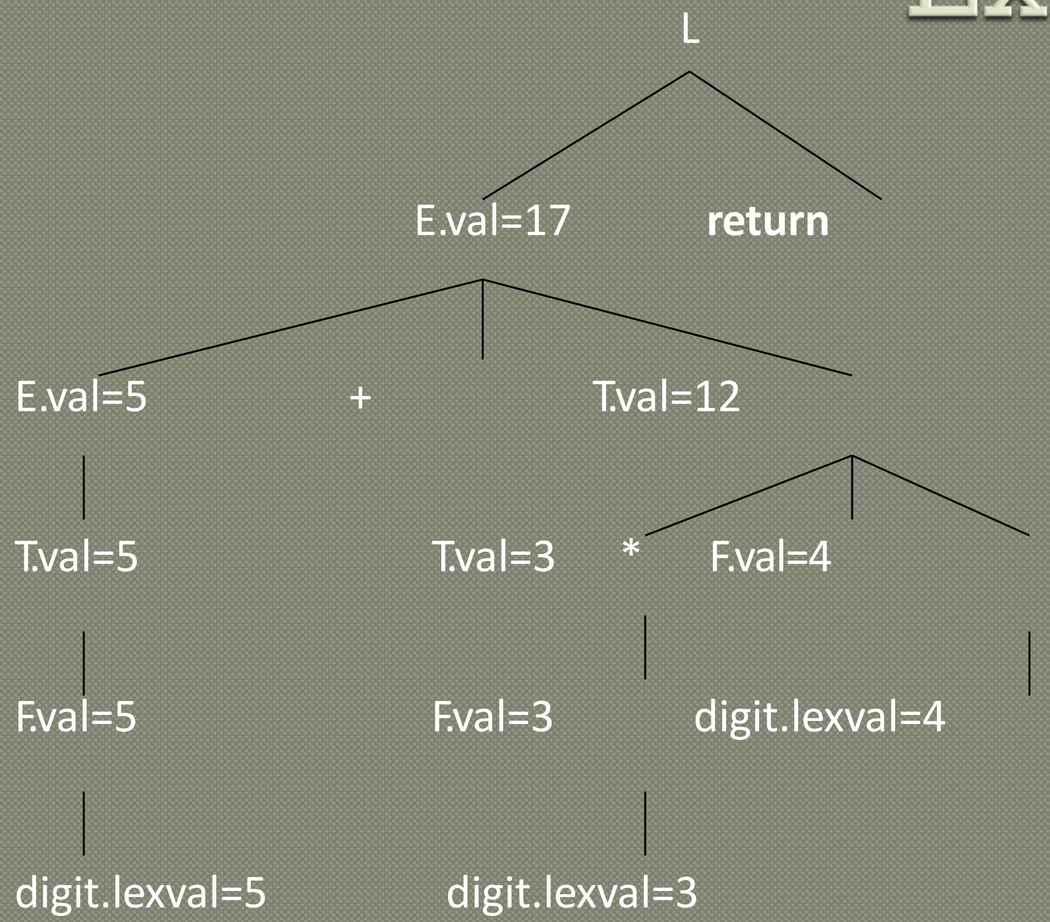
L → E return
$E \rightarrow E_1 + T$
E → T
$T \rightarrow T_1 * F$
T → F
F → (E)
F → digit

- Semantic Rules
print(E.val)
    - $E.val = E_1.val + T.val$
- E.val = T.val
    - $T.val = T_1.val * F.val$
- T.val = F.val
- F.val = E.val
- F.val = digit.lexval

- **Symbols E, T, and F are associated with a synthesized attribute *val*.**
- **The token digit has a synthesized attribute *lexval* (it is assumed that it is evaluated by the lexical analyzer).**

Annotated Parse Tree -- Example

Input: 5+3*4



L

E.val=17 **return**

E.val=5 + T.val=12

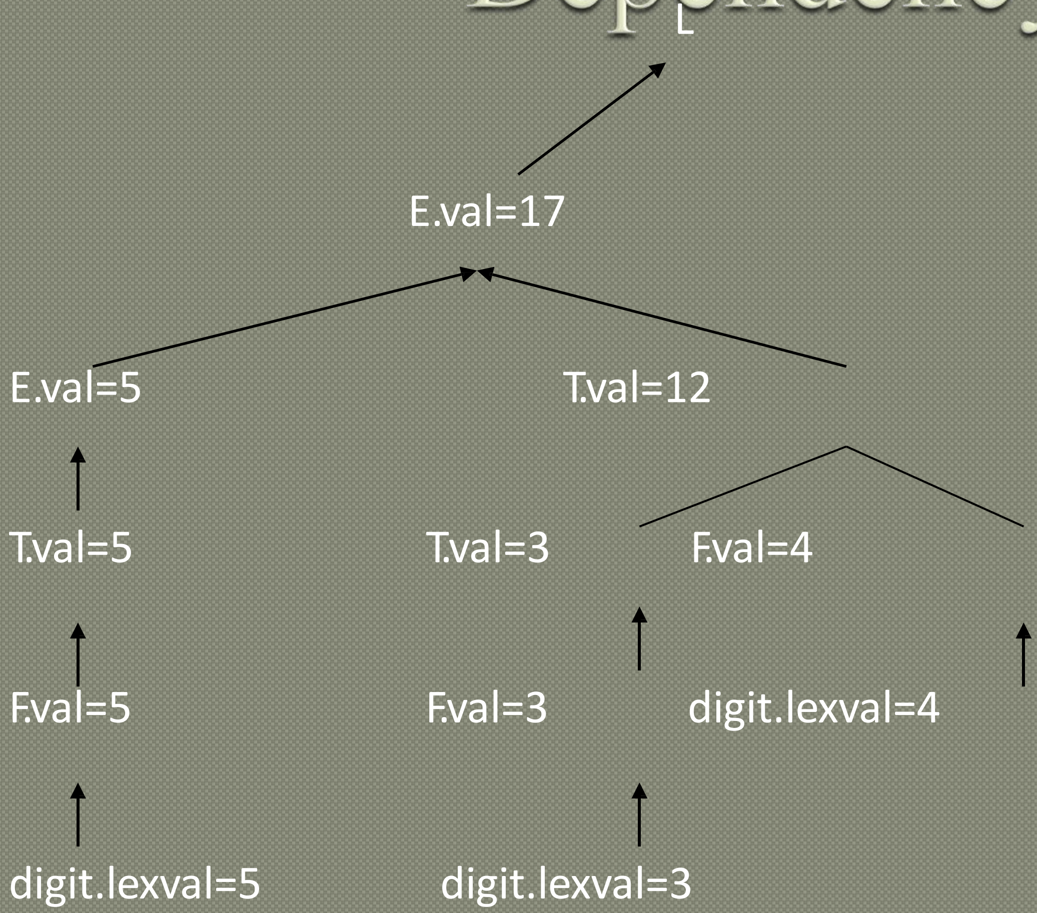
T.val=5 T.val=3 * F.val=4

F.val=5 F.val=3 digit.lexval=4

digit.lexval=5 digit.lexval=3

Dependency Graph

Input: 5+3*4



Syntax-Directed Definition – Example2

Production	Semantic Rules
$E \rightarrow E_1 + T$ T.code	E.loc=newtemp(), E.code = E_1.code || || add E_1.loc,T.loc,E.loc
$E \rightarrow T$	E.loc = T.loc, E.code=T.code
$T \rightarrow T_1 * F$ F.code	T.loc=newtemp(), T.code = T_1.code || || mult T_1.loc,F.loc,T.loc
$T \rightarrow F$	T.loc = F.loc, T.code=F.code
$F \rightarrow (E)$	F.loc = E.loc, F.code=E.code
$F \rightarrow$ id	F.loc = id.name, F.code=“”

Syntax-Directed Definition – Example2

- **Symbols E, T, and F are associated with synthesized attributes *loc* and *code*.**
- **The token id has a synthesized attribute *name* (it is assumed that it is evaluated by the lexical analyzer).**
- **It is assumed that || is the string concatenation operator.**

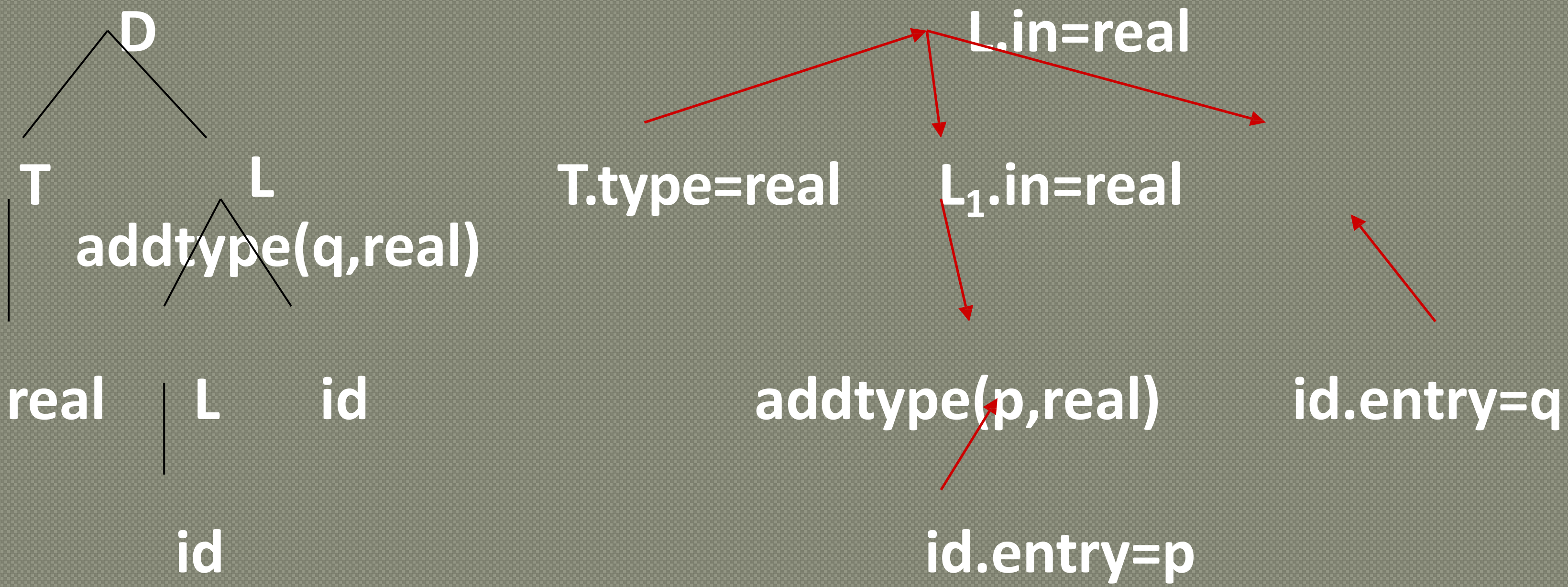
Syntax-Directed Definition – Inherited Attributes

Production	**Semantic Rules**
D → T L	L.in = T.type
T → int	T.type = integer
T → real	T.type = real
L → L_1 id	L_1.in = L.in, addtype(id.entry,L.in)
L → id	addtype(id.entry,L.in)

- Symbol T is associated with a synthesized attribute *type*.
- Symbol L is associated with an inherited attribute *in*.

A Dependency Graph – Inherited Attributes

Input: `real p q`



D

T

L

real

L

id

id

L.in=real

T.type=real

L_1.in=real

addtype(q,real)

addtype(p,real)

id.entry=q

id.entry=p

parse tree

dependency graph

S-Attributed Definitions

- **Syntax-directed definitions are used to specify syntax-directed translations.**
- **To create a translator for an arbitrary syntax-directed definition can be difficult.**
- **We would like to evaluate the semantic rules during parsing (i.e. in a single pass, we will parse and we will also evaluate semantic rules during the parsing).**
- **We will look at two sub-classes of the syntax-directed definitions:**
  - **S-Attributed Definitions: only synthesized attributes used in the syntax-directed definitions.**
  - **L-Attributed Definitions: in addition to synthesized attributes, we may also use inherited attributes in a restricted fashion.**

S-Attributed Definitions

- **To implement S-Attributed Definitions and L-Attributed Definitions are easy (we can evaluate semantic rules in a single pass during the parsing).**
- **Implementations of S-attributed Definitions are a little bit easier than implementations of L-Attributed Definitions**

Bottom-Up Evaluation of S-Attributed Definitions

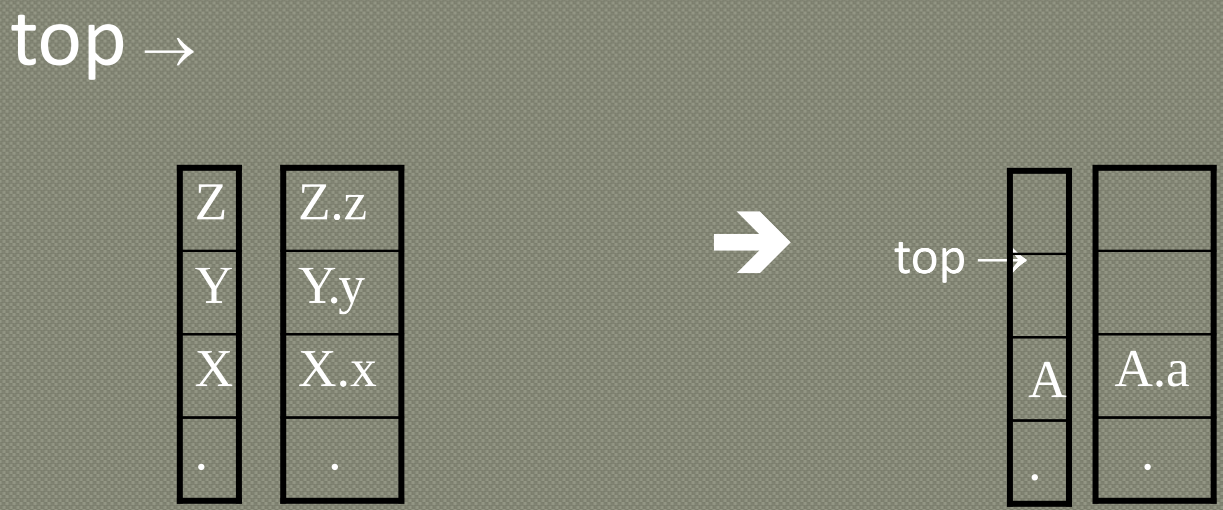
- We put the values of the synthesized attributes of the grammar symbols into a parallel stack.
  - When an entry of the parser stack holds a grammar symbol X (terminal or non-terminal), the corresponding entry in the parallel stack will hold the synthesized attribute(s) of the symbol X.

- We evaluate the values of the attributes during reductions.

Attributed Definitions

A → XYZ A.a=f(X.x,Y.y,Z.z) where all attributes are synthesized.

stack parallel-stack



top →

Z Z.z

Y Y.y

X X.x

. .

→

top →

A A.a

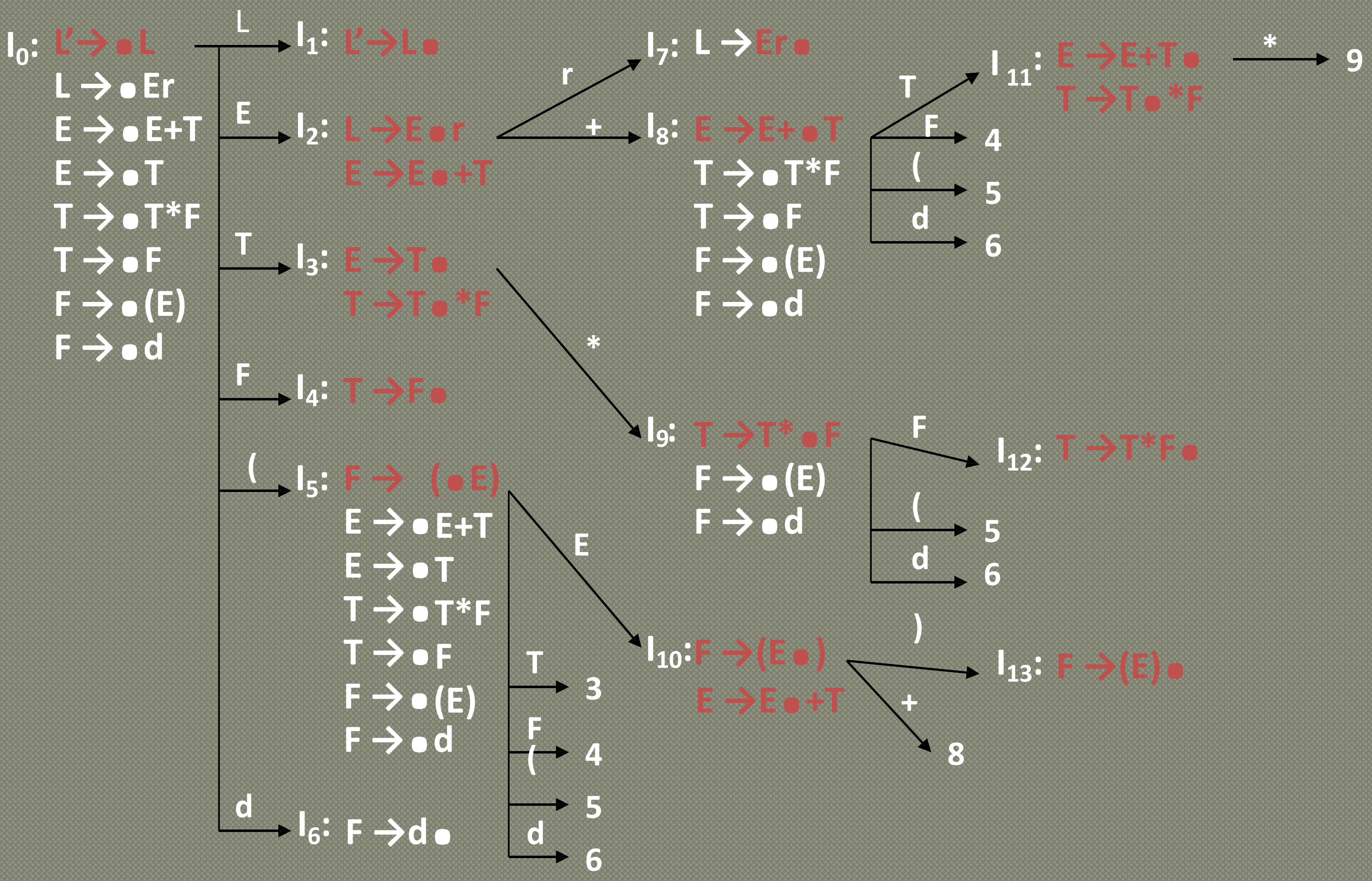
. .

Bottom-Up Eval. of S-Attributed Definitions (cont.)

Production	Semantic Rules
$L \rightarrow E$ return	print(val[top-1])
$E \rightarrow E_1 + T$	val[ntop] = val[top-2] + val[top]
$E \rightarrow T$	
$T \rightarrow T_1 * F$	val[ntop] = val[top-2] * val[top]
$T \rightarrow F$	
$F \rightarrow (E)$	val[ntop] = val[top-1]
$F \rightarrow$ digit	

- **At each shift of digit, we also push digit.lexval into *val-stack*.**
- **At all other shifts, we do not put anything into *val-stack* because other terminals do not have attributes (but we increment the stack pointer for *val-stack*).**

for The Grammar



I_0: L'→•L
L→•Er
E→•E+T
E→•T
T→•T*F
T→•F
F→•(E)
F→•d

I_1: L'→L•

I_2: L→E•r
E→E•+T

I_3: E→T•
T→T•*F

I_4: T→F•

I_5: F→ (•E)
E→•E+T
E→•T
T→•T*F
T→•F
F→•(E)
F→•d

I_6: F→d•

I_7: L→Er•

I_8: E→E+•T
T→•T*F
T→•F
F→•(E)
F→•d

I_9: T→T*•F
F→•(E)
F→•d

I_{10}: F→(E•)
E→E•+T

I_{11}: E→E+T•
T→T•*F

I_{12}: T→T*F•

I_{13}: F→(E)•

Bottom-Up Evaluation -- Example

0E2+8F4	5-3	*4r	T→F	T.val=F.val – do nothing
0E2+8T11	5-3	*4r	s9	push empty slot into val-stack
0E2+8T11*9	5-3-	4r	s6	d.lexval(4) into val-stack
0E2+8T11*9d6	5-3-4	r	F→d	F.val=d.lexval – do nothing
0E2+8T11*9F12	5-3-4	r	T→T*F	T.val=T_1.val*F.val
0E2+8T11	5-12	r	E→E+T	E.val=E_1.val*T.val
0E2	17	r	s7	push empty slot into val-stack

Top-Down Evaluation (of S-Attributed Definitions)

Productions	Semantic Rules
$A \rightarrow B$	print(B.n0), print(B.n1)
$B \rightarrow 0\ B_1$	$B.n0=B_1.n0+1$, $B.n1=B_1.n1$
$B \rightarrow 1\ B_1$	$B.n0=B_1.n0$, $B.n1=B_1.n1+1$
$B \rightarrow \varepsilon$	$B.n0=0$, $B.n1=0$

where B has two synthesized attributes (n0 and n1).

Top-Down Evaluation (of S-Attributed Definitions)

- Remember that: In a recursive predicate parser, each non-terminal corresponds to a procedure.

```
procedure A() {
    call B();                                                         A→
    B
}
procedure B() {
    if (currtoken=0)  { consume 0; call B(); }                        B →
    0 B
                                                                      B →
    else if (currtoken=1) { consume 1; call B(); }
    1 B
                                                                      B → ε
    else if (currtoken=$)  {}    // $ is end-marker
    else error("unexpected token");
}
```

Top-Down Evaluation (of S-Attributed Definitions)

Synthesized attributes of non-terminal B are the output parameters of procedure B.

All the semantic rules can be evaluated at the end of parsing of production rules

```
procedure A() {
    int n0,n1;
    call B(&n0,&n1);
    print(n0);   print(n1);
}
procedure B(int *n0, int *n1) {
    if (currtoken=0)
        {int a,b; consume 0; call B(&a,&b); *n0=a+1; *n1=b;}
    else if (currtoken=1)
        { int a,b; consume 1; call B(&a,&b); *n0=a; *n1=b+1; }
    else if (currtoken=$) {*n0=0; *n1=0; }    // $ is end-marker
    else error(“unexpected token”);
}
```

L-Attributed Definitions

- **S-Attributed Definitions can be efficiently implemented.**
- **We are looking for a larger (larger than S-Attributed Definitions) subset of syntax-directed definitions which can be efficiently evaluated.**

  ➔ L-Attributed Definitions

- **L-Attributed Definitions can always be evaluated by the depth first visit of the parse tree.**
- **This means that they can also be evaluated during the parsing.**

L-Attributed Definitions

- **A syntax-directed definition is L-attributed if each inherited attribute of X_j, where $1 \leq j \leq n$, on the right side of $A \rightarrow X_1X_2...X_n$ depends only on:**
  1. **The attributes of the symbols $X_1,...,X_{j-1}$ to the left of X_j in the production and**
  2. **the inherited attribute of A**

- **Every S-attributed definition is L-attributed, the restrictions only apply to the inherited attributes (not to synthesized attributes).**

A Definition which is NOT L-Attributed

Productions	**Semantic Rules**
A → LM	L.in=l(A.i), M.in=m(L.s), A.s=f(M.s)
A → QR	R.in=r(A.in), Q.in=q(R.s), A.s=f(Q.s)

- **This syntax-directed definition is not L-attributed because the semantic rule Q.in=q(R.s) violates the restrictions of L-attributed definitions.**
- **When Q.in must be evaluated before we enter to Q because it is an inherited attribute.**
- **But the value of Q.in depends on R.s which will be available after we return from R. So, we are not be able to evaluate the value of Q.in before we enter to Q.**

Translation Schemes

- **In a syntax-directed definition, we do not say anything about the evaluation times of the semantic rules (when the semantic rules associated with a production should be evaluated?).**

- **A translation scheme is a context-free grammar in which:**
  - **attributes are associated with the grammar symbols and**
  - **semantic actions enclosed between braces {} are inserted within the right sides of productions.**

- ***Ex:*** **A → { ... } X { ... } Y { ... }**

  Semantic Actions

Translation Schemes

- **When designing a translation scheme, some restrictions should be observed to ensure that an attribute value is available when a semantic action refers to that attribute.**
- **These restrictions (motivated by L-attributed definitions) ensure that a semantic action does not refer to an attribute that has not yet computed.**
- **In translation schemes, we use *semantic action* terminology instead of *semantic rule* terminology used in syntax-directed definitions.**
- **The position of the semantic action on the right side indicates when that semantic action will be evaluated.**

Translation Schemes for S-attributed Definitions

- If our syntax-directed definition is S-attributed, the construction of the corresponding translation scheme will be simple.
- Each associated semantic rule in a S-attributed syntax-directed definition will be inserted as a semantic action into the end of the right side of the associated production.

Production	Semantic Rule	
$E \rightarrow E_1 + T$	$E.val = E_1.val + T.val$	➔ a production of

a syntax directed definition

$\Downarrow$

$E \rightarrow E_1 + T \;\{\, E.val = E_1.val + T.val \,\}$ ➔ the production of

the

A Translation Scheme Example

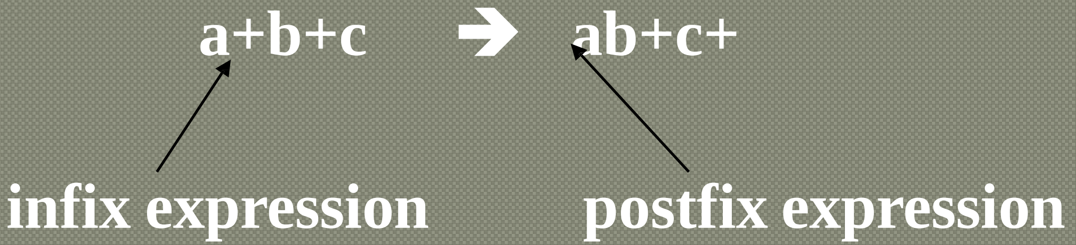
- **A simple translation scheme that converts infix expressions to the corresponding postfix expressions.**

E → T R

R → + T { print(“+”) } R_1

R → ε

T → id { print(id.name) }

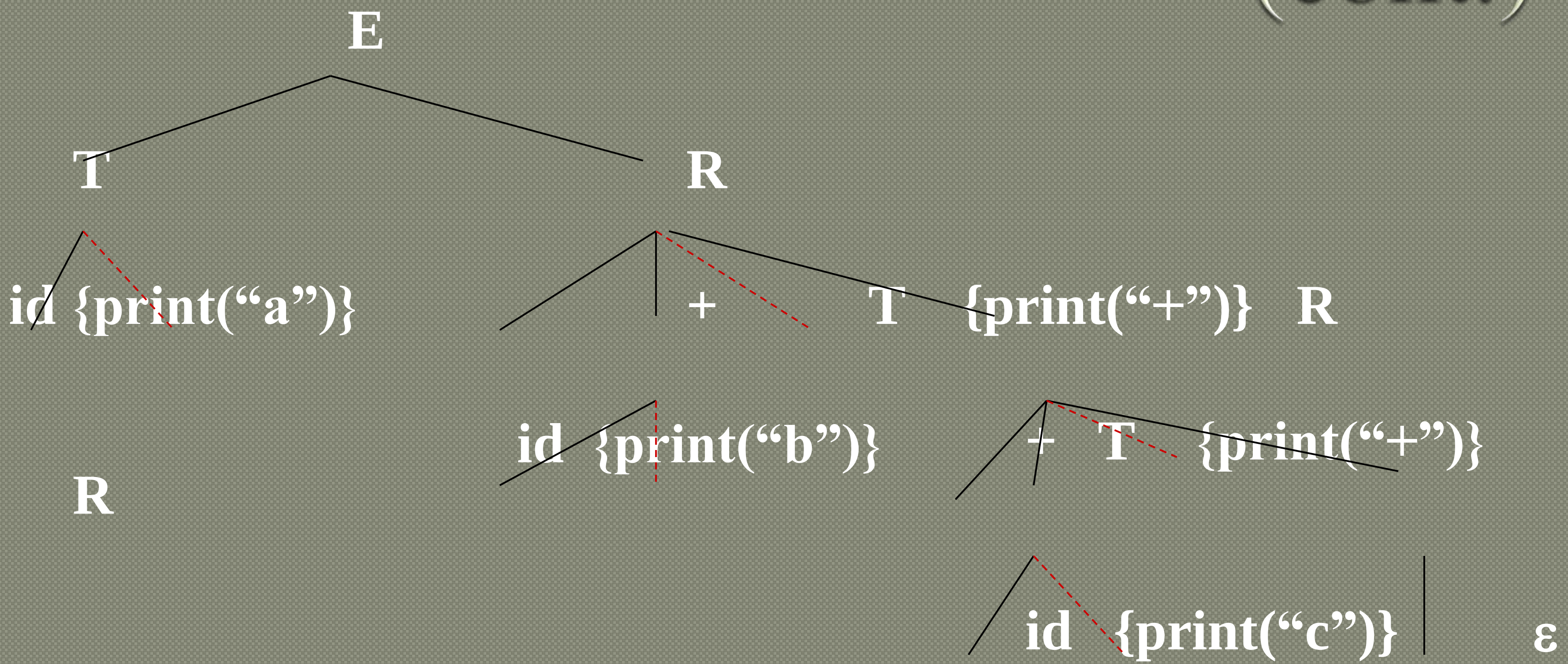


a+b+c ➔ ab+c+

infix expression

postfix expression

A Translation Scheme Example (cont.)



The depth first traversal of the parse tree (executing the semantic actions in that order) will produce the postfix representation of the infix expression.

Inherited Attributes in Translation Schemes

- **If a translation scheme has to contain both synthesized and inherited attributes, we have to observe the following rules:**

  1. **An inherited attribute of a symbol on the right side of a production must be computed in a semantic action before that symbol.**
  2. **A semantic action must not refer to a synthesized attribute of a symbol to the right of that semantic action.**
  3. **A synthesized attribute for the non-terminal on the left can only be computed after all attributes it references have been computed (we normally put this semantic action at the end of the right side of the production).**

- **With a L-attributed syntax-directed definition, it is always possible to construct a corresponding translation scheme which satisfies these three conditions (This may not be possible for a general syntax-directed translation).**

Top-Down Translation

- **We will look at the implementation of L-attributed definitions during predictive parsing.**
- **Instead of the syntax-directed translations, we will work with translation schemes.**
- **We will see how to evaluate inherited attributes (in L-attributed definitions) during recursive predictive parsing.**
- **We will also look at what happens to attributes during the left-recursion elimination in the left-recursive grammars.**

A Translation Scheme with Inherited Attributes

D → T id { addtype(id.entry,T.type), L.in = T.type } L

T → int { T.type = integer }

T → real { T.type = real }

L → id { addtype(id.entry,L.in), L_1.in = L.in } L_1

L → ε

- **This is a translation scheme for an L-attributed definitions.**

Predictive Parsing (of Inherited Attributes)

```
procedure D() {
    int Ttype,Lin,identry;
    call T(&Ttype);  consume(id,&identry);
    addtype(identry,Ttype);  Lin=Ttype;
    call L(Lin);                              a synthesized attribute
    (an output parameter)
}
procedure T(int *Ttype) {
    if (currtoken is int) { consume(int); *Ttype=TYPEINT; }
    else if (currtoken is real) { consume(real);
    *Ttype=TYPEREAL; }
    else { error("unexpected type"); }        an inherited attribute
    (an input parameter)
}
```

Predictive Parsing (of Inherited Attributes)

```
procedure L(int Lin) {
    if (currtoken is id) { int L1in,identry;
    consume(id,&identry);
                              addtype(identry,Lin); L1in=Lin; call
    L(L1in); }
    else if (currtoken is endmarker)  { }
    else { error(“unexpected token”); }
}
```

Translation Scheme - Intermediate Code Generation

E → T { A.in=T.loc } A { E.loc=A.loc}

A → + T { A_1.in=newtemp(); emit(add,A.in,T.loc,A_1.in) }

A_1 { A.loc =A_1.loc}

A → ε { A.loc = A.in}

T → F { B.in=F.loc } B { T.loc=B.loc }

B → * F { B_1.in=newtemp(); emit(mult,B.in,F.loc,B_1.in) }

B_1 { B.loc = B_1.loc}

B → ε { B.loc = B.in }

F → (E) { F.loc = E.loc }

F → id { F.loc = id.name }

Predictive Parsing – Intermediate Code Generation

```
procedure  E(char  **Eloc)  {
    char  *Ain,  *Tloc,  *Aloc;
    call T(&Tloc);  Ain=Tloc;
    call A(Ain,&Aloc); *Eloc=Aloc;
}
procedure A(char *Ain, char **Aloc) {
    if (currtok is +) {
        char *A1in, *Tloc, *A1loc;
       consume(+);   call T(&Tloc);  A1in=newtemp();
    emit("add",Ain,Tloc,A1in);
       call A(A1in,&A1loc);   *Aloc=A1loc;
    }
    else { *Aloc = Ain }
}
```

Predictive Parsing (cont.)

```
procedure T(char **Tloc) {
    char *Bin, *Floc, *Bloc;
    call F(&Floc);  Bin=Floc;
    call B(Bin,&Bloc); *Tloc=Bloc;
}
procedure B(char *Bin, char **Bloc) {
    if (currtok is *) {
       char *B1in, *Floc, *B1loc;
      consume(+);  call F(&Floc);  B1in=newtemp();
    emit(“mult”,Bin,Floc,B1in);
      call B(B1in,&B1loc);  Bloc=B1loc;
    }
```

Predictive Parsing (cont.)

```
else { *Bloc = Bin }
}
procedure F(char **Floc) {
   if (currtok is “(“) { char *Eloc; consume(“(“);
   call E(&Eloc); consume(“)”);  *Floc=Eloc }
   else { char *idname; consume(id,&idname);
   *Floc=idname }
}
```

Bottom-Up Evaluation of Inherited Attributes

- Using a top-down translation scheme, we can implement any L-attributed definition based on a LL(1) grammar.
- Using a bottom-up translation scheme, we can also implement any L-attributed definition based on a LL(1) grammar (each LL(1) grammar is also an LR(1) grammar).
- In addition to the L-attributed definitions based on LL(1) grammars, we can implement some of L-attributed definitions based on LR(1) grammars (not all of them) using the bottom-up translation scheme.

Removing Embedding Semantic Actions

- **In bottom-up evaluation scheme, the semantic actions are evaluated during the reductions.**
- **During the bottom-up evaluation of S-attributed definitions, we have a parallel stack to hold synthesized attributes.**
- ***Problem*: where are we going to hold inherited attributes?**
- ***A Solution*:**

Removing Embedding Semantic Actions

- – We will convert our grammar to an equivalent grammar to guarantee to the followings.
- – All embedding semantic actions in our translation scheme will be moved into the end of the production rules.
- – All inherited attributes will be copied into the synthesized attributes (most of the time synthesized attributes of new non-terminals).
- – Thus we will be evaluate all semantic actions during reductions, and we find a place to store an inherited attribute.

Removing Embedding Semantic Actions

- **To transform our translation scheme into an equivalent translation scheme:**

1. **Remove an embedding semantic action S_i, put new a non-terminal M_i instead of that semantic action.**
2. **Put that semantic action S_i into the end of a new production rule $M_i \rightarrow \varepsilon$ for that non-terminal M_i.**
3. **That semantic action S_i will be evaluated when this new production rule is reduced.**
4. **The evaluation order of the semantic rules are not changed by this transformation.**

Removing Embedding Semantic Actions

$A \rightarrow \{S_1\}\ X_1\ \{S_2\}\ X_2\ \ldots\ \{S_n\}\ X_n$

$\Downarrow$ **remove embedding semantic actions**

$A \rightarrow M_1\ X_1\ M_2\ X_2\ \ldots\ M_n\ X_n$

$M_1 \rightarrow \varepsilon\ \{S_1\}$

$M_2 \rightarrow \varepsilon\ \{S_2\}$

.

.

$M_n \rightarrow \varepsilon\ \{S_n\}$

Removing Embedding Semantic Actions

$E \rightarrow T\,R$

$R \rightarrow +\ T$ { print("+") } R_1

$R \rightarrow \varepsilon$

$T \rightarrow$ id { print(id.name) }

⇓ remove embedding semantic actions

$E \rightarrow T\,R$

$R \rightarrow +\ T\ M\ R_1$

$R \rightarrow \varepsilon$

$T \rightarrow$ id { print(id.name) }

$M \rightarrow \varepsilon$ { print("+") }

Translation with Inherited Attributes

- **Let us assume that every non-terminal A has an inherited attribute A.i, and every symbol X has a synthesized attribute X.s in our grammar.**
- **For every production rule $A \rightarrow X_1 X_2 \ldots X_n$,**
  - **introduce new marker non-terminals $M_1, M_2, \ldots, M_n$ and**
  - **replace this production rule with $A \rightarrow M_1 X_1 M_2 X_2 \ldots M_n X_n$**
  - **the synthesized attribute of X_i will be not changed.**
  - **the inherited attribute of X_i will be copied into the synthesized attribute of M_i by the new semantic action added at the end of the new production rule $M_i \rightarrow \varepsilon$.**
  - **Now, the inherited attribute of X_i can be found in the synthesized attribute of M_i (which is immediately available in the stack.**

Inherited Attributes

$S \rightarrow \{A.i=1\}\ A\ \{S.s=k(A.i,A.s)\}$

$A \rightarrow \{B.i=f(A.i)\}\ B\ \{C.i=g(A.i,B.i,B.s)\}\ C\ \{A.s= h(A.i,B.i,B.s,C.i,C.s)\}$

$B \rightarrow b\ \{B.s=m(B.i,b.s)\}$

$C \rightarrow c\ \{C.s=n(C.i,c.s)\}$

$S \rightarrow \{M_1.i=1\}\ M_1\ \{A.i=M_1.s\}\ A\ \{S.s=k(M_1.s,A.s)\}$

$A \rightarrow \{M_2.i=f(A.i)\}\ M_2\ \{B.i=M_2.s\}\ B\ \{M_3.i=g(A.i,M_2.s,B.s)\}\ M_3\ \{C.i=M_3.s\}\ C\ \{A.s= h(A.i, M_2.s,B.s, M_3.s,C.s)\}$

$B \rightarrow b\ \{B.s=m(B.i,b.s)\}$

$C \rightarrow c\ \{C.s=n(C.i,c.s)\}$

$M_1 \rightarrow \varepsilon\ \{M_1.s=M_1.i\}$

$M_2 \rightarrow \varepsilon\ \{M_2.s=M_2.i\}$

$M_3 \rightarrow \varepsilon\ \{M_3.s=M_3.i\}$

Actual Translation Scheme

$S \rightarrow \{M_1.i=1\}\ M_1\ \{A.i=M_1.s\}\ A\{S.s=k(M_1.s,A.s)\}$

$A \rightarrow \{M_2.i=f(A.i)\}\ M_2\ \{B.i=M_2.s\}\ B\ \{M_3.i=g(A.i,M_2.s,B.s)\}\ M_3$
$\{C.i=M_3.s\}\qquad C\ \{A.s=h(A.i, M_2.s,B.s, M_3.s,C.s)\}$

$B \rightarrow b\ \{B.s=m(B.i,b.s)\}$

$C \rightarrow c\ \{C.s=n(C.i,c.s)\}$

$M_1 \rightarrow \varepsilon\ \{M_1.s= M_1.i\}$

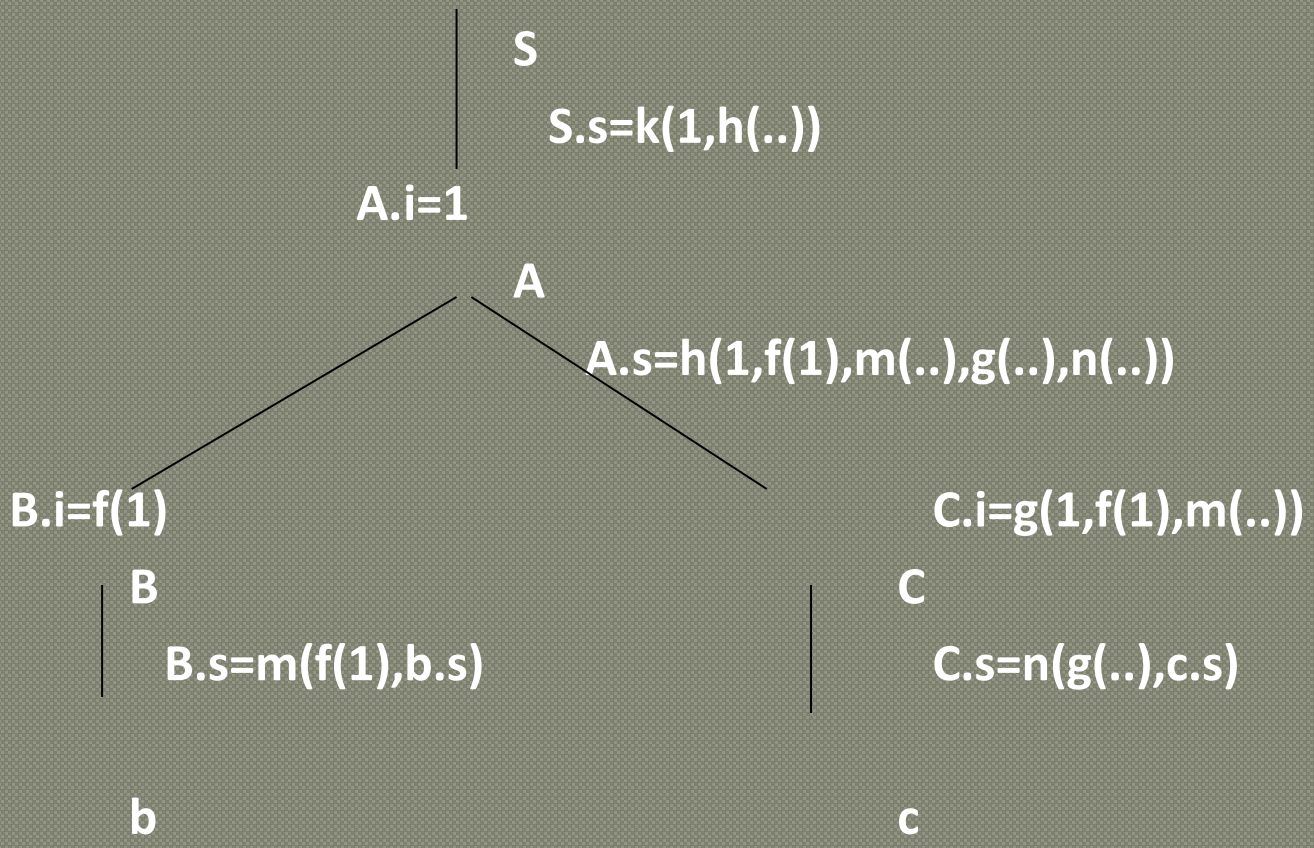
$M_2 \rightarrow \varepsilon\ \{M_2.s=M_2.i\}$

$M_3 \rightarrow \varepsilon\ \{M_3.s=M_3.i\}$

Actual Translation Scheme

$S \rightarrow M_1\ A$	{ s[ntop]=k(s[top-1],s[top]) }
$M_1 \rightarrow \varepsilon$	{ s[ntop]=1 }
$A \rightarrow M_2\ B\ M_3\ C$	{ s[ntop]=h(s[top-4],s[top-3],s[top-2], s[top-1],s[top]) }
$M_2 \rightarrow \varepsilon$	{ s[ntop]=f(s[top]) }
$M_3 \rightarrow \varepsilon$	{ s[ntop]=g(s[top-2],s[top-1],s[top])}
$B \rightarrow b$	{ s[ntop]=m(s[top-1],s[top]) }
$C \rightarrow c$	{ s[ntop]=n(s[top-1],s[top]) }

Evaluation of Attributes



S

S.s=k(1,h(..))

A.i=1

A

A.s=h(1,f(1),m(..),g(..),n(..))

B.i=f(1)

C.i=g(1,f(1),m(..))

B

C

B.s=m(f(1),b.s)

C.s=n(g(..),c.s)

b

c

Evaluation of Attributes

stack	**input**	**s-attribute stack**
	bc$	
M_1	bc$	1
$M_1 M_2$	bc$	1 f(1)
$M_1 M_2$ b	c$	1 f(1) b.s
$M_1 M_2$ B	c$	1 f(1) m(f(1),b.s)
$M_1 M_2$ B M_3	c$	1 f(1) m(f(1),b.s) g(1,f(1),m(f(1),b.s))
$M_1 M_2$ B M_3 c	$	1 f(1) m(f(1),b.s) g(1,f(1),m(f(1),b.s)) c.s
$M_1 M_2$ B M_3 C	$	1 f(1) m(f(1),b.s) g(1,f(1),m(f(1),b.s)) n(g(..),c.s)
M_1 A	$	1 h(f(1),m(..),g(..),n(..))
S	$	k(1,h(..))

Problems

- All L-attributed definitions based on LR grammars cannot be evaluated during bottom-up parsing.

$S \rightarrow \{ L.i=0 \}\ L$

$L \rightarrow \{ L_1.i=L.i+1 \}\ L_1\ 1$

$L \rightarrow \varepsilon\ \{ print(L.i) \}$

➔ this translations scheme cannot be implemented during the bottom-up parsing

$S \rightarrow M_1\ L$

$L \rightarrow M_2\ L_1\ 1$

$L \rightarrow \varepsilon \quad \{ print(s[top]) \}$

$M_1 \rightarrow \varepsilon \quad \{ s[ntop]=0 \}$

$M_2 \rightarrow \varepsilon \quad \{ s[ntop]=s[top]+1 \}$

➔ But since $L \rightarrow \varepsilon$ will be reduced first by the bottom-up parser, the translator cannot know the number of 1s.

Problems

- **The modified grammar cannot be LR grammar anymore.**

$L \rightarrow Lb$ $\qquad\qquad\qquad\qquad$ $L \rightarrow M\,Lb$

$L \rightarrow a$ ➔ $L \rightarrow a$ $\qquad$ NOT LR-grammar

$M \rightarrow \varepsilon$

$S' \rightarrow .L, \$$
$L \rightarrow .\ M\ L\ b, \$$
$L \rightarrow .\ a, \$$
$M \rightarrow .,a$ ➔ shift/reduce conflict

Intermediate Code Generation

- ***Intermediate codes*** **are machine independent codes, but they are close to machine instructions.**

- **The given program in a source language is converted to an equivalent program in an intermediate language by the intermediate code generator.**

- **Intermediate language can be many different languages, and the designer of the compiler decides this intermediate language.**
  - **syntax trees can be used as an intermediate language.**

Intermediate Code Generation

- postfix notation can be used as an intermediate language.
- three-address code (Quadraples) can be used as an intermediate language
  - we will use quadraples to discuss intermediate code generation
  - quadraples are close to machine instructions, but they are not actual machine instructions.
- some programming languages have well defined intermediate languages.
  - java – java virtual machine
  - prolog – warren abstract machine
  - In fact, there are byte-code emulators to execute instructions in these intermediate languages.

Three-Address Code (Quadraples)

- A quadraple is:

  ```
  x := y op z
  ```

  where x, y and z are names, constants or compiler-generated temporaries; `op` is any operator.

- But we may also the following notation for quadraples (much better notation because it looks like a machine code instruction)

  ```
  op  y,z,x
  ```

  apply operator `op` to `y` and `z`, and store the result in `x`.
- We use the term "three-address code" because each statement usually contains three addresses (two for operands, one for the result).

Three-Address Statements

Binary Operator: `op y,z,result` or `result := y op z`

where `op` is a binary arithmetic or logical operator. This binary operator is applied to `y` and `z`, and the result of the operation is stored in `result`.

```
Ex:         add  a,b,c
            gt   a,b,c
            addr a,b,c
            addi a,b,c
```

Unary Operator: `op y,,result` or `result := op y`

where `op` is a unary arithmetic or logical operator. This unary operator is applied to `y`, and the result of the operation is stored in `result`.

```
Ex:         uminus      a,,c
            not         a,,c
            int to real a,,c
```

Three-Address Statements (cont.)

Move Operator: `mov y,,result` or `result := y`

where the content of `y` is copied into `result`.

Ex:
```
mov    a,,c
movi   a,,c
movr   a,,c
```

Unconditional Jumps: `jmp ,,L` or `goto L`

We will jump to the three-address code with the label `L`, and the execution continues from that statement.

Ex:
```
jmp   ,,L1 // jump to L1
jmp   ,,7  // jump to the statement 7
```

Three-Address Statements (cont.)

Conditional Jumps: `jmp`*`relop`* `y,z,L` or `if y` *`relop`* `z goto L`

We will jump to the three-address code with the label `L` if the result of `y relop z` is true, and the execution continues from that statement. If the result is false, the execution continues from the statement following this conditional jump statement.

```
Ex:     jmpgt    y,z,L1        // jump to L1 if y>z
              jmpgte   y,z,L1        // jump to L1 if y>=z
              jmpe     y,z,L1        // jump to L1 if y==z
              jmpne    y,z,L1        // jump to L1 if y!=z
```

Three-Address Statements (cont.)

Our relational operator can also be a unary operator.

```
jmpnz   y,,L1 // jump to L1 if y is not zero
jmpz    y,,L1  // jump to L1 if y is zero
jmpt    y,,L1  // jump to L1 if y is true
jmpf    y,,L1  // jump to L1 if y is false
```

Three-Address Statements (cont.)

Procedure Parameters: **param x,, or param x**

Procedure Calls: **call p,n, or call p,n**

where x is an actual parameter, we invoke the procedure p with n parameters.

Ex: **param x_1,,**

param x_2,,

➔ p(x_1,…,x_n)

param x_n,,

call p,n,

f(x+1,y) ➔ add x,1,t1

param t1,,

param y,,

call f,2,

Three-Address Statements (cont.)

***Indexed Assignments*:**

move y[i],,x or x := y[i]

move x,,y[i] or y[i] := x

Address and Pointer Assignments:

moveaddr y,,x or x := &y

movecont y,,x or x := *y

Syntax-Directed Translation into Three-Address Code

S → id := E S.code = E.code || gen(‘mov’ E.place ‘,’ id.place)

E → E_1 + E_2 E.place = newtemp();
E.code = E_1.code || E_2.code || gen(‘add’ E_1.place ‘,’ E_2.place ‘,’ E.place)

E → E_1 * E_2 E.place = newtemp();
E.code = E_1.code || E_2.code || gen(‘mult’ E_1.place ‘,’ E_2.place ‘,’ E.place)

E → - E_1 E.place = newtemp();
E.code = E_1.code || gen(‘uminus’ E_1.place ‘,’ E.place)

E → (E_1) E.place = E_1.place;
E.code = E_1.code

E → id E.place = id.place;
E.code = null

Syntax-Directed Translation (cont.)

```
S → while E do $S_1$  S.begin = newlabel();
                      S.after = newlabel();
                      S.code = gen(S.begin “:”)  ||  E.code  ||
                               gen(‘jmpf’ E.place ‘,’ S.after)  || $S_1$.code  ||
                               gen(‘jmp’ ‘,’ S.begin)  ||
                               gen(S.after ‘:”)
S → if E then $S_1$ else $S_2$    S.else = newlabel();
                      S.after = newlabel();
                      S.code = E.code  ||
                               gen(‘jmpf’ E.place ‘,’ S.else)  || $S_1$.code  ||
                               gen(‘jmp’ ‘,’ S.after) ||
                               gen(S.else ‘:”) || $S_2$.code ||
                               gen(S.after ‘:”)
```

Produce Three-Address Code

S → id := E

- { p= lookup(id.name);
  - if (p is not nil) then emit(‘mov’ E.place ‘,,’ p)

E → E_1 + E_2

  - else error(“undefined-variable”) }
- { E.place = newtemp();

E → E_1 * E_2

  - emit(‘add’ E_1.place ‘,’ E_2.place ‘,’ E.place) }
- { E.place = newtemp();
  - emit(‘mult’ E_1.place ‘,’ E_2.place ‘,’ E.place) }

E → - E_1 { E.place = newtemp();
emit(‘uminus’ E_1.place ‘,,’ E.place) }

E → (E_1) { E.place = E_1.place; }

E → id { p= lookup(id.name);
if (p is not nil) then E.place = id.place
else error(“undefined-variable”) }

Translation Scheme with Locations

```
S → id := { E.inloc = S.inloc } E
    { p = lookup(id.name);
      if (p is not nil) then  { emit(E.outloc ‘mov’ E.place ‘,’ p);
          S.outloc=E.outloc+1 }
          else { error(“undefined-variable”); S.outloc=E.outloc
    } }

E → { $E_1$.inloc = E.inloc } $E_1$ + { $E_2$.inloc = $E_1$.outloc } $E_2$
      { E.place = newtemp();  emit($E_2$.outloc ‘add’ $E_1$.place ‘,’
    $E_2$.place ‘,’             E.place); E.outloc=$E_2$.outloc+1 }

E → { $E_1$.inloc = E.inloc } $E_1$ + { $E_2$.inloc = $E_1$.outloc } $E_2$
      { E.place = newtemp();  emit($E_2$.outloc ‘mult’ $E_1$.place ‘,’
    $E_2$.place ‘,’             E.place); E.outloc=$E_2$.outloc+1 }
```

Translation Scheme with Locations

E → - { E_1.inloc = E.inloc } E_1
{ E.place = newtemp(); emit(E_1.outloc ‘uminus’
E_1.place ‘,’ E.place);
E.outloc=E_1.outloc+1 }

E → (E_1){ E.place = E_1.place; E.outloc=E_1.outloc+1 }

E → id { E.outloc = E.inloc; p= lookup(id.name);
if (p is not nil) then E.place = id.place
else error(“undefined-variable”) }

Boolean Expressions

E $\rightarrow$ { E_1.inloc = E.inloc } E_1 and { E_2.inloc = E_1.outloc } E_2
{ E.place = newtemp(); emit(E_2.outloc ‘and’ E_1.place ‘,’ E_2.place ‘,’ E.place); E.outloc=E_2.outloc+1 }

E $\rightarrow$ { E_1.inloc = E.inloc } E_1 or { E_2.inloc = E_1.outloc } E_2
{ E.place = newtemp(); emit(E_2.outloc ‘and’ E_1.place ‘,’ E_2.place ‘,’ E.place); E.outloc=E_2.outloc+1 }

E $\rightarrow$ not { E_1.inloc = E.inloc } E_1
{ E.place = newtemp(); emit(E_1.outloc ‘not’ E_1.place ‘,,’ E.place); E.outloc=E_1.outloc+1 }

E $\rightarrow$ { E_1.inloc = E.inloc } E_1 relop { E_2.inloc = E_1.outloc } E_2
{ E.place = newtemp();
emit(E_2.outloc relop.code E_1.place ‘,’ E_2.place ‘,’ E.place); E.outloc=E_2.outloc+1 }

Translation Scheme(cont.)

```
S → while { E.inloc = S.inloc } E do
    { emit(E.outloc ‘jmpf’ E.place ‘,,’ ‘NOTKNOWN’);
      $S_1$.inloc=E.outloc+1;  }  $S_1$
    { emit($S_1$.outloc ‘jmp’ ‘,,’S.inloc);
      S.outloc=$S_1$.outloc+1;
      backpatch(E.outloc,S.outloc); }
```

Translation Scheme(cont.)

S → if { E.inloc = S.inloc } E then

  { emit(E.outloc ‘jmpf’ E.place ‘„’‘NOTKNOWN’);

    S_1.inloc=E.outloc+1; } S_1 else

  { emit(S_1.outloc ‘jmp’ ‘„’‘NOTKNOWN’);

    S_2.inloc=S_1.outloc+1;

    backpatch(E.outloc,S_2.inloc); } S_2

  { S.outloc=S_2.outloc;

    backpatch(S_1.outloc,S.outloc); }

Three Address Codes - Example

```
x:=1;
y:=x+10;
while (x<y) {
    x:=x+1;
    if (x%2==1) then y:=y+1;
    else y:=y-2;
}
```

➔

```
01: mov   1,,x
02: add   x,10,t1
03: mov   t1,,y
04: lt    x,y,t2
05: jmpf  t2,,17
06: add   x,1,t3
07: mov   t3,,x
```

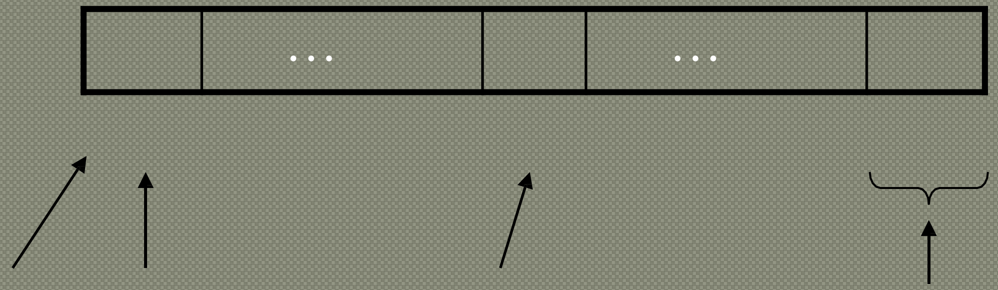
Three Address Codes - Example

```
08: mod   x,2,t4
09: eq    t4,1,t5
10: jmpf  t5,,14
11: add   y,1,t6
12: mov   t6,,y
13: jmp   ,,16
14: sub   y,2,t7
15: mov   t7,,y
16: jmp   ,,4
17 :
```

Arrays

- **Elements of arrays can be accessed quickly if the elements are stored in a block of consecutive locations.**

A one-dimensional array A:



base_A low i width

base_A is the address of the first location of the array A,

width is the width of each array element.

low is the index of the first array element

location of A[i] ➔ base_A+(i-low)*width

Arrays (cont.)

$base_A$+(i-low)*width

can be re-written as i*width + ($base_A$-low*width)

should be computed at run-time can be computed at compile-time

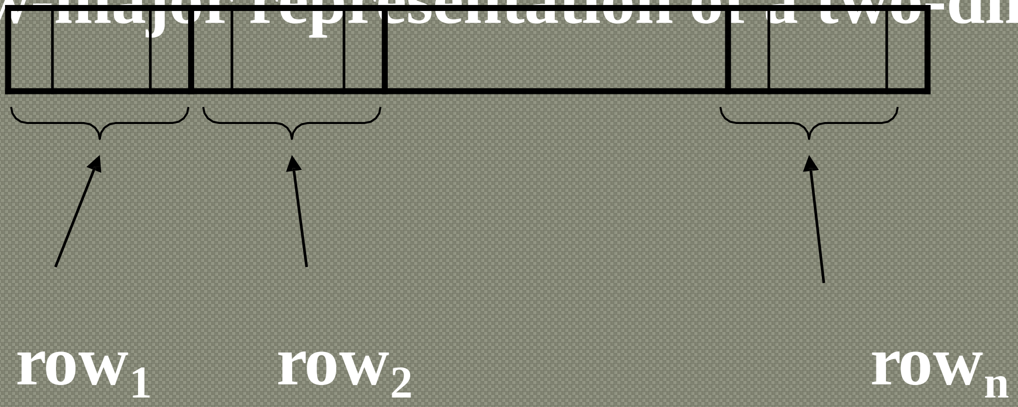
- So, the location of A[i] can be computed at the run-time by evaluating the formula i*width+c where c is ($base_A$-low*width) which is evaluated at compile-time.

- Intermediate code generator should produce the code to evaluate this formula i*width+c (one multiplication and one addition operation).

Two-Dimensional Arrays

- A two-dimensional array can be stored in
  - either row-major (*row-by-row*) or
  - column-major (*column-by-column*).
- Most of the programming languages use row-major method.

- Row-major representation of a two-dimensional array:



Two-Dimensional Arrays (cont.)

- **The location of $A[i_1,i_2]$ is**

  $$base_A + ((i_1-low_1)*n_2+i_2-low_2)*width$$

  $base_A$ is the location of the array A.

  low_1 is the index of the first row

  low_2 is the index of the first column

  n_2 is the number of elements in each row

  width is the width of each array element
- **Again, this formula can be re-written as**

$$\underbrace{((i_1*n_2)+i_2)*width}\ +\ \underbrace{(base_A-((low_1*n_1)+low_2)*width)}$$

should be computed at run-time can be computed at compile-time

Multi-Dimensional Arrays

- In general, the location of $A[i_1,i_2,...,i_k]$ is

  $((... ((i_1*n_2)+i_2) ...)*n_k+i_k)*width + (base_A-((...((low_1*n_1)+low_2)...)*n_k+low_k)*width)$

- So, the intermediate code generator should produce the codes to evaluate the following formula (to find the location of $A[i_1,i_2,...,i_k]$) :

  $((... ((i_1*n_2)+i_2) ...)*n_k+i_k)*width + c$

- To evaluate the $((... ((i_1*n_2)+i_2) ...)*n_k+i_k$ portion of this formula, we can use the recurrence equation:

  $e_1 = i_1$

  $e_m = e_{m-1} * n_m + i_m$

Translation Scheme for Arrays

- **If we use the following grammar to calculate addresses of array elements, we need inherited attributes.**

L → id | id [Elist]
Elist → Elist , E | E

- **Instead of this grammar, we will use the following grammar to calculate addresses of array elements so that we do not need inherited attributes (we will use only synthesized attributes).**

L → id | Elist]
Elist → Elist , E | id [E

Translation Scheme for Arrays (cont.)

```
S → L := E          { if (L.offset is null)  emit(‘mov’ E.place ‘„’ L.place)
                       else emit(‘mov’ E.place ‘„’ L.place ‘[‘ L.offset ‘]’) }

$E \rightarrow E_1 + E_2$        { E.place = newtemp();
                       emit(‘add’ $E_1$.place ‘,’ $E_2$.place ‘,’ E.place) }

$E \rightarrow ( E_1 )$          { E.place = $E_1$.place; }

E → L               { if (L.offset is null)  E.place = L.place)
                       else { E.place = newtemp();
                                  emit(‘mov’ L.place ‘[‘ L.offset ‘]’ ‘„’ E.place)
    } }
```

Translation Scheme for Arrays (cont.)

L $\rightarrow$ id { L.place = id.place; L.offset = null; }

L $\rightarrow$ Elist]

{ L.place = newtemp(); L.offset = newtemp();

emit(‘mov’ c(Elist.array) ‘,,’ L.place);

emit(‘mult’ Elist.place ‘,’ width(Elist.array) ‘,’L.offset)

}

Elist $\rightarrow$ Elist$_1$, E

{ Elist.array = Elist$_1$.array ; Elist.place = newtemp(); Elist.ndim = Elist$_1$.ndim + 1;

emit(‘mult’ Elist$_1$.place ‘,’ limit(Elist.array,Elist.ndim) ‘,’ Elist.place);

emit(‘add’ Elist.place ‘,’ E.place ‘,’Elist.place); }

Elist $\rightarrow$ id [E

{Elist.array = id.place ; Elist.place = E.place; Elist.ndim = 1; }

Translation Scheme for Arrays – Example 1

- **A one-dimensional double array A : 5..100**

  ➔ n_1=95 width=8 (double) low_1=5

- **Intermediate codes corresponding to x := A[y]**

```
mov     c,,t1        // where c=base_A-(5)*8
mult  y,8,t2
mov   t1[t2],,t3
mov   t3,,x
```

Translation Scheme for Arrays – Example2

- A two-dimensional `int` array `A` : 1..10x1..20
  ➔ n_1=10 n_2=20 width=4 (integers) low_1=1 low_2=1

- Intermediate codes corresponding to `x := A[y,z]`

```
mult  y,20,t1
add   t1,z,t1
mov          c,,t2              // where c=base_A-
(1*20+1)*4
mult  t1,4,t3
mov   t2[t3],,t4
mov   t4,,x
```

Translation Scheme for Arrays – Example3

- A three-dimensional `int` array `A : 0..9x0..19x0..29`
  ➔ n_1=10 n_2=20 n_3=30 width=4 (integers) low_1=0 low_2=0 low_3=0

- Intermediate codes corresponding to `x := A[w,y,z]`

```
mult   w,20,t1
add    t1,y,t1
mult   t1,30,t2
add    t2,z,t2
mov     c,,t3              // where c=base_A-((0*20+0)*30+0)*4
mult   t2,4,t4
mov    t3[t4],,t5
mov    t5,,x
```

Declarations

P → M D

M → € { offset=0 }

D → D ; D

D → id : T { enter(id.name,T.type,offset);
offset=offset+T.width }

T → int { T.type=int; T.width=4 }

T → real { T.type=real; T.width=8 }

T → array[num] of T_1 { T.type=array(num.val,T_1.type);
T.width=num.val*T_1.width }

T → ↑ T_1 { T.type=pointer(T_1.type); T.width=4 }

where *enter* crates a symbol table entry with given values.

Nested Procedure Declarations

- **For each procedure we should create a symbol table.**

mktable(previous) – create a new symbol table where previous is the parent symbol table of this new symbol table

enter(symtable,name,type,offset) – create a new entry for a variable in the given symbol table.

enterproc(symtable,name,newsymbtable) – create a new entry for the procedure in the symbol table of its parent.

Nested Procedure Declarations

addwidth(symtable,width) – puts the total width of all entries in the symbol table into the header of that table.

- **We will have two stacks:**
  - **tblptr – to hold the pointers to the symbol tables**
  - **offset – to hold the current offsets in the symbol tables in tblptr stack.**

Nested Procedure Declarations

P → M D { addwidth(top(tblptr),top(offset)); pop(tblptr); pop(offset) }

M → € { t=mktable(nil); push(t,tblptr); push(0,offset) }

D → D ; D

D → proc id N D ; S
{ t=top(tblptr); addwidth(t,top(offset));
pop(tblptr); pop(offset);
enterproc(top(tblptr),id.name,t) }

D → id : T { enter(top(tblptr),id.name,T.type,top(offset));
top(offset)=top(offset)+T.width }

N → € { t=mktable(top(tblptr)); push(t,tblptr); push(0,offset) }

Intermediate Code Generation

Intermediate Code Generation

- **Translating source program into an “intermediate language.”**
  - **Simple**
  - **CPU Independent,**
  - **…yet, close in spirit to machine language.**
- **Or, depending on the application other intermediate languages may be used, but in general, we opt for simple, well structured intermediate forms.**
- **(and this completes the “Front-End” of Compilation).**

Benefits

1. **Retargeting is facilitated**
2. **Machine independent Code Optimization can be applied.**

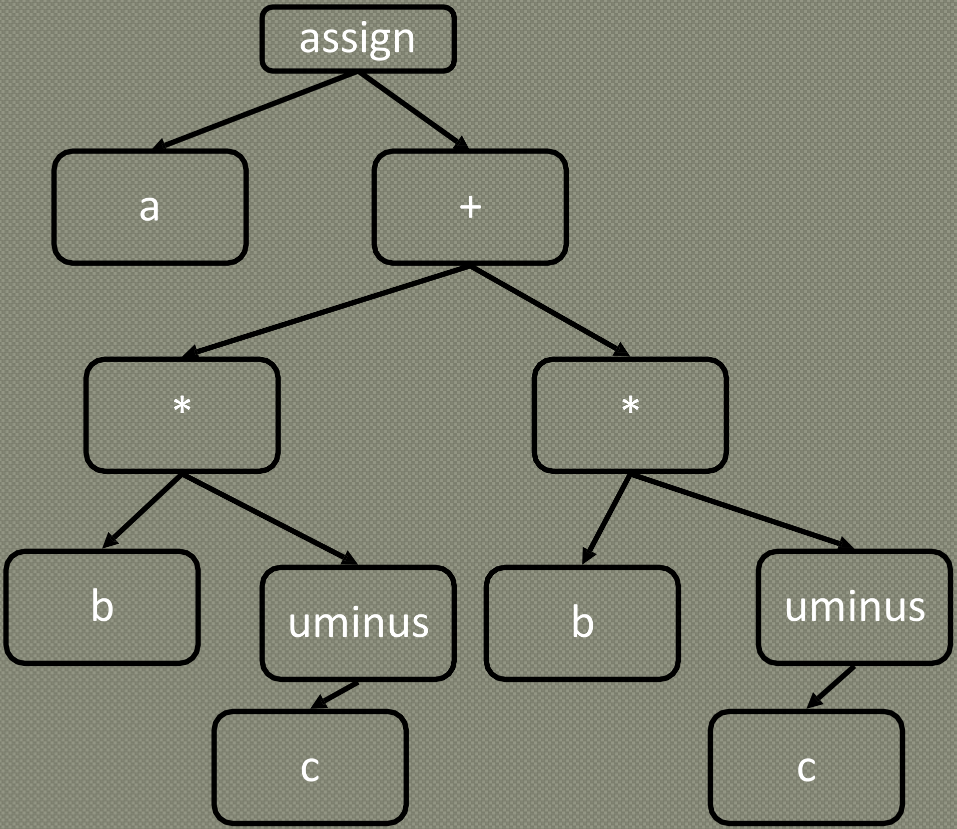
Intermediate Code Generation (II)

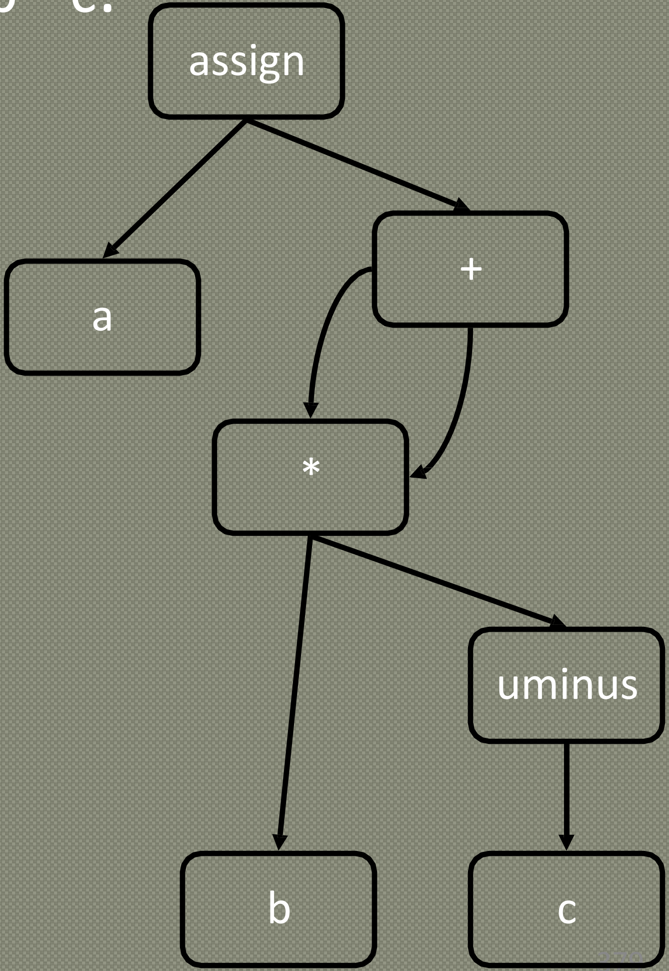
- *Intermediate codes* are machine independent codes, but they are close to machine instructions.
- The given program in a source language is converted to an equivalent program in an intermediate language by the intermediate code generator.
- Intermediate language can be many different languages, and the designer of the compiler decides this intermediate language.

  - syntax trees can be used as an intermediate language.
  - postfix notation can be used as an intermediate language.
  - three-address code (Quadraples) can be used as an intermediate language
    - we will use quadraples to discuss intermediate code generation
    - quadraples are close to machine instructions, but they are not actual machine instructions.

Types of Intermediate Languages

- Graphical Representations.
  - Consider the assignment a:=b*-c+b*-c:





Syntax Dir. Definition for Assignment Statements

PRODUCTION **Semantic Rule**

S → id := E { S.*nptr* = *mknode* (‘assign’, *mkleaf*(id, id.*entry*), E.*nptr*) }

E → E_1 + E_2

E → E_1 * E_2

E → - E_1

E → (E_1)

E → id

- {E.*nptr* = *mknode*(‘+’, E_1.*nptr*,E_2.*nptr*) }
- {E.*nptr* = *mknode*(‘*’, E_1.*nptr*,E_2.*nptr*) }
- {E.*nptr* = *mknode*(‘uminus’,E_1.*nptr*) }
- {E.*nptr* = E_1.*nptr* }
- {E.*nptr* = *mkleaf*(id, id.*entry*) }

Three Address Code

- **Statements of general form x:=y op z**

- **No built-up arithmetic expressions are allowed.**

- **As a result, x:=y + z * w**
  should be represented as
  t_1:=z * w
  t_2:=y + t_1
  x:=t_2

Three Address Code

- **Observe that given the syntax-tree or the dag of the graphical representation we can easily derive a three address code for assignments as above.**

- **In fact three-address code is a linearization of the tree.**

- **Three-address code is useful: related to machine-language/ simple/ optimizable.**

Example of 3-address code

t_1:= - c
t_2:= b * t_1
t_3:= - c
t_4:= b * t_3
t_5:= $_2$ + t_4
a:= t_5

t_1:=- c
t_2:= b *
t_1
t_5:= t_2 +
t_2
a:= t_5

Types of Three-Address Statements.

Assignment Statement:	x:=y op z
Assignment Statement:	**x:=op z**
Copy Statement:	**x:=z**
Unconditional Jump:	**goto L**
Conditional Jump:	**if x relop y goto L**
Stack Operations:	**Push/pop**

More Advanced:

Procedure:

```
param $x_1$
param $x_2$
…
param $x_n$
call p,n
```

Types of Three-Address Statements.

Index Assignments:

```
x:=y[i]
x[i]:=y
```

Address and Pointer Assignments:

```
x:=&y
x:=*y
*x:=y
```

Syntax-Directed Translation into 3-address code.

- First deal with assignments.
- Use attributes
  - E.*place:* the name that will hold the value of E
    - Identifier will be assumed to already have the place attribute defined.
  - E.*code:*hold the three address code statements that evaluate E (this is the `translation’ attribute).
- Use function *newtemp* that returns a new temporary variable that we can use.
- Use function *gen* to generate a single three address statement given the necessary information (variable names and operations).

Syntax-Dir. Definition for 3-address code

PRODUCTION	Semantic Rule
$S \rightarrow$ id := E	{ S.*code* = E.*code*||*gen*(id.*place* ‘=’ E.*place* ‘;’) }
$E \rightarrow E_1 + E_2$	{E.*place*= *newtemp* ; E.*code* = E$_1$.*code* || E$_2$.*code* || || *gen*(E.*place*‘:=’E$_1$.*place*‘+’E$_2$.*place*) }
$E \rightarrow E_1 * E_2$	{E.*place*= *newtemp* ; E.*code* = E$_1$.*code* || E$_2$.*code* || || *gen*(E.*place*‘=’E$_1$.*place*‘*’E$_2$.*place*) }
$E \rightarrow - E_1$	{E.*place*= *newtemp* ; E.*code* = E$_1$.*code* || || *gen*(E.*place* ‘=’ ‘uminus’ E$_1$.*place*) }
$E \rightarrow (E_1)$	{E.*place*= E$_1$.*place* ; E.*code* = E$_1$.*code*}
$E \rightarrow$ id	{E.*place* = id.*entry* ; E.*code* = ‘’}

e.g. **a := b * - (c+d)**

What about things that are not assignments?

- E.g. while statements of the form “while E do S” (intepreted as while the value of E is not 0 do S)

Extension to the previous syntax-dir. Def.

PRODUCTION

S → while E do S_1

Semantic Rule

***S.begin* = *newlabel*;**

***S.after* = *newlabel* ;**

What about things that are not assignments?(cont)

S.*code*= *gen*(S.*begin* ‘:’)
|| E.*code*
|| *gen*(‘if’ *E*.place ‘=’ ‘0’ ‘goto’ S.*after*)
|| S_1.*code*
|| gen(‘goto’ S.*begin*)
|| gen(S.*after* ‘:’)

Implementations of 3-address statements

- Quadruples

t_1:=- c
t_2:=b * t_1
t_3:=- c
t_4:=b * t_3
t_5:=t_2 + t_4
a:=t_5

	op	*arg1*	*arg2*	*result*
(0)	uminus	c		t_1
(1)	*	b	t_1	t_2
(2)	uminus	c		
(3)	*	b	t_3	t_4
(4)	+	t_2	t_4	t_5
(5)	:=	t_5		a

Temporary names must be entered into the symbol table as they are created.

Implementations of 3-address statements, II

- Triples

t_1:=- c

t_2:=b * t_1

t_3:=- c

t_4:=b * t_3

t_5:=t_2 + t_4

a:=t_5

	op	*arg1*	*arg2*
(0)	**uminus**	**c**	
(1)	*****	**b**	**(0)**
(2)	**uminus**	**c**	
(3)	*****	**b**	**(2)**
(4)	**+**	**(1)**	**(3)**
(5)	**assign**	**a**	**(4)**

Temporary names are not entered into the symbol table.

Other types of 3-address statements

- e.g. ternary operations like
  x[i]:=y x:=y[i]
- require two or more entries. e.g.

	op	*arg1*	*arg2*
(0)	**[ ] =**	**x**	**i**
(1)	**assign**	**(0)**	**y**

	op	*arg1*	*arg2*
(0)	**[ ] =**	**y**	**i**
(1)	**assign**	**x**	**(0)**

Implementations of 3-address statements, III

- **Indirect Triples**

	op		*op*	*arg1*	*arg2*
(0)	(14)	(14)	uminus	c	
(1)	(15)	(15)	*	b	(14)
(2)	(16)	(16)	uminus	c	
(3)	(17)	(17)	*	b	(16)
(4)	(18)	(18)	+	(15)	(17)
(5)	(19)	(19)	assign	a	(18)

Dealing with Procedures

P → procedure id ‘;’ block ‘;’

<u>Semantic Rule</u>

***begin* = *newlabel*;**

Enter into symbol-table in the entry of the procedure name the begin label.

P.*code* = *gen*(*begin* ‘:’) || block.*code* || gen(‘pop’ return_address) || gen(“goto return_address”)

S → call id

<u>Semantic Rule</u>

Look up symbol table to find procedure name. Find its begin label called proc_begin

return* = *newlabel;

S.*code* = *gen*(‘push’return); *gen*(goto proc_begin) || *gen*(return “:”)

Declarations

Using a global variable *offset*

PRODUCTION	Semantic Rule
P → M D	**{ }**
M → ε	**{*offset*:=0 }**
D → id : T	**{ *addtype*(id.*entry*, T.*type*, *offset*) *offset*:=*offset* + T.*width* }**
T → char	**{T.*type* = *char*; T.*width* = 4; }**
T → integer	**{T.*type* = *integer* ; T.*width* = 4; }**
T → array [num] of T_1	**{T.*type*=*array*(1..num.*val*,T_1.*type*) T.*width* = num.val * T_1.*width*}**
T → ^T_1	**{T.*type* = *pointer*(T_1.*type*); T_1.*width* = 4}**

Nested Procedure Declarations

- For each procedure we should create a symbol table.

mktable(previous) – create a new symbol table where previous is the parent symbol table of this new symbol table

enter(symtable,name,type,offset) – create a new entry for a variable in the given symbol table.

enterproc(symtable,name,newsymbtable) – create a new entry for the procedure in the symbol table of its parent.

addwidth(symtable,width) – puts the total width of all entries in the symbol table into the header of that table.

- We will have two stacks:
  - tblptr – to hold the pointers to the symbol tables
  - offset – to hold the current offsets in the symbol tables in tblptr stack.

Keeping Track of Scope Information

Consider the grammar fraction:

P → D

D → D ; D | id : T | proc id ; D ; S

Each procedure should be allowed to use independent names.

Nested procedures are allowed.

Keeping Track of Scope Information

(a translation scheme)

P → M D { *addwidth*(*top*(tblptr), *top*(offset));

pop(tblptr); _pop_(offset) }

M → ε { t:=*mktable*(null); *push*(t, tblptr);

push(0,

offset)}

D → D_1 ; D_2 ...

D → proc id ; N D ; S { t:=*top*(tblpr);

addwidth(t,*top*(offset));

pop(tblptr); _pop_(offset);

enterproc(*top*(tblptr), id.name, t)}

N → ε {t:=*mktable*(*top*(tblptr)); *push*(t,tblptr);

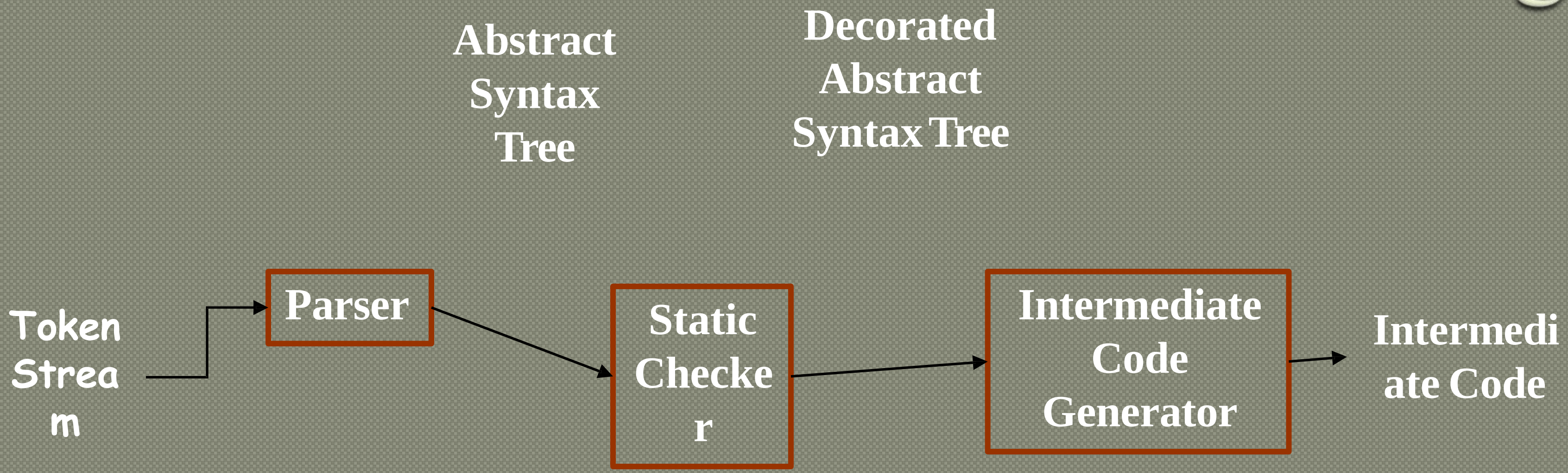
Keeping Track of Scope Information

D → id : T {*enter*(*top*(tblptr), id.name, T.*type*, *top*(offset);
**   *top*(offset):=*top*(offset) + T.*width***

Example: proc func1; D; proc func2 D; S; S

Type Checking

Static Checking



Abstract Syntax Tree

Decorated Abstract Syntax Tree

Token Stream

Parser

Static Checker

Intermediate Code Generator

Intermediate Code

- Static (Semantic) Checks
  - Type checks: operator applied to incompatible operands?
  - Flow of control checks: break (outside while?)
  - Uniqueness checks: labels in case statements
  - Name related checks: same name?

Type Checking

- **Problem: Verify that a type of a construct matches that expected by its context.**

- **Examples:**
  - **mod requires integer operands (PASCAL)**
  - *** (dereferencing) – applied to a pointer**
  - **a[i] – indexing applied to an array**
  - **f(a1, a2, …, an) – function applied to correct arguments.**
- **Information gathered by a type checker:**
  - **Needed during code generation.**

Type Systems

- **A collection of rules for assigning type expressions to the various parts of a program.**

- **Based on: Syntactic constructs, notion of a type.**

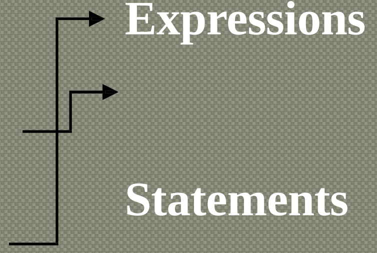
- **Example: If both operators of “+”, “-”, “*” are of type integer then so is the result.**

- **Type Checker: An implementation of a type system.**
  - **Syntax Directed.**

- **Sound Type System: eliminates the need for checking type errors during run time.**

Type Expressions

- **Implicit Assumptions:**
  - **Each program has a type**
  - **Types have a structure**



Expressions

Statements

Basic Types

Boolean

Real

Enumerations

Void

Variables

Character

Integer

Sub-ranges

Error

Names

Type Constructors

Arrays

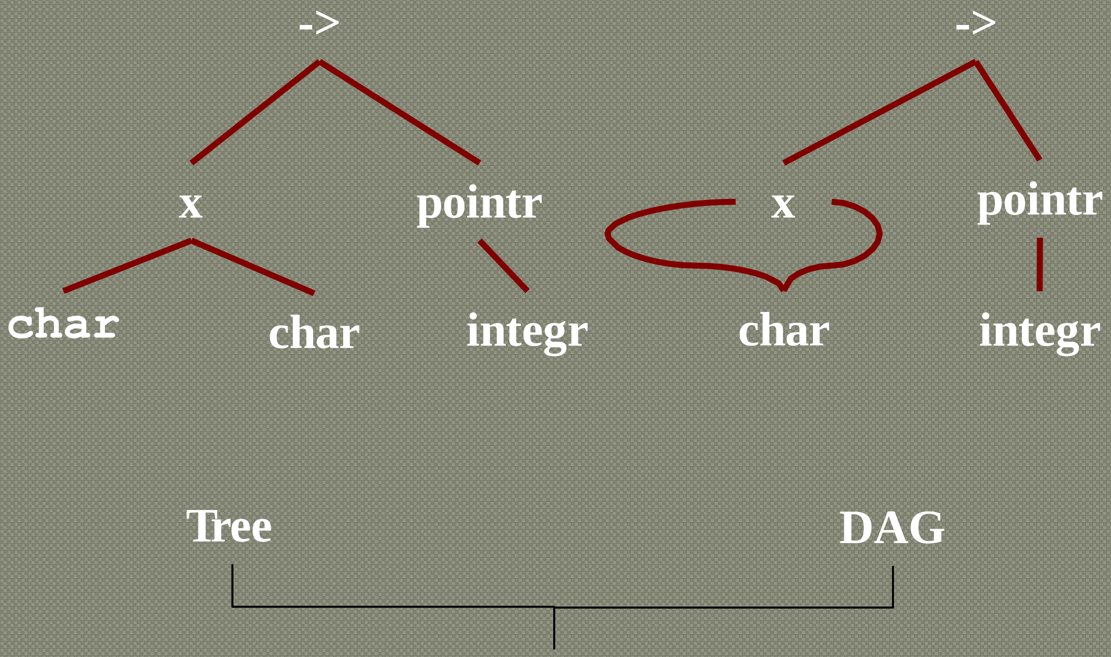
Records

Sets

Pointers

Functions

Representation of Type Expressions



->

x

pointr

char

char

integr

->

x

pointr

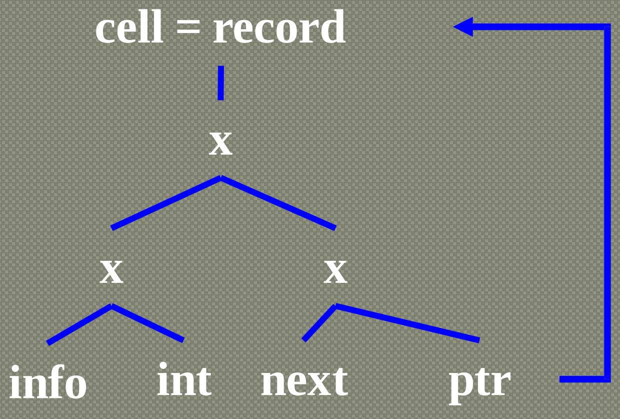
char

integr

Tree

DAG

(char x char)-> pointer (integer)



cell = record

x

x

x

info

int

next

ptr

```
struct cell {
        int info;
        struct cell * next;
};
```

Type Expressions Grammar

```
Type ->    int | float | char | …
           | void
           | error
           | name
           | variable
           | array( size, Type)
           | record( (name, Type)*)
           | pointer( Type)
           | tuple((Type)*)
           | arrow(Type, Type)
```

Basic Types

Structured Types

A Simple Typed Language

```
Program -> Declaration; Statement
Declaration -> Declaration; Declaration
                        | id: Type
Statement -> Statement; Statement
                        | id := Expression
                        | if Expression then Statement
                        | while Expression do Statement
Expression -> literal | num | id
                  | Expression mod Expression
                  | E[E] | E ↑ | E (E)
```

Type Checking Expressions

E -> int_const { E.type = int }

E -> float_const { E.type = float }

E -> id { E.type = sym_lookup(id.entry, type) }

E -> E1 + E2 {E.type = if E1.type ∉{int, float} | E2.type ∈ {int, float}

then error

else if E1.type == E2.type == int

then int

else float }

Type Checking Expressions

E -> E1 [E2]   {E.type = if E1.type = array(S, T) &
E2.type = int then T else error}

E -> *E1 {E.type = if E1.type = pointer(T) then T else error}

E -> &E1 {E.type = pointer(E1.tye)}

E -> E1 (E2)   {E.type = if (E1.type = arrow(S, T) &
E2.type = S, then T else err}

E -> (E1, E2) {E.type = tuple(E1.type, E2.type)}

Type Checking Statements

S -> id := E {S.type := if id.type = E.type then void else error}

S -> if E then S1 {S.type := if E.type = boolean then S1.type else error}

S -> while E do S1 {S.type := if E.type = boolean then S1.type}

S -> S1; S2 {S.type := if S1.type = void ∧ S2.type = void then void else error}

Equivalence of Type Expressions

Problem: When in E1.type = E2.type?

- We need a precise definition for type equivalence
- Interaction between type equivalence and type representation

Example:

```
type vector = array [1..10] of real
type weight = array [1..10] of real
var x, y: vector; z: weight
```

Name Equivalence: When they have the same name.

- x, y have the same type; z has a different type.

Structural Equivalence: When they have the same structure.

- x, y, z have the same type.

Structural Equivalence

- **Definition: by Induction**
  - **Same basic type(basis)**
  - **Same constructor applied to SE Type(induction step)**
  - **Same DAG Representation**

- **In Practice: modifications are needed**
  - **Do not include array bounds – when they are passed as parameters**
  - **Other applied representations (More compact)**

- **Can be applied to: Tree/ DAG**
  - **Does not check for cycles**
  - **Later improve it.**

Algorithm Testing Structural Equivalence

```
function stequiv(s, t): boolean
{
    if (s & t are of the same basic type) return true;
          if (s = array(s1, s2) & t = array(t1, t2))
          return equal(s1, t1) & stequiv(s2, t2);
          if (s = tuple(s1, s2) & t = tuple(t1, t2))
          return stequiv(s1, t1) & stequiv(s2, t2);
    if (s = arrow(s1, s2) & t = arrow(t1, t2))
          return stequiv(s1, t1) & stequiv(s2, t2);
    if (s = pointer(s1) & t = pointer(t1))
          return stequiv(s1, t1);
}
```

Types

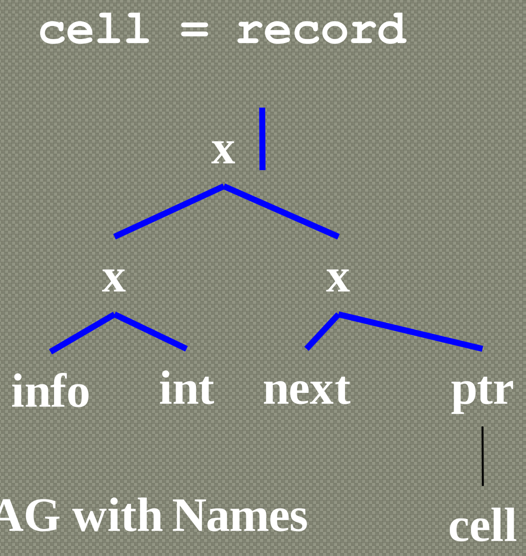
Where: Linked Lists, Trees, etc.

How: records containing pointers to similar records

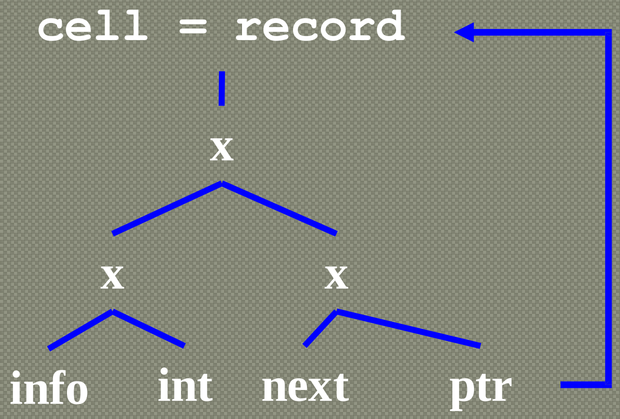
Example: type link = ↑ cell;

cell = record info: int; next = link end

Representation:



DAG with Names



Substituting names out (cycles)

Recursive Types in C

- **C Policy: avoid cycles in type graphs by:**
  - **Using structural equivalence for all types**
  - **Except for records -> name equivalence**

- **Example:**
  - **struct cell {int info; struct cell * next;}**

- **Name use: name cell becomes part of the type of the record.**
  - **Use the acyclic representation**
  - **Names declared before use – except for pointers to records.**
  - **Cycles – potential due to pointers in records**
  - **Testing for structural equivalence stops when a record constructor is reached ~ same named record type?**

Overloading Functions & Operators

- **Overloaded Symbol: one that has different meanings depending on its context**

- **Example: Addition operator +**

- **Resolving (operator identification): overloading is resolved when a unique meaning is determined.**

- **Context: it is not always possible to resolve overloading by looking only the arguments of a function**
  - **Set of possible types**
  - **Context (inherited attribute) necessary**

Overloading Example

```
function “*” (i, j: integer) return complex;
function “*” (x, y: complex) return complex;
* Has the following types:
    arrow(tuple(integer, integer), integer)
    arrow(tuple(integer, integer), complex)
    arrow(tuple(complex, complex), complex)
int i, j;
k = i * j;
```

Narrowing Down Types

E’ -> E {E’.types = E. types
E.unique = if E’.types = {t} then t else error}

E -> id {E.types = lookup(id.entry)}

E -> E1(E2) {E.types = {s’ | ∃ s ∈ E2.types and S->s’ ∈ E1.types}

t = E.unique

S = {s | s ∈ E2.types and S->t ∈E1.types}

E2.unique = if S ={s} the S else error

E1.unique = if S = {s} the S->t else error

Polymorphic Functions

- Defn: a piece of code (functions, operators) that can be executed with arguments of different types.

- Examples: Built in Operator indexing arrays, pointer manipulation

- Why use them: facilitate manipulation of data structures regardless of types.

- Example HL:

```
fun length(lptr) = if null (lptr) then 0
                                        else length(+l(lptr)) + 1
```

A Language for Polymorphic Functions

- P -> D ; E
- D -> D ; D | id : Q
- Q -> ∀ α. Q | T
- T -> arrow (T, T) | tuple (T, T)
    - | unary (T) | (T)
    - | basic
    - | α
- E -> E (E) | E, E | id

Type Variables

- **Why: variables representing type expressions allow us to talk about unknown types.**
  - **Use Greek alphabets α, β, γ …**
- **Application: check consistent usage of identifiers in a language that does not require identifiers to be declared before usage.**
  - **A type variable represents the type of an undeclared identifier.**
- **Type Inference Problem: Determine the type of a language constant from the way it is used.**
  - **We have to deal with expressions containing variables.**

Examples of Type Inference

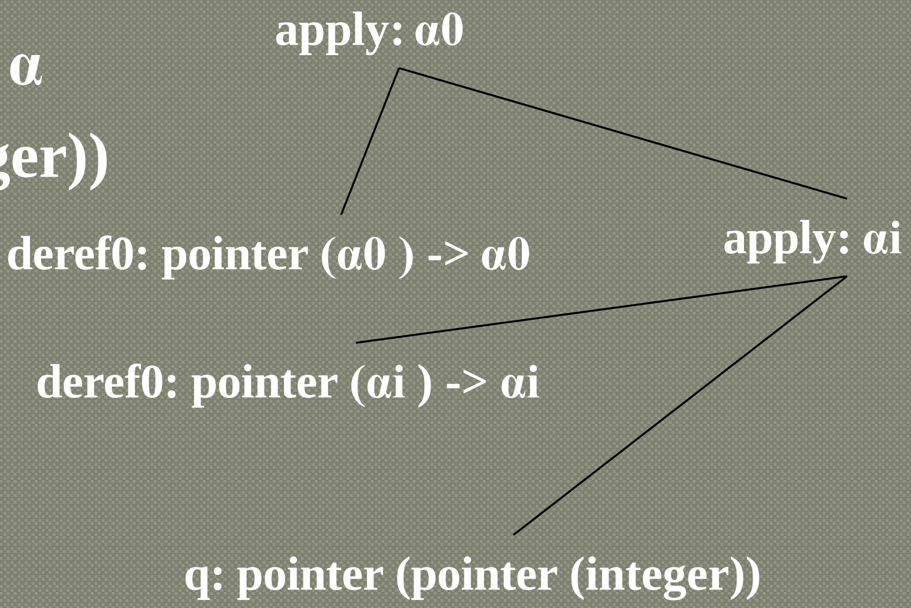
- Type link ↑ cell;
- Procedure mlist (lptr: link; procedure p);
- { while lptr <> null { p(lptr); lptr := lptr ↑ .next} }
- Hence: p: link -> void
- Function deref (p){ return p ↑; }
- P: β, β = pointer(α)
- Hence deref: $\forall$ α. pointer(α) -> α

Program in Polymorphic Language

deref: ∀ α. pointer(α) -> α

q: pointer (pointer (integer))

deref (deref((q))



apply: α0

deref0: pointer (α0) -> α0

apply: αi

deref0: pointer (αi) -> αi

q: pointer (pointer (integer))

Notation:

-> arrow

x tuple

Subsripts i and o distinguish between the inner and outer occurrences of deref, respectively.

Type Checking Polymorphic Functions

- Distinct occurrences of a p.f. in the same expression need not have arguments of the same type.
  - – deref (deref (q))
  - – Replace α with fresh variable and remove ∀ (αi, αo)

- The notion of type equivalence changes in the presence of variables.
  - – Use unification: check if s and t can be made structurally equivalent by replacing type vars by the type expression.

- We need a mechanism for recording the effect of unifying two expressions.
  - – A type variable may occur in several type expressions.

Substitutions and Unification

- **Substitution: a mapping from type variables to type expressions.**

```
Function subst (t: type Expr): type Expr { S
    if (t is a basic type) return t;
    if (t is a basic variable) return S(t); --identify if t ∉ S
    if (t is t1 -> t2) return subst(t1) -> subst (t2); }
```

- **Instance: $S(t)$ is an instance of t written $S(t) < t$.**
  - **Examples: pointer (integer) < pointer (α) , int -> real ≠ α-> α**

- **Unify: $t1 \approx t2$ if $\exists S.\ S(t1) = S(t2)$**

- **Most General Unifier S: A substitution S:**
  - **$S(t1) = S(t2)$**
  - **$\forall S'.\ S'(t1) = S'(t2)$ ➔ $\forall t.\ S'(t) < S(t)$.**

Polymorphic Type checking Translation Scheme

E -> E1 (E2) { p := mkleaf(newtypevar); unify
(E1.type, mknode('->', E2.type,p);
E.type = p}

E -> E1, E2 {E.type := mknode('x', E1.type, E2.type); }

E -> id { E.type := fresh (id.type) }

fresh (t): replaces bound vars in t by fresh vars. Returns pointer to a node representing result.type.

fresh(∀ α.pointer(α) -> α) = pointer(α1) -> α1.

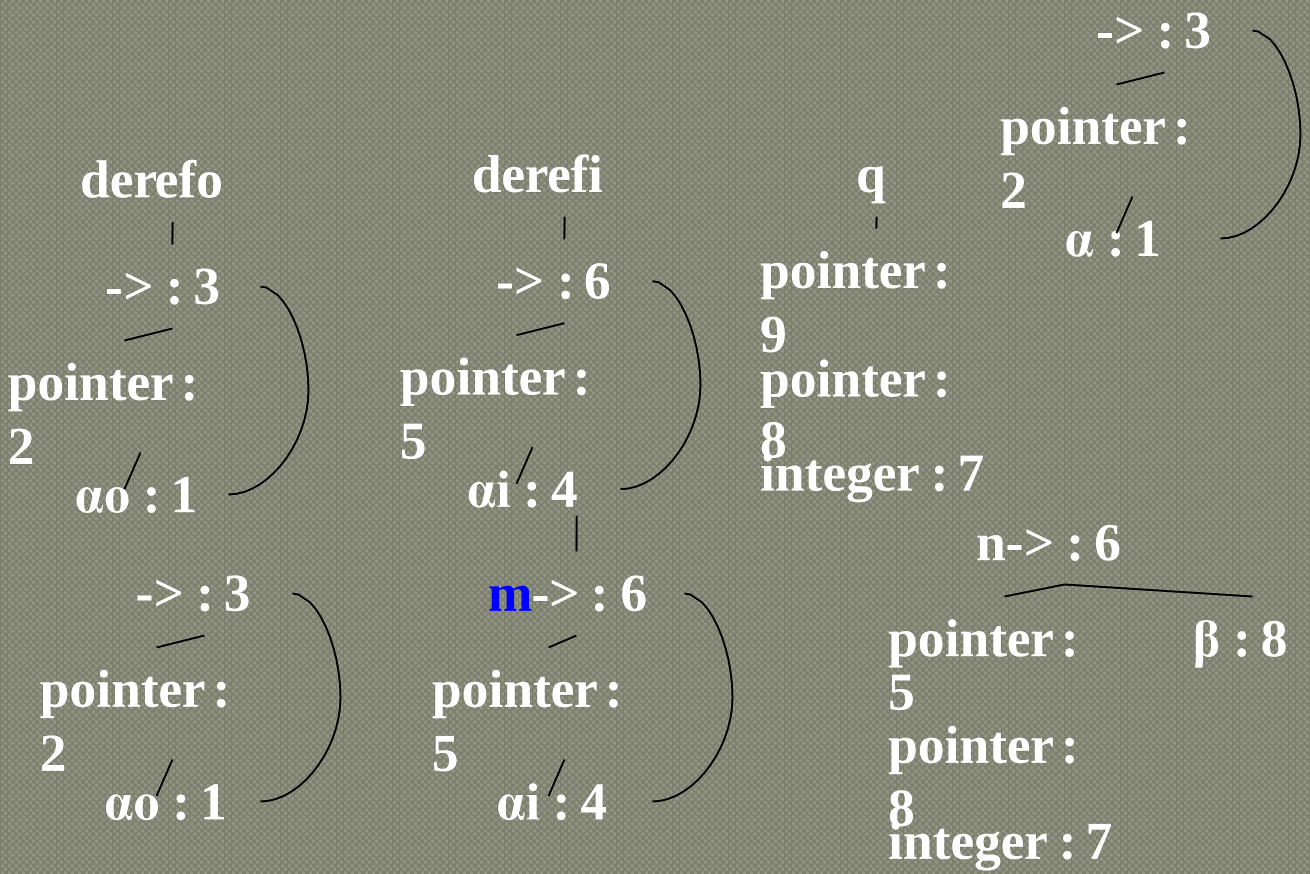
unify (m, n): unifies expressions represented by m and n.

- Side-effect: keep track of substitution
- Fail-to-unify: abort type checking.

PType Checking Example

Given: derefo (derefi (q)) q = pointer (pointer (int))

Bottom Up: fresh (∀α. Pointer(α) -> α)



UNIT-5

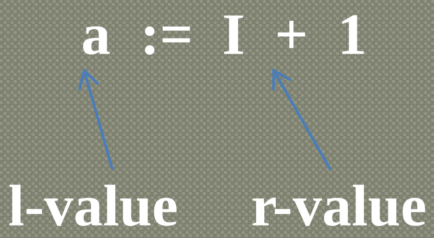
SYMBOL TABLE

Symbol tables

Def : Symbol table is a data structure used by compiler to keep track of semantics of variable .

L-value and r – value : the l and r prefixes come from left and right side assignment .

Ex:



a := I + 1

l-value r-value

Symbol table entries

- Variable names
- Constants
- Procedure names
- Function names
- Literal constants and strings
- Compiler generated temporaries
- Labels in source languages

Compiler uses following types of information from symbol table

1.Data type

2.Name

3.Declaring procedures

4.Offset in storage

Symbol table entries

5.If structure are record then pointer to structure variable.

6.For parameters, whether parameter passing is

by value or reference.

7.Number and type of arguments passed to the function.

8.Base address.

How to store names in symbol table

- **There are two types of name representation.**
- **Fixed length names :**
- **A fixed space for each name is allocated in symbol table .**

name										**attribute**
C	A	L	C	U	L	A	T	E		
S	U	m								
a										
b										

Variable length name

- **Variable length**

name		attribute
Starting index	length	
0	10	
10	4	
14	2	
16	2	

Symbol table management

- **Data structure for symbol table :**

1 . Linear list

2 . Arrays

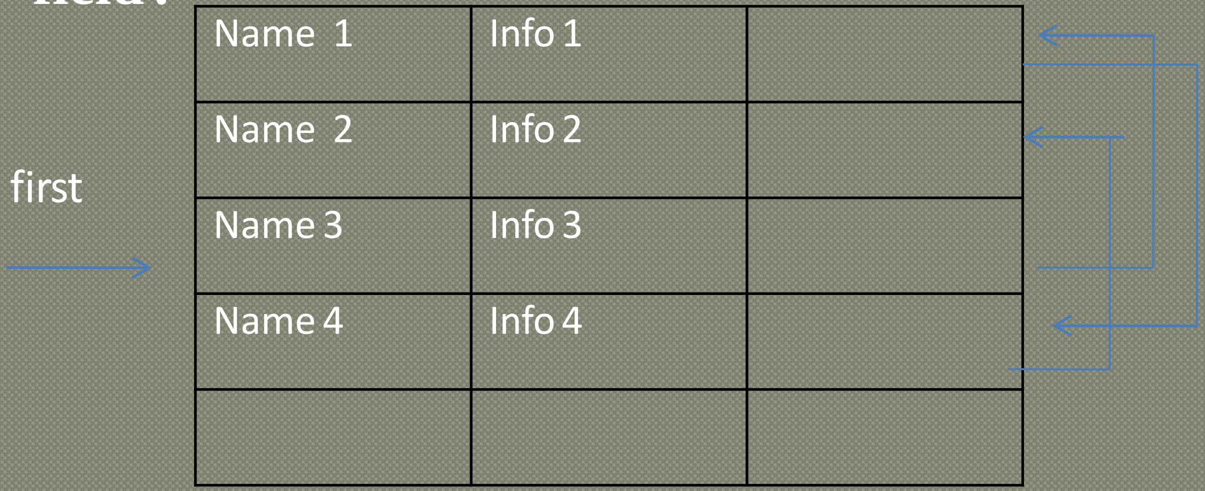
The pointer variable is maintained at the end of all stored records .

Name 1	Info 1
Name2	Info 2
Name 3	Info 3
. . .	. . .
Name n	Info n

Available
(start of empty
slot

Symbol table management

- **Self organization list :**
- **This symbol table implementation is using linked list .**
- **We search the records in the order pointed by the link of link field .**



first

Name 1	Info 1	
Name 2	Info 2	
Name 3	Info 3	
Name 4	Info 4	

A pointer first is maintained to point to first record of symbol table
The reference to these names can be name 3,name 1,name 4,name2 .

Symbol table management

- **Self organization list :**
- When the name is referenced or created it is moved to the front of the list .
- The most frequently referred names will tend to be front of the list . Hence access time to most frequently referred names will be the least .

Symbol table management

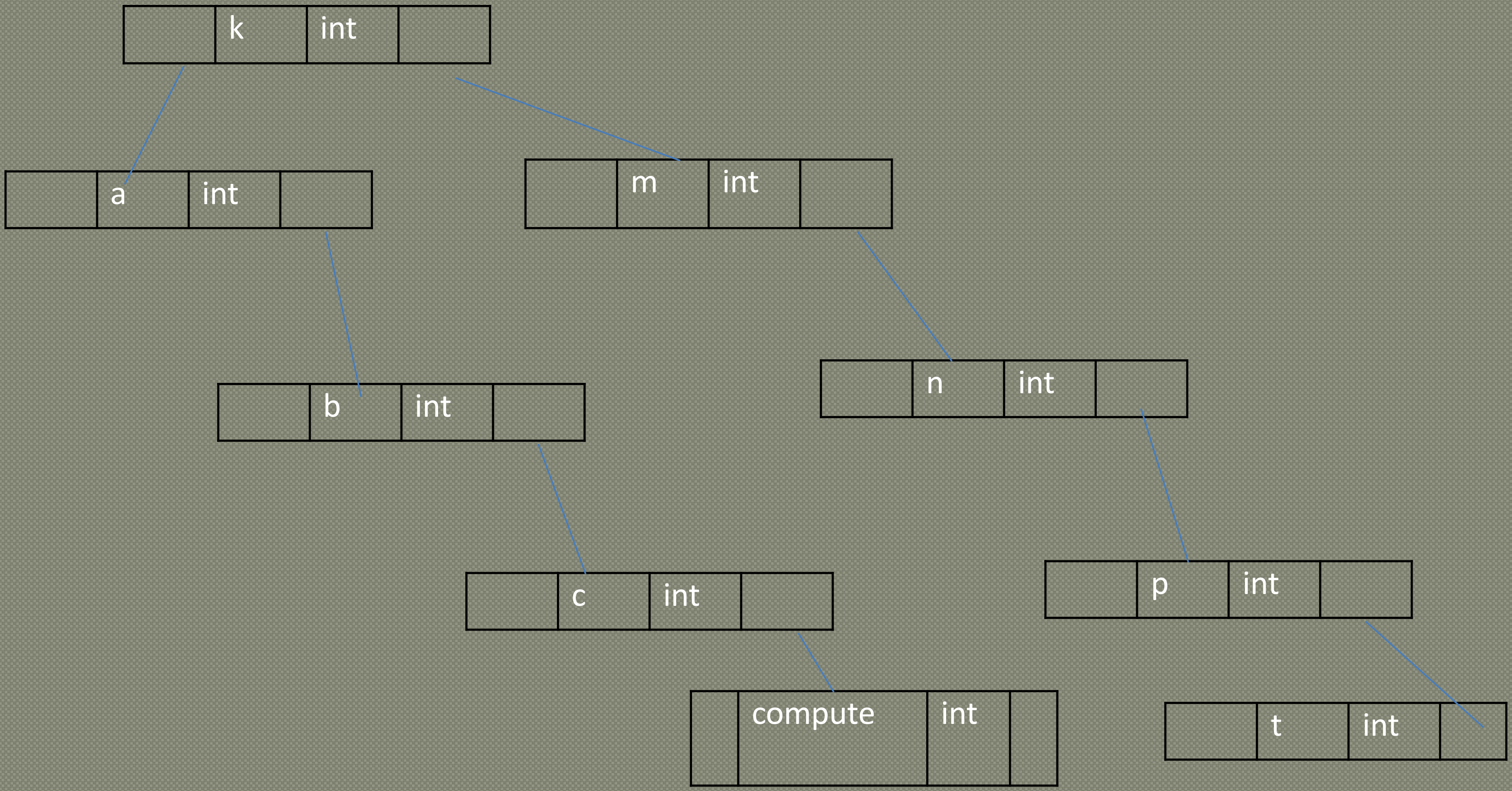
- Binary trees

Left node	Symbols	Information	Right child

- Ex:
- Int m,n,p;
- Int compute(int a,int b,intc)
- {
- T=a+b*c;
- Return(t);
- }
- Main()
- {
- Int k;
- K=compute(10,20,30)
- }

Symbol table management

- **Binary tree structure organization**



Symbol table management

- **Binary tree structure organization**
- **Advantages :**
- **Insertion of any symbol is efficient.**
- **Any symbol can be searched efficiently using binary searching method.**
- **Disadvantages :**
- **This structure consumes lot of space in storing left pointer,right pointer and null pointers.**

Symbol table management

- **Hash tables :**
- **It is used to search the records of symbol table .**
- **In hashing two tables are maintained a hash table and symbol table .**
- **Hash table contains of k entries from 0,1 to k-1 . These entries are basically pointers to symbol table pointing to the names of symbol table .**
- **To determine where the name is in symbol table ,we use a hash function 'h' such that h(name) will result any integer between 0 to k-1 . We can search any name by**

position = h(name)

Using this position we can obtain the exact locations of name in symbol table .

Symbol table management

- **Hash tables :**

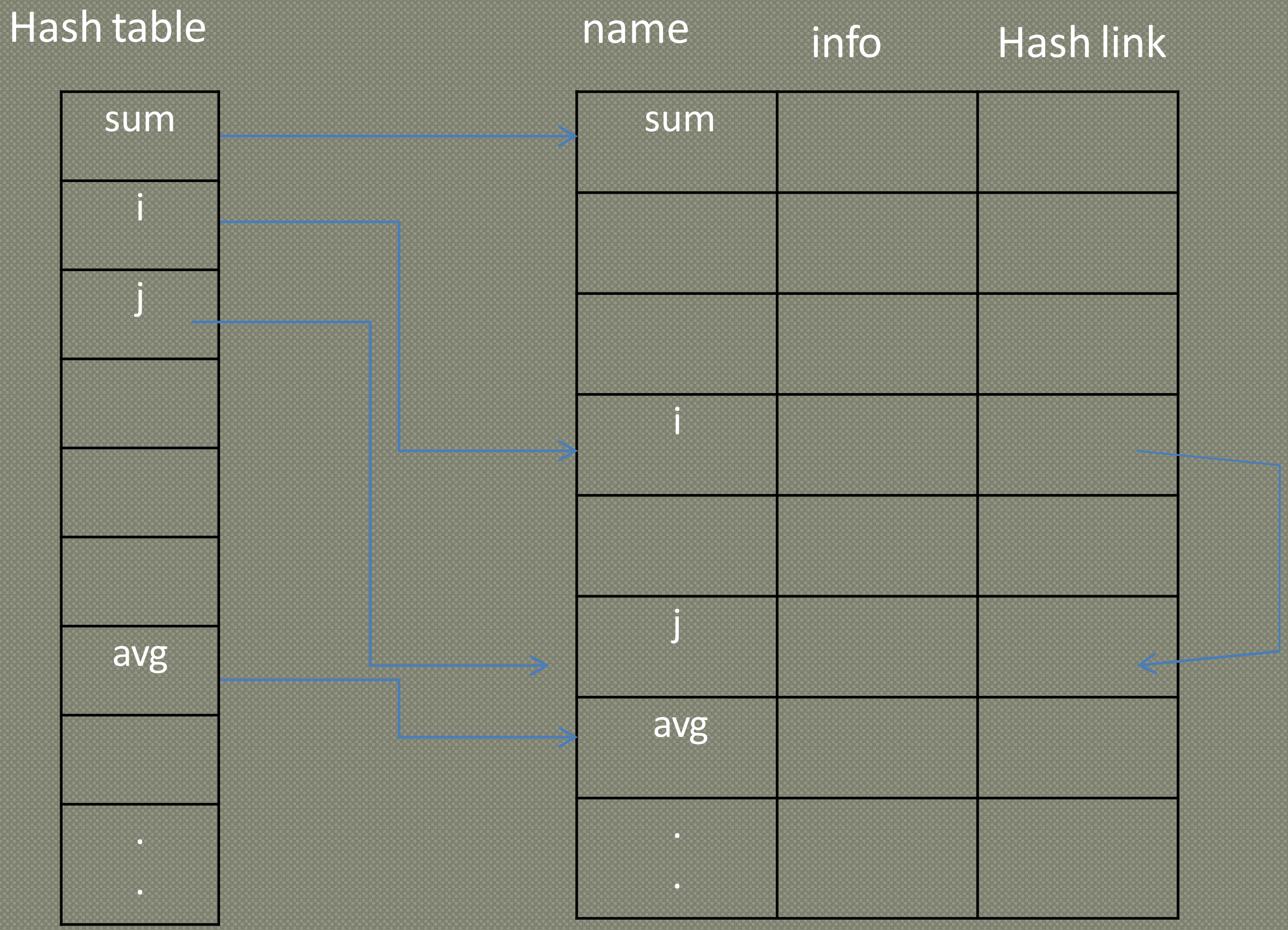
The hash function should result in uniform distribution of names in symbol table .

The hash function should be such that there will be minimum number of collision . Collision is such a situation where hash function results in same location for storing the names .

Various collision resolution techniques are open addressing, chaining , rehashing .

Symbol table management

- **Hash tables :**



Symbol table management

- **Hash tables :**
- **The advantages of hash table is quick search is possible .**
- **The disadvantage is that hashing is complicated to implement . Some extra space is required . Obtaining scope of variables is very difficult .**

Symbol table management

- **HEAPALLOCATION:**
- **The stack allocation strategy cannot be used if either of the following is possible:**
- **1. The values of local names must be retained when activation ends.**
- **2. A called activation outlives the caller.**
-
- **In each of the above cases, the deallocation of activation records need not occur in a last- in first-out fashion, so storage cannot be organized as a stack.**
- **Heap allocation parcels out pieces contiguous storage, as needed for activation records or other objects. Pieces may be deallocated in any order, so over** time the heap will consist of alternate areas that are free and in use.

Storage allocation stratagies

- 1.code area
- 2.Static data area
- 3.Stack area
- 4.heap area
- There are 3 different storage allocation strategies based on this division of run time storage .the strategies are
- 1.Static allocation :At compile time
- 2.Stack allocation : A stack is used to manage the run time manage .
- 3 . Heap allocation : heap is used to manage the dynamic memory allocation .

Static Allocation

- **Done at compile time**
  - Literals (and constants) bound to values
  - Variables bound to addresses
- **Compiler notes undefined symbols**
  - Library functions
  - Global Variables and System Constants
- Linker (and loader if DLLs used) resolve undefined references.

Stack Based Allocation

- **Stack Layout determined at compile time**
  - Variables bound to offsets from top of stack.
    - Layout called stack frame or activation record
  - Compilers use registers
- **Function parameters and results need consistent treatment across modules**
  - C/C++ use prototypes
  - Eiffel/Java/Oberon use single definition

Heap Allocation

- Heap provides dynamic memory management.
  - Not to be confused with binary heap or binomial heap data structures.
  - Under the hood, may periodically need to request additional memory from the O/S.
    - Requested large regions (requests are expensive).
  - Done using a library (e.g. C)
  - Or as part of the language (C++, Java, Lisp).

Activation Record

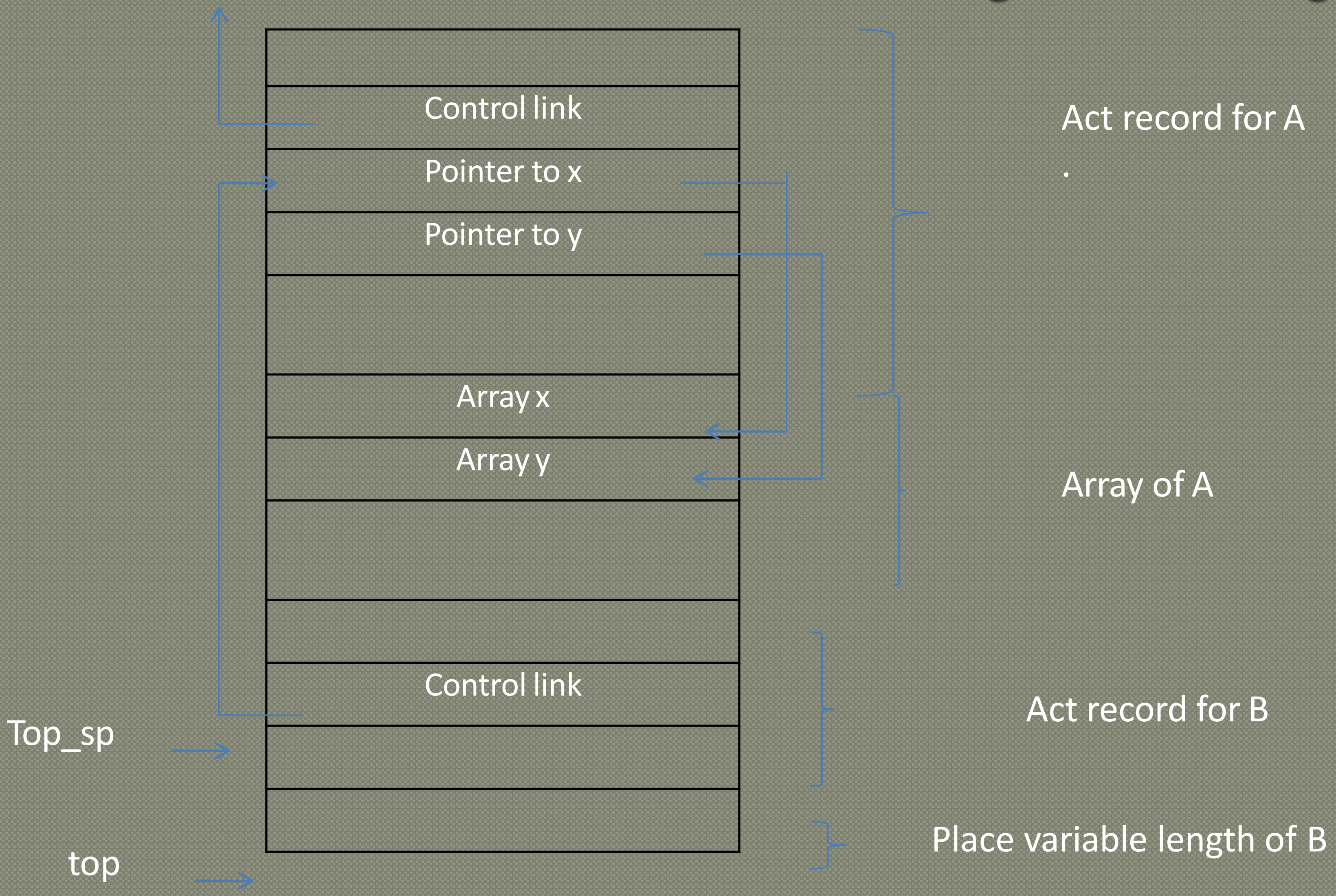
model of Activation Record

Return value
Actual parameters
Control link(dynamic link)
Access link (static link)
Saved machine status
Local variables
Temporaries

Activation Record

- Temporary values : These values are needed during the evaluation of expressions .
- Local variables : the local data is a data that is local to the execution of procedure is stored in this field of activation record .
- Saved machine registers : the status of machine just before the procedure is called .this field contains the machine registers and program counter .
- Control link : this field is optional .it points the activation record of the calling procedure . This link is also called dynamic link .
- Access link : this field is optional . It refers the non local data in other activation record . This field is also called static link field .
- Actual parameters : this contains the information about the actual parameters .
- Return values : this field is used to store the result of a function call .

Storage variable length data



Block structer and non block structure storage allocation

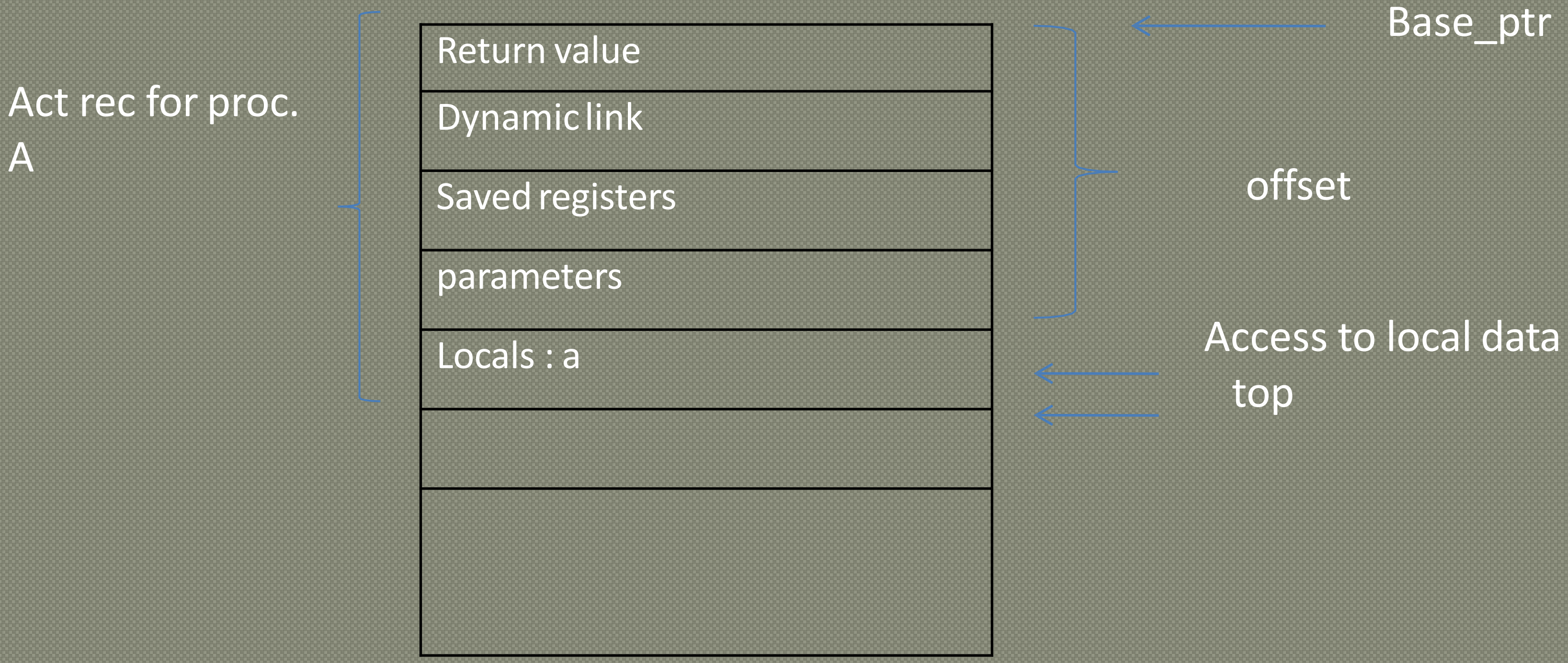
- The storage allocation can be done for 2 types of data variables
- 1. Local data
- 2. Non local data .
- Local data can be accessed with the help of activation record .
- Non local data can be accessed using scope information .
- The block structured storage allocation can be done using static scope or lexical scope .
- The non block structured can be done by dynamic scope .

Local data

- **Reference to any variable x in procedure = Base pointer pointing to start of procedure + Offset of variable x from base pointer .**
- **Ex: consider following program**
- **procedure A**
- **int a;**
- **procedure B**
- **int b;**
- **body of B;**
- **body of A;**

Local data

- **The contents of stack along with base pointer and offset are as shown below .**



Act rec for proc. A

Return value

Dynamic link

Saved registers

parameters

Locals : a

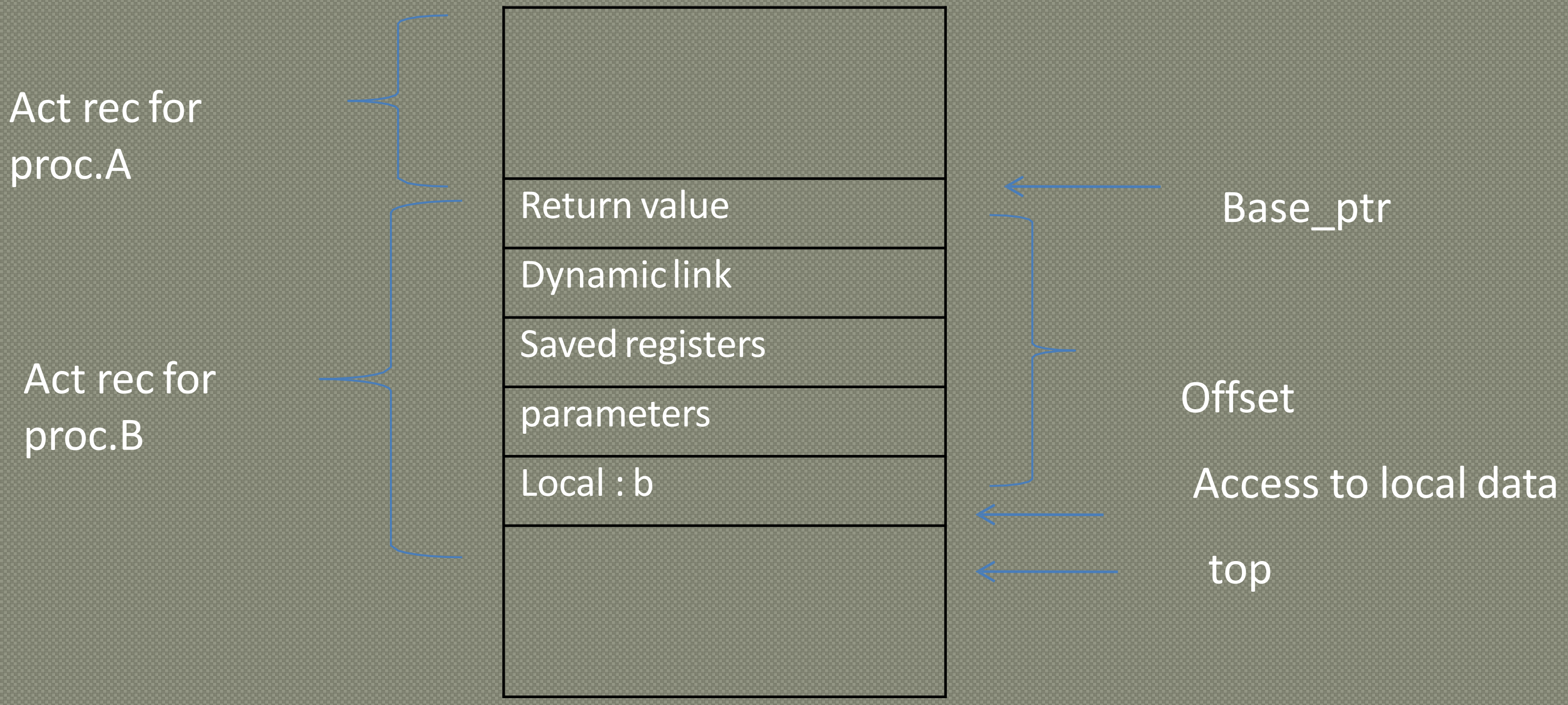
Base_ptr

offset

Access to local data

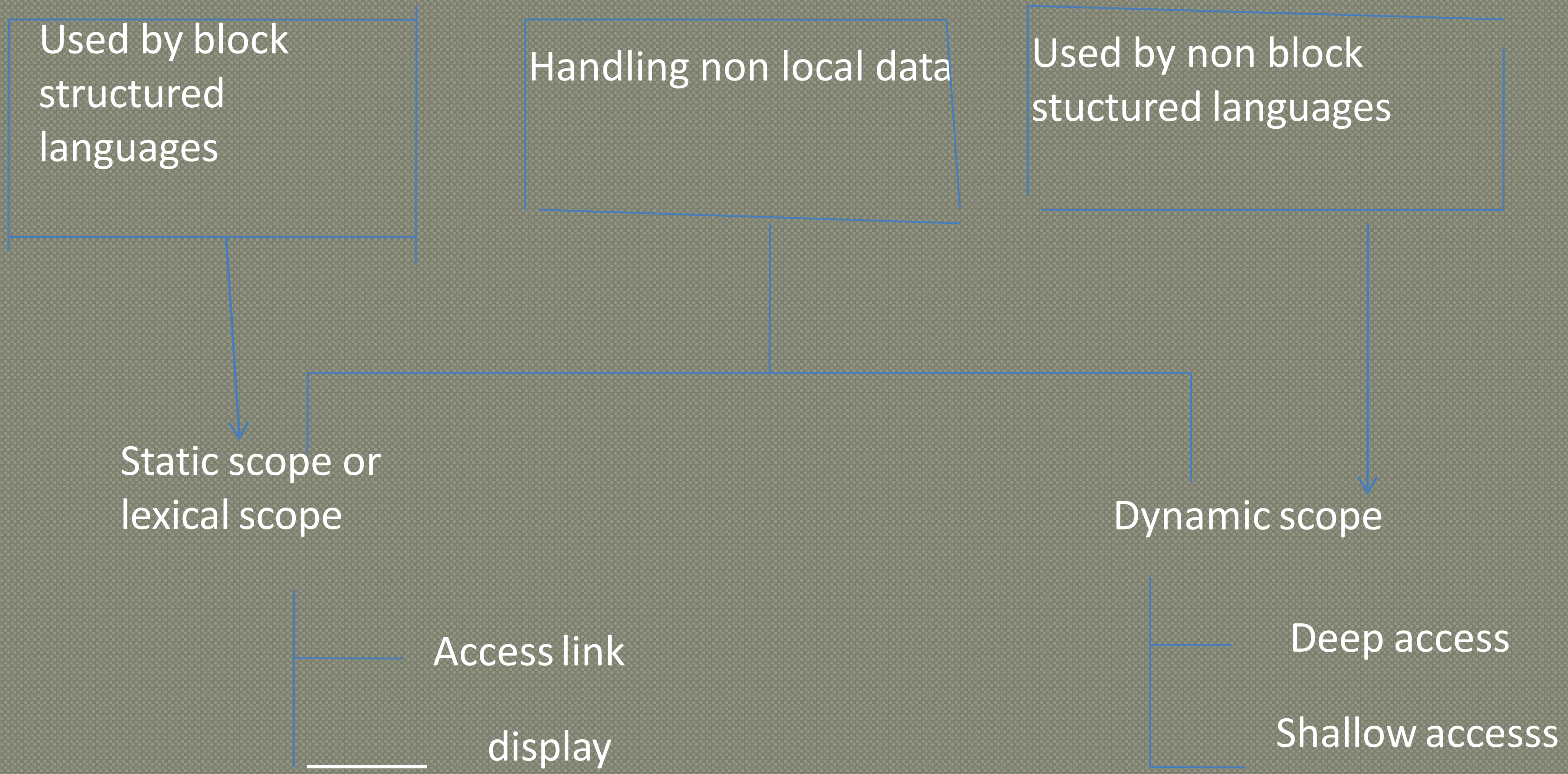
top

Local data


Act rec for proc.A
Act rec for proc.B
Return value
Dynamic link
Saved registers
parameters
Local : b
Base_ptr
Offset
Access to local data
top

Access to non local names

access



Used by block structured languages

Handling non local data

Used by non block stuctured languages

Static scope or lexical scope

Dynamic scope

Access link

display

Deep access

Shallow accesss

Scope rules

Static scope rule : Is also called as lexical scope .

- **In this type the scope is determined by examining the program test . PASCAL,C and ADA are the languages that use the static scope .**

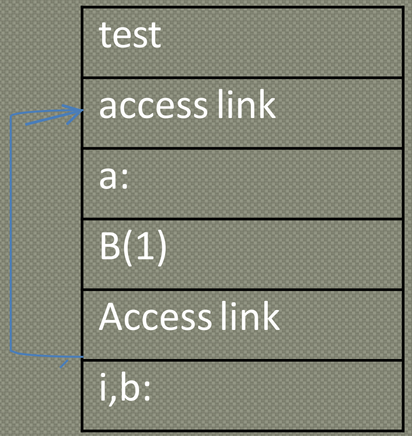
Dynamic scope : for non block structured languages this dynamic scope allocation rules are used .

- **Ex: LISP and SNOBOL**

Static scope or lexical scope

Access link

- **By using pointers to each record.**
- **These pointers are called access links.**



test

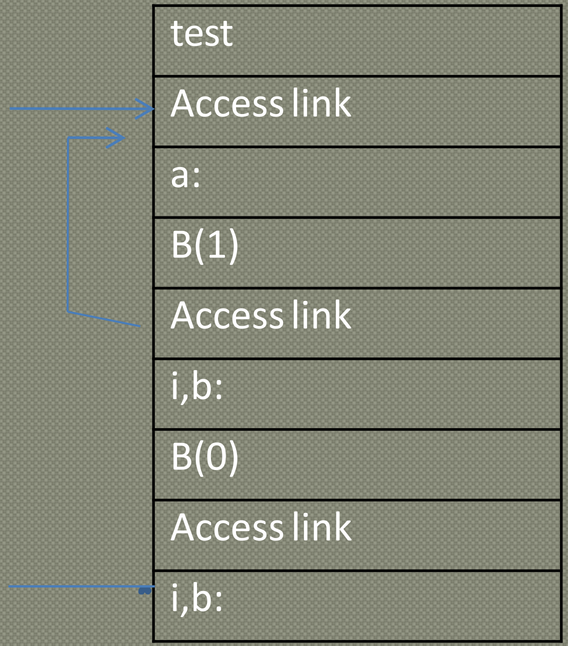
access link

a:

B(1)

Access link

i,b:



test

Access link

a:

B(1)

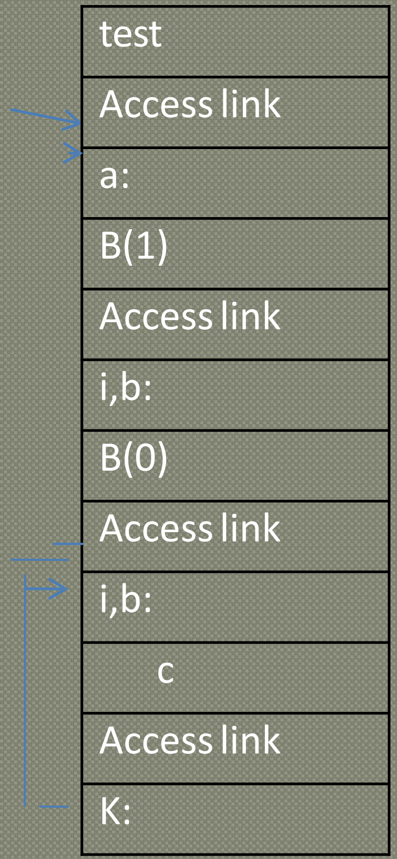
Access link

i,b:

B(0)

Access link

i,b:



test

Access link

a:

B(1)

Access link

i,b:

B(0)

Access link

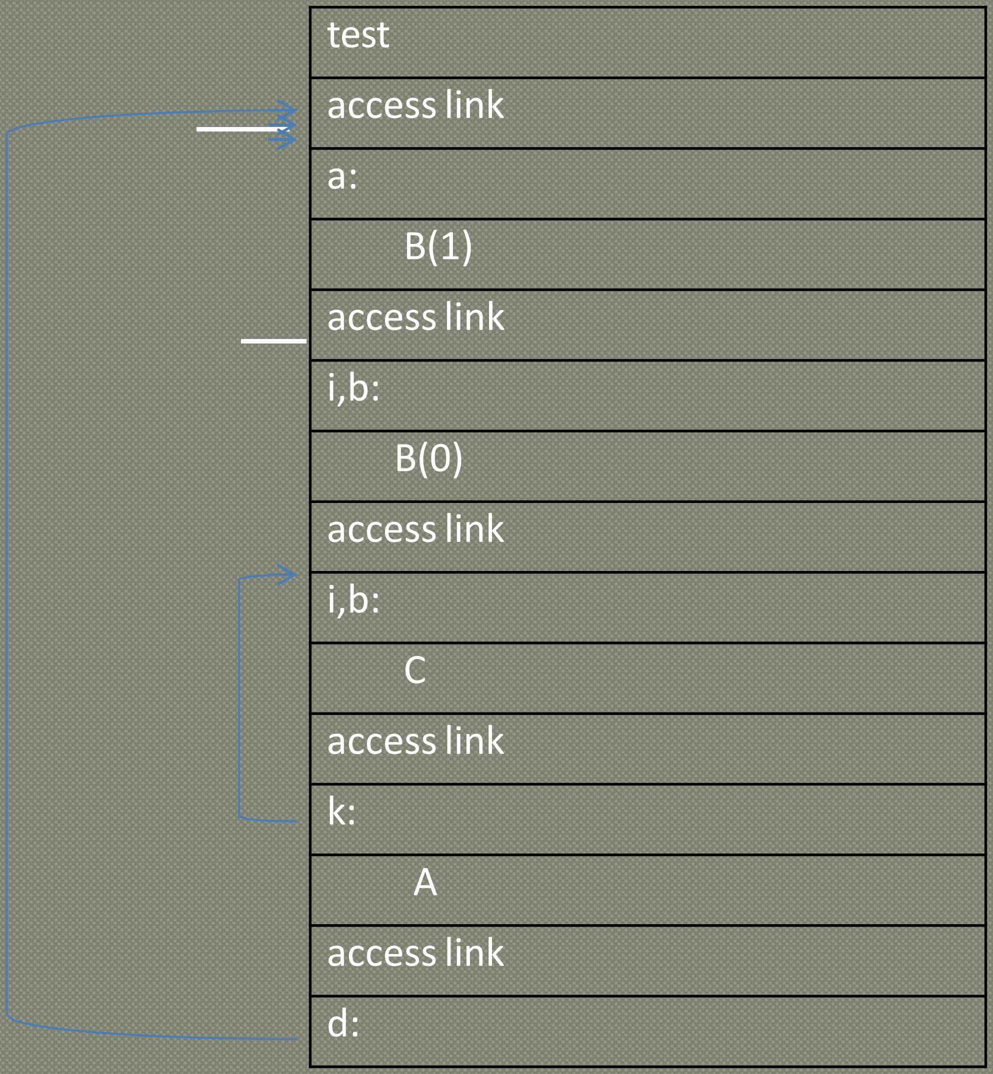
i,b:

c

Access link

K:

Static scope or lexical scope access link



test

access link

a:

B(1)

access link

i,b:

B(0)

access link

i,b:

C

access link

k:

A

access link

d:

Static scope or lexical scope

- **Displays :**
- **It is expensive to traverse down access link every time when a particular local variable is accessed . To speed up the access to non local can be achieved by maintaining an array of pointers called display.**
- **In display**
- **An array of pointers to activation record is maintained.**
- **Array is indexing by nesting level.**
- **The pointers points to only accessible activation record.**
- **The display changes when a new activation occurs and it must be reset when control returns from the new activation.**

Storage allocation for non block structured languages

- **Dynamic scope :**
- **1. deep access : the idea is keep a stack of active variables, use control links instead of access links and when you want to find a variable then search the stack from top to bottom looking for most recent activation record that contains the space for desired variables . This method of accessing non local variables is called Deep access .**
- **In this method a symbol table is needed to be used at run time .**
- **Shallow access : the idea is to keep a central storage with one slot for every variable name . If the names are not created at run time then that storage layout can be fixed at compile time otherwise when new activation of procedure occures,then that procedure changes the storage entries for its locals at entry and exit .**

Comparison of Deep and Shallow access

- **Deep access takes longer time to access the non locals while Shallow access allows fast access .**
- **Shallow access has a overhead of handling procedure entry and exit .**
- **Deep access needs a symbol table at run time .**

UNIT-6

Code Optimization

Introduction

- **Concerns with machine-independent code optimization**
  - **90-10 rule: execution spends 90% time in 10% of the code.**
    - **It is moderately easy to achieve 90% optimization. The rest 10% is very difficult.**
    - **Identification of the 10% of the code is not possible for a compiler – it is the job of a profiler.**
- **In general, loops are the hot-spots**

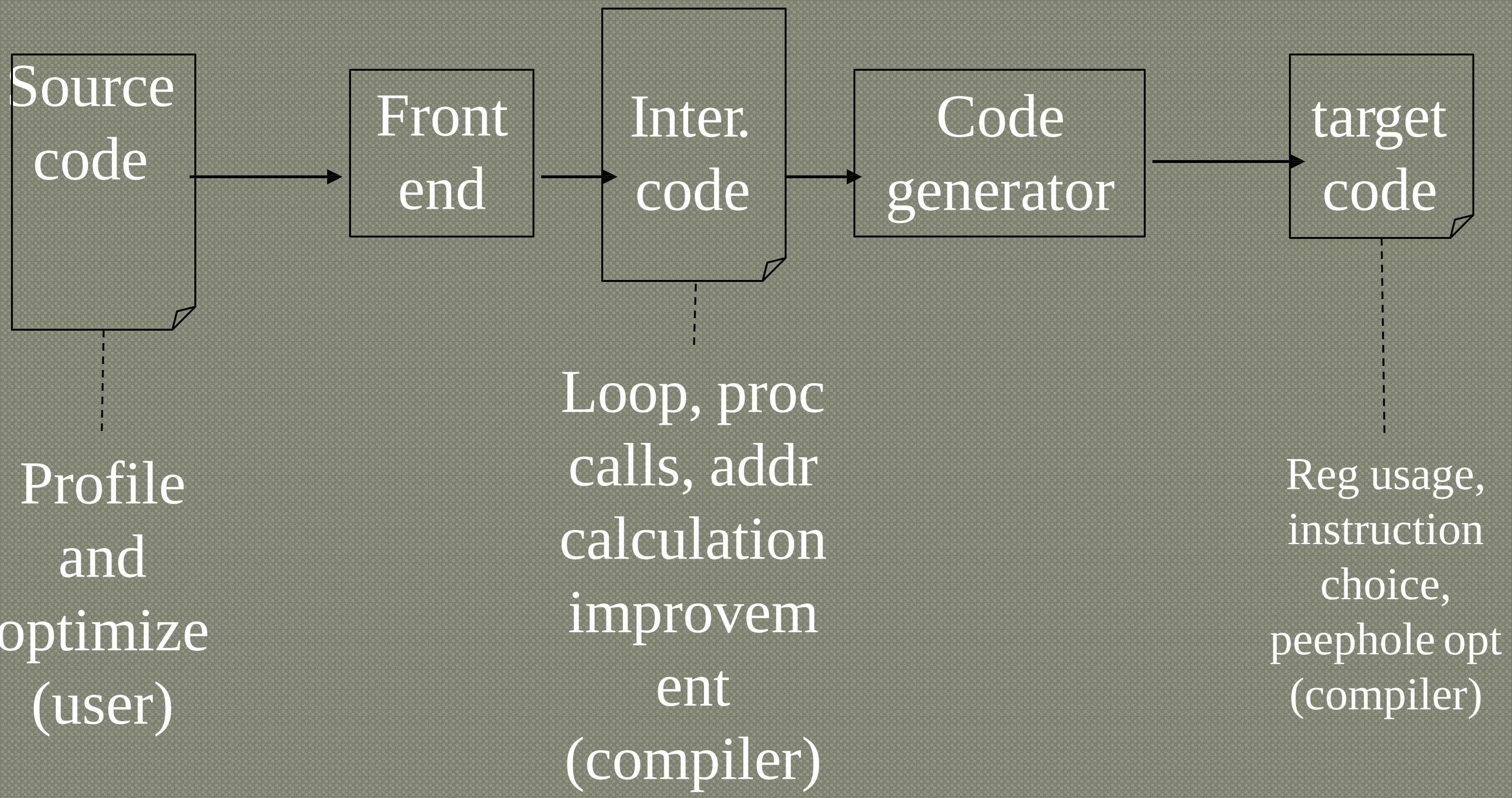
Introduction

- Criterion of code optimization
  - Must preserve the semantic equivalence of the programs
  - The algorithm should not be modified
  - Transformation, on average should speed up the execution of the program
  - Worth the effort: Intellectual and compilation effort spend on insignificant improvement.

    Transformations are simple enough to have a good effect

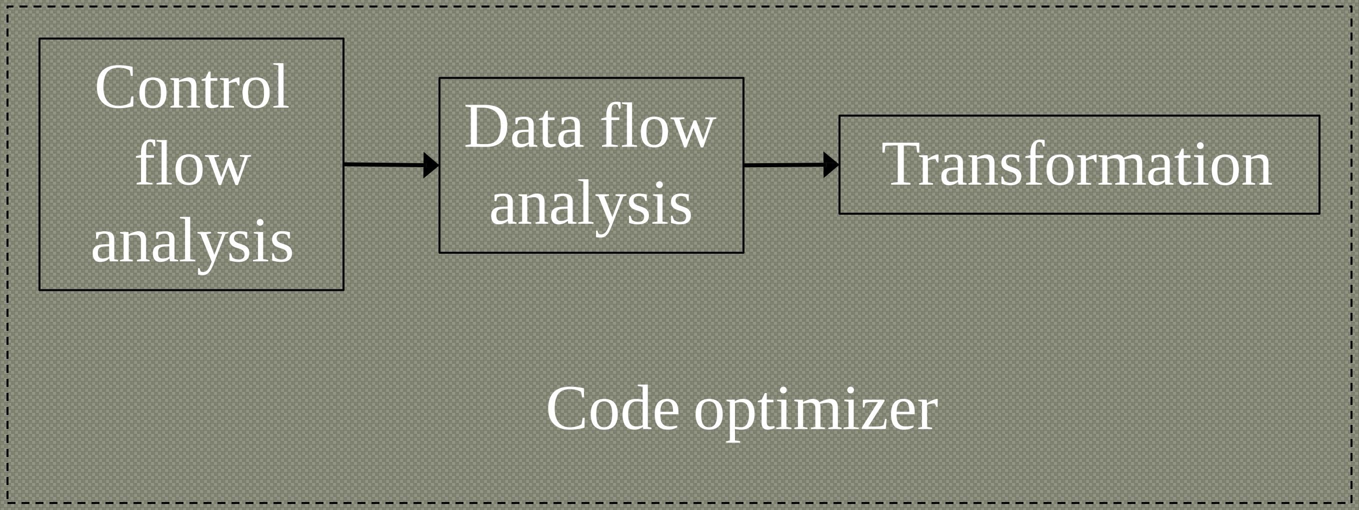
Introduction

- **Optimization can be done in almost all phases of compilation.**


Source code
Front end
Inter. code
Code generator
target code
Profile and optimize (user)
Loop, proc calls, addr calculation improvem ent (compiler)
Reg usage, instruction choice, peephole opt (compiler)

Introduction

- **Organization of an optimizing compiler**



Classifications of Optimization techniques

- Peephole optimization
- Local Optimization
- Global Optimization
  - Inter-procedural
  - Intra-procedural
- Loop Optimization

Factors influencing Optimization

- The target machine: machine dependent factors can be parameterized to compiler for fine tuning

- Architecture of Target CPU:
  - Number of CPU registers
  - RISC vs CISC
  - Pipeline Architecture
  - Number of functional units

- Machine Architecture
  - Cache Size and type
  - Cache/Memory transfer rate

Themes behind Optimization Techniques

- **Avoid redundancy:** something already computed need not be computed again
- **Smaller code:** less work for CPU, cache, and memory!
- **Less jumps:** jumps interfere with code pre-fetch
- **Code locality:** codes executed close together in time is generated close together in memory – increase locality of reference
- **Extract more information about code:** More info – better code generation

Redundancy elimination

- Redundancy elimination = determining that two computations are equivalent and eliminating one.

- There are several types of redundancy elimination:

  - Value numbering
    - Associates symbolic values to computations and identifies expressions that have the same value

  - Common subexpression elimination
    - Identifies expressions that have operands with the same name

Redundancy elimination

- Constant/Copy propagation
  - Identifies variables that have constant/copy values and uses the constants/copies in place of the variables.

- Partial redundancy elimination
  - Inserts computations in paths to convert partial redundancy to full redundancy.

Optimizing Transformations

- **Common sub expression elimination**
- **Code motion**
- **Strength reduction**
- **Dead code elimination**
- **Copy propagation**
- **Loop optimization**
- **Compile time evalution**
- **Induction variables and strength reduction**

Compile-Time Evaluation

- Expressions whose values can be pre-computed at the compilation time

- Two ways:
  - Constant folding
  - Constant propagation

Compile-Time Evaluation

- **Constant folding: Evaluation of an expression with constant operands to replace the expression with single value**

- **<u>Example:</u>**

```
area := (22.0/7.0) * r ** 2

area := 3.14286 * r ** 2
          ↓
```

Compile-Time Evaluation

- **Constant Propagation: Replace a variable with constant which has been assigned to it earlier.**

- **Example:**

```
 pi := 3.14286
 area = pi * r ** 2
                         area = 3.14286 * r ** 2
```

Constant Propagation

- What does it mean?
  - Given an assignment x = c, where c is a constant, replace later uses of x with uses of c, provided there are no intervening assignments to x.
    - Similar to copy propagation
    - Extra feature: It can analyze constant-value conditionals to determine whether a branch should be executed or not.
- When is it performed?
  - Early in the optimization process.
- What is the result?
  - Smaller code
  - Fewer registers

Common Sub-expression Evaluation

- **Identify common sub-expression present in different expression, compute once, and use the result in all the places.**
  - **The *definition* of the variables involved should not change**

Example:

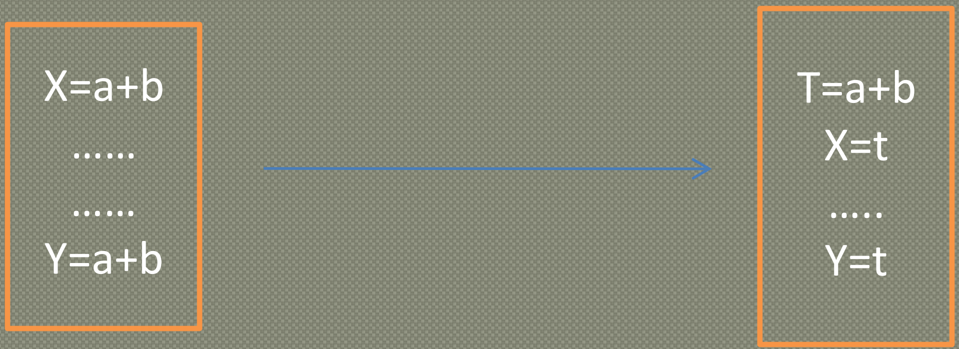
```
a := b * c         temp := b * c
…                       a := temp
…                       …
x := b * c + 5  x := temp + 5
```

Common Subexpression Elimination

- Local common subexpression elimination
  - Performed within basic blocks
  - Algorithm sketch:
    - Traverse BB from top to bottom
    - Maintain table of expressions evaluated so far
      - if any operand of the expression is redefined, remove it from the table

Common Subexpression Elimination

- Modify applicable instructions as you go
  - generate temporary variable, store the expression in it and use the variable next time the expression is encountered.



```
X=a+b
……
……
Y=a+b
```

```
T=a+b
X=t
…..
Y=t
```

Common Subexpression Elimination

```
c = a + b
d = m * n
e = b + d
f = a + b
g = - b
h = b + a
a = j + a
k = m * n
j = b + d
a = - b
if m * n go to L
```

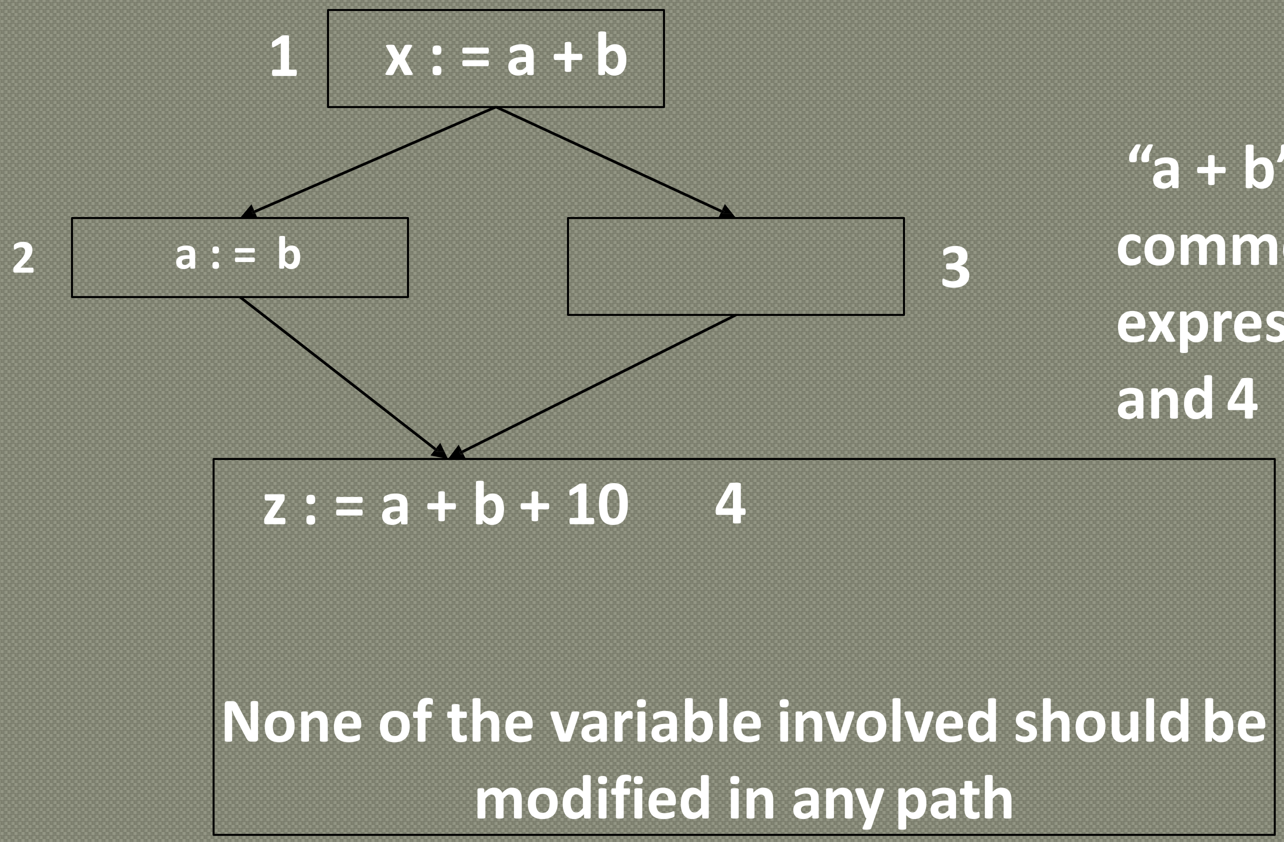
```
t1 = a + b
c = t1
t2 = m * n
d = t2
t3 = b + d
e = t3
f = t1
g = -b
h = t1 /* commutative */
a = j + a
k = t2
j = t3
a = -b
if t2 go to L
```

the table contains quintuples:
(pos, opd1, opr, opd2, tmp)

Common Subexpression Elimination

- Global common subexpression elimination
  - Performed on flow graph
  - Requires available expression information
    - In addition to finding what expressions are available at the endpoints of basic blocks, we need to know where each of those expressions was most recently evaluated (which block and which position within that block).

Common Sub-expression Evaluation



1 x : = a + b

2 a : = b

3

z : = a + b + 10 4

None of the variable involved should be modified in any path

"a + b" is not a common sub-expression in 1 and 4

Code Motion

- **Moving code from one part of the program to other without modifying the algorithm**
  - **Reduce size of the program**
  - **Reduce execution frequency of the code subjected to movement**

Code Motion

1. *Code Space reduction*: Similar to common sub-expression elimination but with the objective to reduce code size.

   Example: Code hoisting

```
                                      temp : = x ** 2
if (a< b) then                        if (a< b) then
   z := x ** 2                          z := temp
else                                  else
   y := x ** 2 + 10        →                   y := temp + 10
```

   "x ** 2" is computed once in both cases, but the code size in the second case reduces.

Code Motion

2 *Execution*

3 *frequency reduction*: reduce execution frequency of partially available expressions (expressions available atleast in one path)

Example:

```
    if (a<b) then                  if (a<b) then
         z = x * 2                   temp = x * 2
                                     z = temp
    else              →                       else
    y = 10                           y = 10
                                     temp = x * 2
    g = x * 2                                 g = temp;
```

Code Motion

- Move expression out of a loop if the evaluation does not change inside the loop.

Example:

```
while ( i < (max-2) ) …
```

Equivalent to:

```
t :=  max - 2
while ( i < t ) …
```

Code Motion

- **Safety of Code movement**

  Movement of an expression *e* from a basic block b_i to another block b_j, is safe if it does not introduce any new occurrence of *e* along any path.

Example: Unsafe code movement

```
                            temp = x * 2
  if (a<b) then             if (a<b) then
     z = x * 2                 z = temp
                  →
  else                      else
     y = 10                    y = 10
```

Strength Reduction

- Replacement of an operator with a less costly one.

Example:

```
                          temp = 5;
    for i=1 to 10 do      for i=1 to 10 do
     …                       …
     x = i * 5               x = temp
                  ⟶
     …                       …
                             temp = temp + 5
     end                  end
```

- Typical cases of strength reduction occurs in address calculation of array references.
- Applies to integer expressions involving induction variables (loop optimization)

Dead Code Elimination

- Dead Code are portion of the program which will not be executed in any path of the program.
  - Can be removed
- Examples:
  - No control flows into a basic block
  - A variable is dead at a point -> its value is not used anywhere in the program
  - An assignment is dead -> assignment assigns a value to a dead variable

Dead Code Elimination

- **<u>Examples</u>:**
- **i=j;**
- **…**
- **X=i+10**
- **The optimization can be performed by**
- **Eliminating the assignment statement**
- **i=j**
-

This assignment statement is called dead assignment .

Copy Propagation

- What does it mean?
  - Given an assignment x = y, replace later uses of x with uses of y, provided there are no intervening assignments to x or y.
- When is it performed?
  - At any level, but usually early in the optimization process.
- What is the result?
  - Smaller code

Copy Propagation

- `f  :=  g` are called copy statements or copies
- Use of `g` for `f`, whenever possible after copy statement

Example:

```
  x[i] = a;                        x[i] = a;
  sum = x[i] + a;                  sum = a + a;
```

- May not appear to be code improvement, but opens up scope for other optimizations.

Local Copy Propagation

- Local copy propagation
  - Performed within basic blocks
  - Algorithm sketch:
    - traverse BB from top to bottom
    - maintain table of copies encountered so far
    - modify applicable instructions as you go

Loop Optimization

- Decrease the number if instruction in the inner loop
- Even if we increase no of instructions in the outer loop
- Techniques:
  - Code motion
  - Induction variable elimination
  - Strength reduction

Peephole Optimization

- Pass over generated code to examine a few instructions, typically 2 to 4
  - Redundant instruction Elimination: Use algebraic identities
  - Flow of control optimization: removal of redundant jumps
  - Use of machine idioms

Redundant instruction elimination

- Redundant load/store: see if an obvious replacement is possible

```
MOV  R0, a
MOV a, R0
```

Can eliminate the second instruction without needing any global knowledge of *a*

- Unreachable code: identify code which will never be executed:

```
#define DEBUG 0
if( DEBUG) {
    print debugging info
}
```

→

```
if (0 != 1) goto L2
print debugging info

 L2:
```

Algebraic identities

- Worth recognizing single instructions with a constant operand:

```
A * 1 = A

A * 0 = 0

A / 1 = A

A * 2 = A + A
```

  More delicate with floating-point

- Strength reduction:

```
A ^ 2 = A * A
```

Objective

- Why would anyone write `X * 1`?
- Why bother to correct such obvious junk code?
- In fact one might write

```
#define MAX_TASKS   1
...
a = b * MAX_TASKS;
```

- Also, seemingly redundant code can be produced by other optimizations. This is an important effect.

The right shift problem

- Arithmetic Right shift:
  - shift right and use sign bit to fill most significant bits

    -5 111111...1111111011

    SAR 111111...1111111101

    which is -3, not -2
  - in most languages -5/2 = -2

Addition chains for multiplication

- If multiply is very slow (or on a machine with no multiply instruction like the original SPARC), decomposing a constant operand into sum of powers of two can be effective:

```
    X * 125    =     x * 128 - x*4 + x
```

  - two shifts, one subtract and one add, which may be faster than one multiply
  - Note similarity with efficient exponentiation method

Folding Jumps to Jumps

- A jump to an unconditional jump can copy the target address

```
          JNE lab1
          . . .
    lab1: JMP lab2
```

Can be replaced by:

```
      JNE lab2
```

As a result, lab1 may become dead (unreferenced)

Jump to Return

- A jump to a return can be replaced by a return

```
             JMP lab1
               ...
      lab1:  RET
```

  - Can be replaced by

```
         RET
```

    lab1 may become dead code

Usage of Machine idioms

- Use machine specific hardware instruction which may be less costly.

i := i + 1

ADD i, #1 INC i

⟶

Local Optimization

Optimization of Basic Blocks

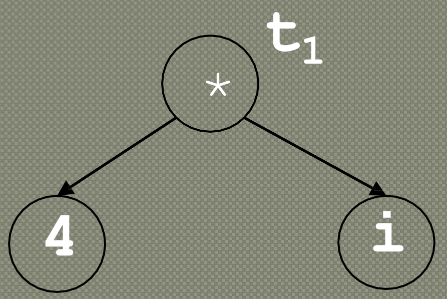
- Many structure preserving transformations can be implemented by construction of DAGs of basic blocks

# of Basic Block (BB)

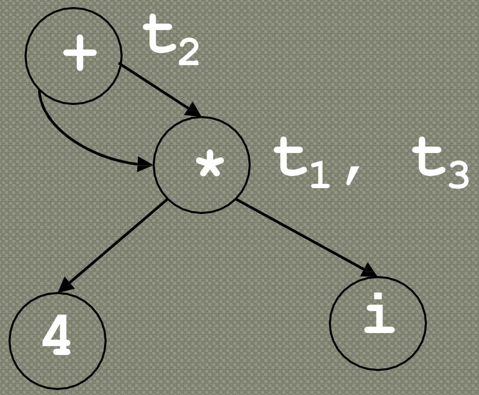
- Leaves are labeled with unique identifier (var name or const)
- Interior nodes are labeled by an operator symbol
- Nodes optionally have a list of labels (identifiers)
- Edges relates operands to the operator (interior nodes are
- operator)
- Interior node represents computed value
- – Identifier in the label are deemed to hold the value

Example: DAG for BB

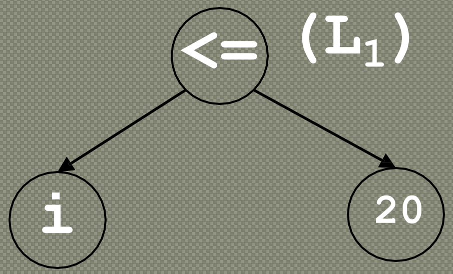
t_1 := 4 * i



t_1 := 4 * i
t_3 := 4 * i
t_2 := t_1 + t_3



if (i <= 20)goto L_1



Construction of DAGs for BB

- I/p: Basic block, *B*
- O/p: A DAG for *B* containing the following information:

1) A label for each node
2) For leaves the labels are ids or consts
3) For interior nodes the labels are operators
4) For each node a list of attached ids (possible empty list, no consts)

Construction of DAGs for BB

- Data structure and functions:
  - Node:
    1) Label: label of the node
    2) Left: pointer to the left child node
    3) Right: pointer to the right child node
    4) List: list of additional labels (empty for leaves)
  - Node (*id*): returns the most recent node created for *id*. Else return *undef*
  - Create(*id,l,r*): create a node with label *id* with *l* as left child and *r* as right child. *l* and *r* are optional params.

Construction of DAGs for BB

- **Method:**

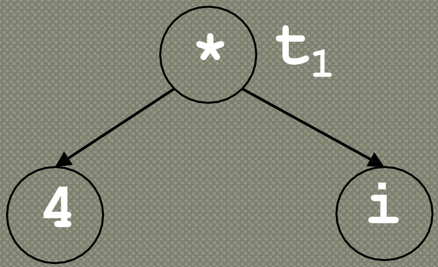
```
For each 3AC, $A$ in $B$
$A$ if of the following forms:
    1. $x := y$ op $z$
    2. $x :=$ op $y$
    3. $x := y$
1.  if (($n_y$ = node($y$)) == *undef*)
        $n_y$ = Create ($y$);
        if ($A$ == type 1)
          and (($n_z$ = node($z$)) == *undef*)
              $n_z$ = Create($z$);
```

Construction of DAGs for BB

```
2.  If (A == type 1)
        Find a node labelled 'op' with left and right as $n_y$ and $n_z$
        respectively [determination of common sub-expression]
        If (not found)  n = Create (op, $n_y$, $n_z$);
    If (A == type 2)
        Find a node labelled 'op' with a single child as $n_y$
        If (not found) n = Create (op, $n_y$);
    If (A == type 3)  n = Node (y);
3.  Remove x from Node(x).list
    Add x in n.list
    Node(x) = n;
```

Example: DAG construction from BB

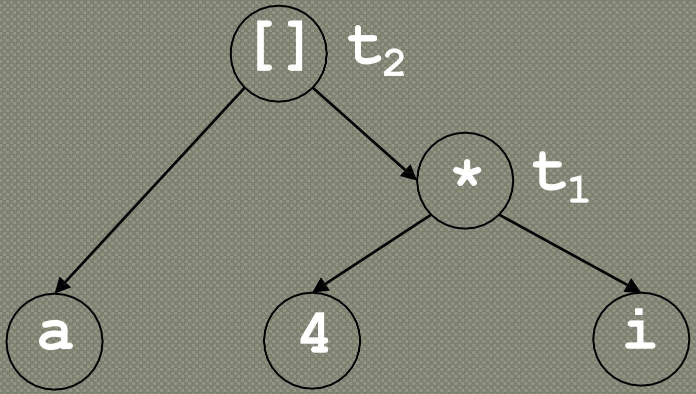
t_1 := 4 * i



Example: DAG construction from BB

t_1 := 4 * i

t_2 := a [t_1]

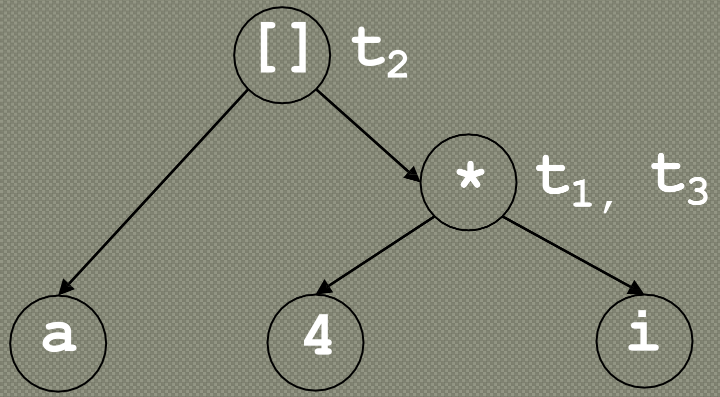


Example: DAG construction from BB

t_1 := 4 * i

t_2 := a [t_1]

t_3 := 4 * i



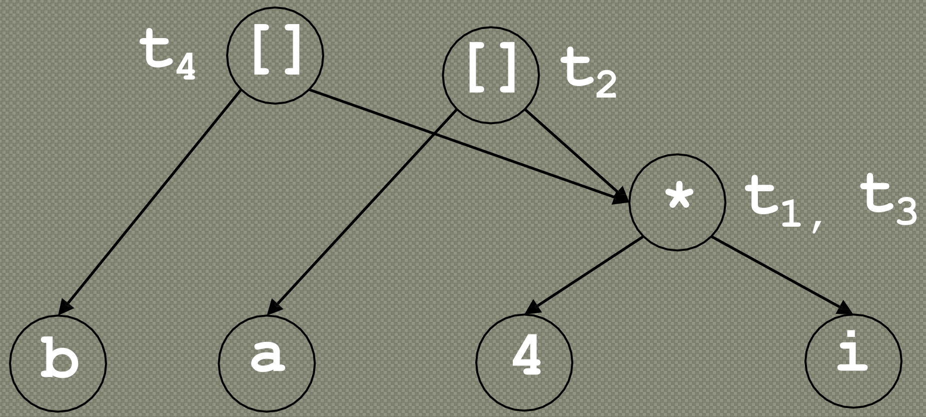
Example: DAG construction from BB

t_1 := 4 * i

t_2 := a [t_1]

t_3 := 4 * i

t_4 := b [t_3]



Example: DAG construction from BB

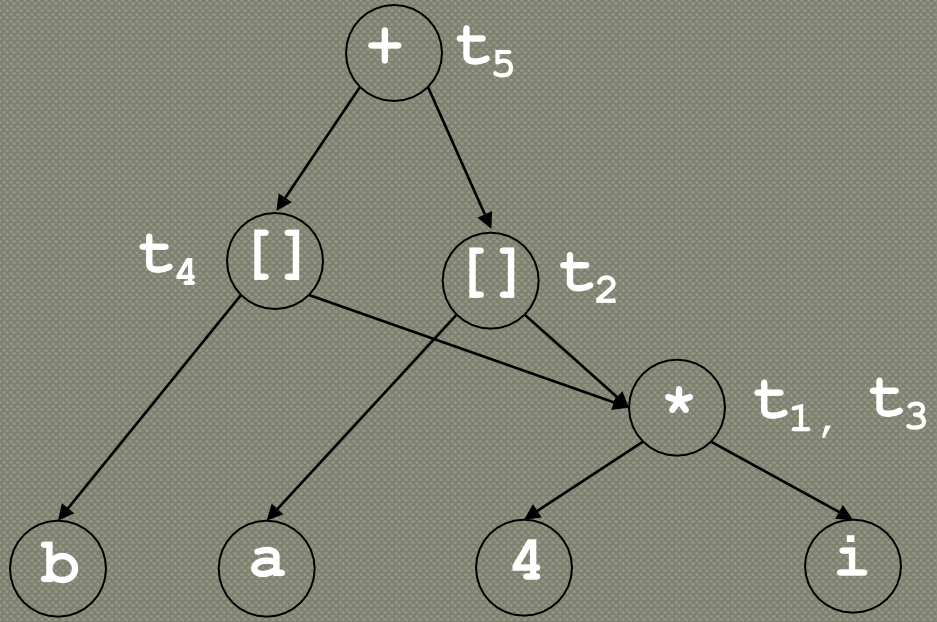
t_1 := 4 * i

t_2 := a [t_1]

t_3 := 4 * i

t_4 := b [t_3]

t_5 := t_2 + t_4



Example: DAG construction from BB

t_1 := 4 * i

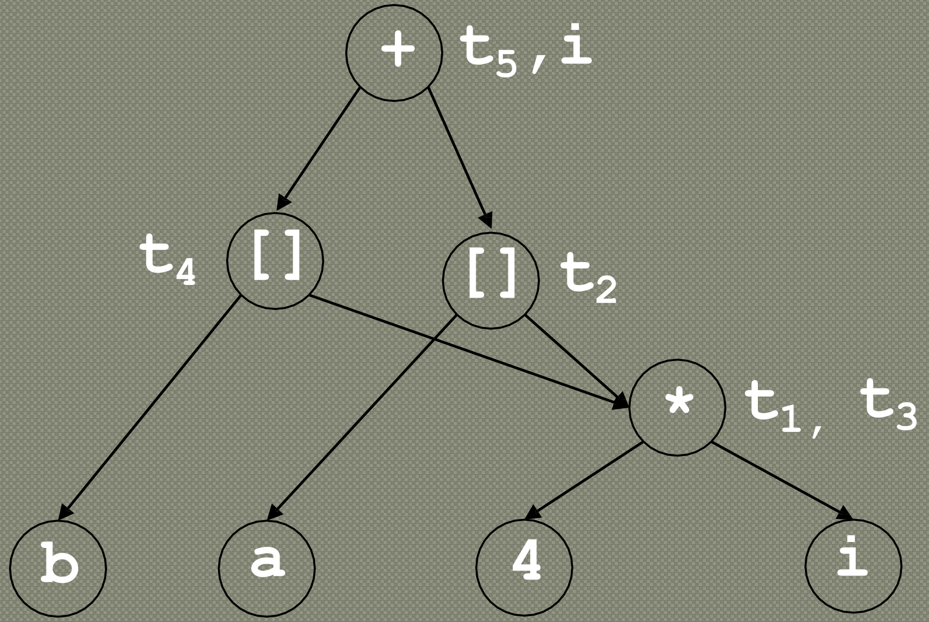
t_2 := a [t_1]

t_3 := 4 * i

t_4 := b [t_3]

t_5 := t_2 + t_4

i := t_5

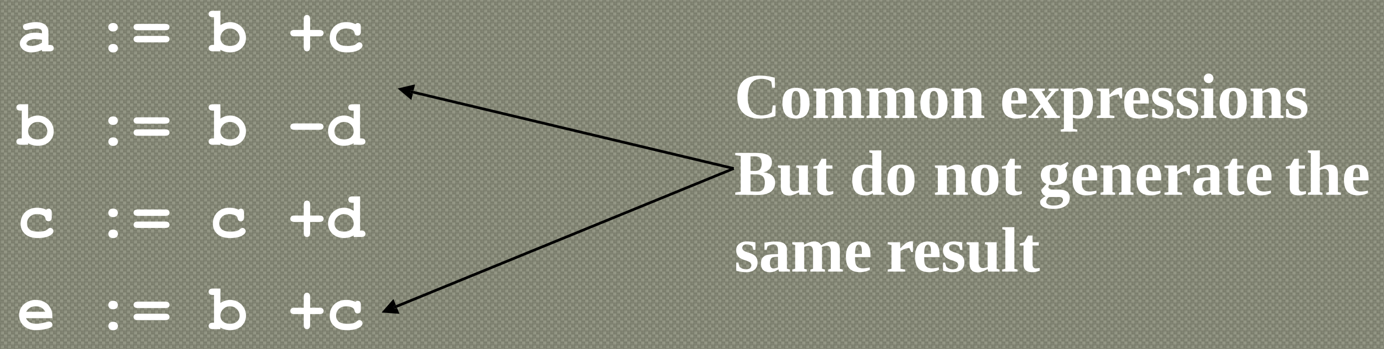


DAG of a Basic Block

- **Observations:**
  - **A leaf node for the initial value of an id**
  - **A node *n* for each statement *s***
  - **The children of node *n* are the last definition (prior to *s*) of the operands of *n***

Optimization of Basic Blocks

- Common sub-expression elimination: by construction of DAG
  - Note: for common sub-expression elimination, we are actually targeting for expressions that compute the same value.



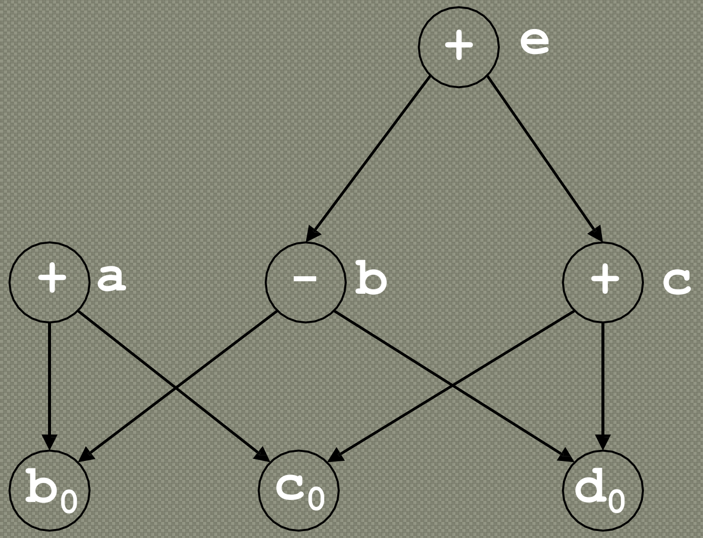
```
a := b +c
b := b -d
c := c +d
e := b +c
```

Common expressions
But do not generate the
same result

Optimization of Basic Blocks

- **DAG representation identifies expressions that yield the same result**

```
a := b + c
b := b - d
c := c + d
e := b + c
```

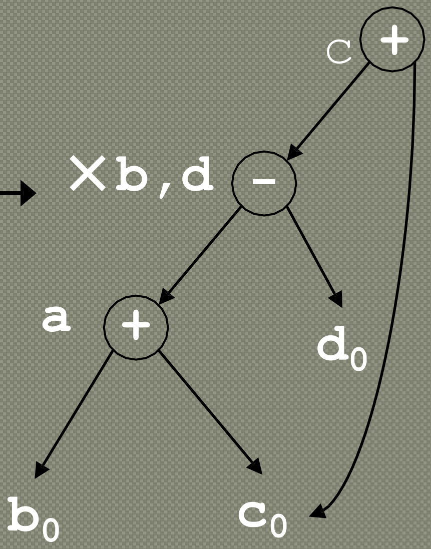


Optimization of Basic Blocks

- **Dead code elimination: Code generation from DAG eliminates dead code.**

```
a := b + c
b := a - d
d := a - d
c := d + c
```

b is not live



```
a := b + c
d := a - d
c := d + c
```

Loop Optimization

Loop Optimizations

- Most important set of optimizations
  - Programs are likely to spend more time in loops
- Presumption: Loop has been identified
- Optimizations:
  - Loop invariant code removal
  - Induction variable strength reduction
  - Induction variable reduction

Loops in Flow Graph

- **Dominators:**

  A node *d* of a flow graph *G* dominates a node *n*, if every path in *G* from the initial node to *n* goes through *d*.

  Represented as: *d dom n*

  Corollaries:

  Every node dominates itself.
  The initial node dominates all nodes in *G.*
  The entry node of a loop dominates all nodes in the loop.

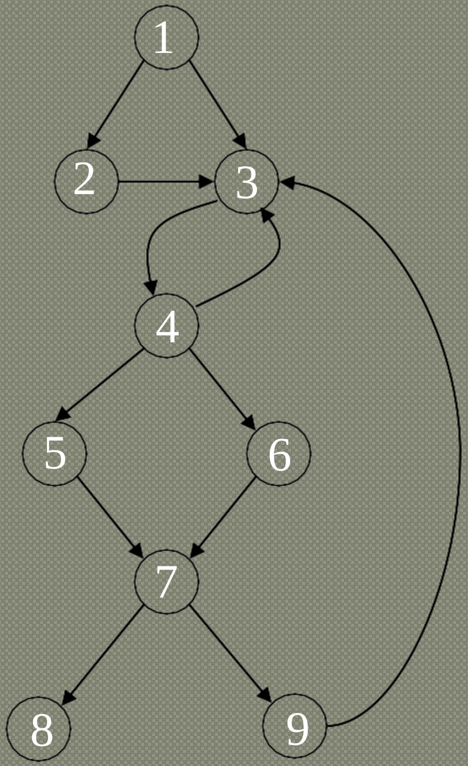
Loops in Flow Graph

- Each node *n* has a unique *immediate dominator m*, which is the last dominator of *n* on any path in *G* from the initial node to *n*.

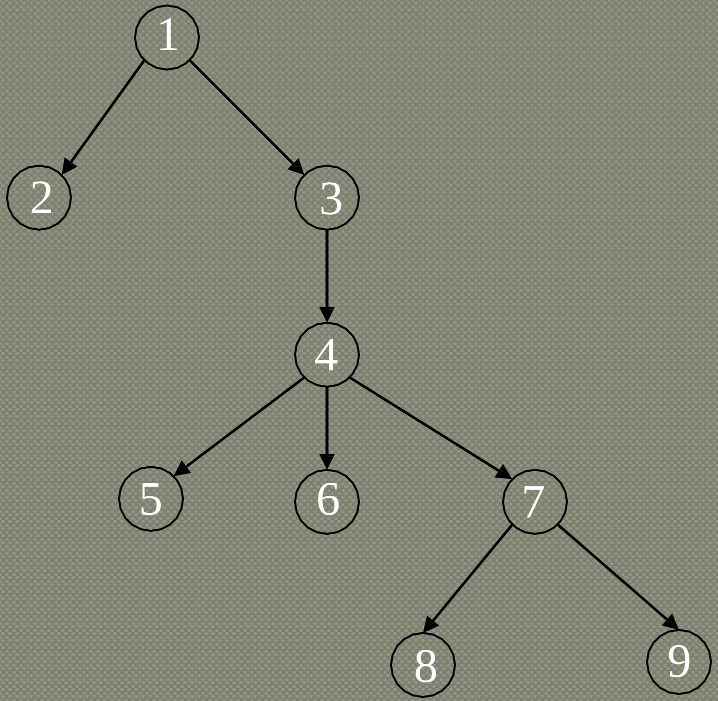
  (d ≠ n) && (d dom n) → d dom m
- Dominator tree (*T*):

  A representation of dominator information of flow graph *G*.
  - The root node of *T* is the initial node of *G*
  - A node *d* in *T* dominates all node in its sub-tree

Example: Loops in Flow Graph



Flow Graph



Dominator Tree

Loops in Flow Graph

- **Natural loops:**
  1. **A loop has a single entry point, called the "header". Header dominates all node in the loop**
  2. **There is at least one path back to the header from the loop nodes (i.e. there is at least one way to iterate the loop)**

- **Natural loops can be detected by *back edges*.**
  - **B*ack edges*: edges where the sink node (head) dominates the source node (tail) in *G***

Loop Optimization

- **Loop interchange: exchange inner loops with outer loops**

- **Loop splitting: attempts to simplify a loop or eliminate dependencies by breaking it into multiple loops which have the same bodies but iterate over different contiguous portions of the index range.**

  – **A useful special case is *loop peeling* - simplify a loop with a problematic first iteration by performing that iteration separately before entering the loop.**

Loop Optimization

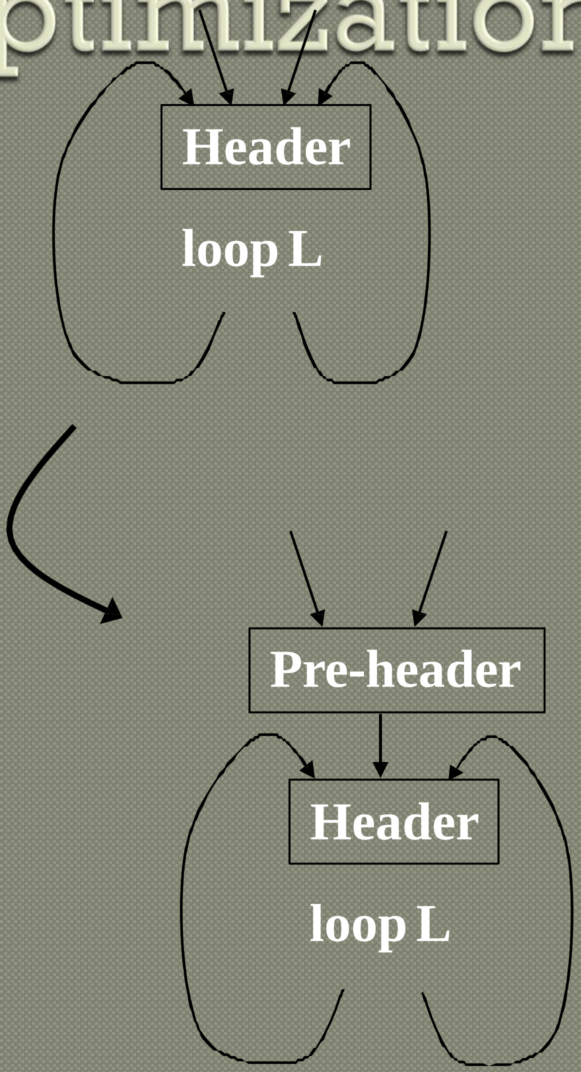
- Loop fusion: two adjacent loops would iterate the same number of times, their bodies can be combined as long as they make no reference to each other's data

- Loop fission: break a loop into multiple loops over the same index range but each taking only a part of the loop's body.

- Loop unrolling: duplicates the body of the loop multiple times

Loop Optimization

- **Pre-Header:**
  - Targeted to hold statements that are moved out of the loop
  - A basic block which has only the header as successor
  - Control flow that used to enter the loop from outside the loop, through the header, enters the loop from the pre-header



Loop Invariant Code Removal

- Move out to pre-header the statements whose source operands do not change within the loop.
  - Be careful with the memory operations
  - Be careful with statements which are executed in some of the iterations

Loop Invariant Code Removal

- Rules: A statement S: *x*:=*y* op *z* is loop invariant:
  - *y* and *z* not modified in loop body
  - S is the only statement to modify *x*
  - For all uses of *x*, *x* is in the available def set.
  - For all exit edge from the loop, S is in the available def set of the edges.
  - If S is a load or store (mem ops), then there is no writes to address(*x*) in the loop.

Loop Invariant Code Removal

- Loop invariant code removal can be done without available definition information.

Rules that need change:

- For all use of *x* is in the available definition set
- For all exit edges, if *x* is live on the exit edges, is in the available definition set on the exit edges

- Approx of First rule:
  - *d* dominates all uses of *x*
- Approx of Second rule
  - *d* dominates all exit basic blocks where *x* is live

Loop Induction Variable

- Induction variables are variables such that every time they change value, they are incremented or decremented.
  - Basic induction variable: induction variable whose only assignments within a loop are of the form:

    `i = i +/- C`, where C is a constant.

  - Primary induction variable: basic induction variable that controls the loop execution

    `(for i=0; i<100; i++)`

    `i` (register holding i) is the primary induction variable.

  - Derived induction variable: variable that is a linear function of a basic induction variable.

Loop Induction Variable

- Basic: r4, r7, r1
- Primary: r1
- Derived: r2

```
       r1 = 0
       r7 = &A

Loop:  r2 = r1 * 4
       r4 = r7 + 3
       r7 = r7 + 1
       r10 = *r2
       r3 = *r4
       r9 = r1 * r3
       r10 = r9 >> 4
       *r2 = r10
       r1 = r1 + 4
       If(r1 < 100) goto Loop
```

Induction Variable Strength Reduction

- Create basic induction variables from derived induction variables.
- Rules: (S: *x* := *y* op *z*)
  - *op* is *, <<, +, or –
  - *y* is a induction variable
  - *z* is invariant
  - No other statement modifies *x*
  - *x* is not *y* or *z*
  - *x* is a register

Induction Variable Strength Reduction

- **Transformation:**

Insert the following into the bottom of pre-header:

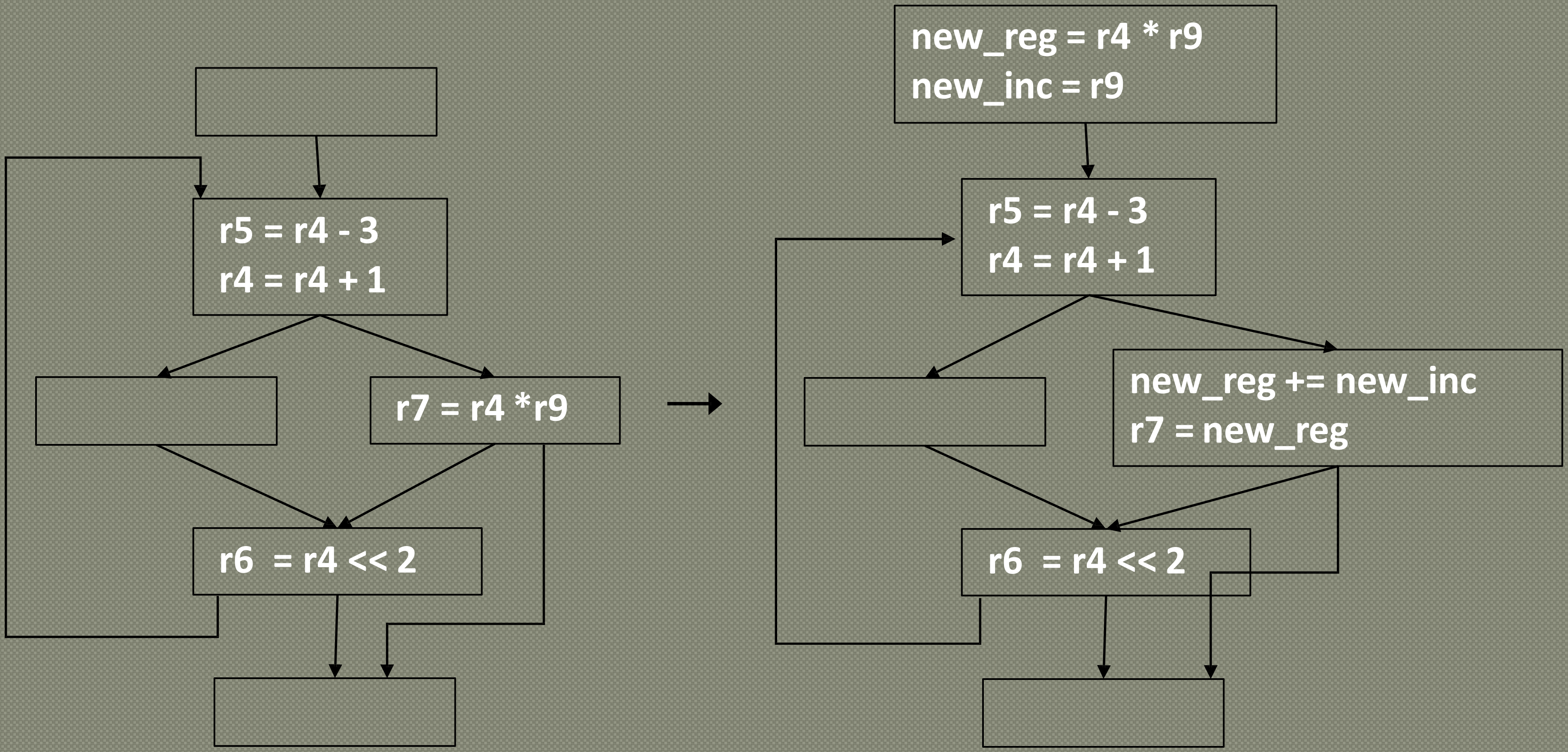
```
    new_reg = expression of target statement S
    if (opcode(S)) is not add/sub, insert to the bottom of the preheader
            new_inc = inc(y,op,z)
    else
            new_inc = inc(x)
    Insert the following at each update of y
            new_reg = new_reg + new_inc
```

Change S: ***x = new_reg***

Function: inc()

Calculate the amount of inc for 1st param.

Example: Induction Variable Strength Reduction

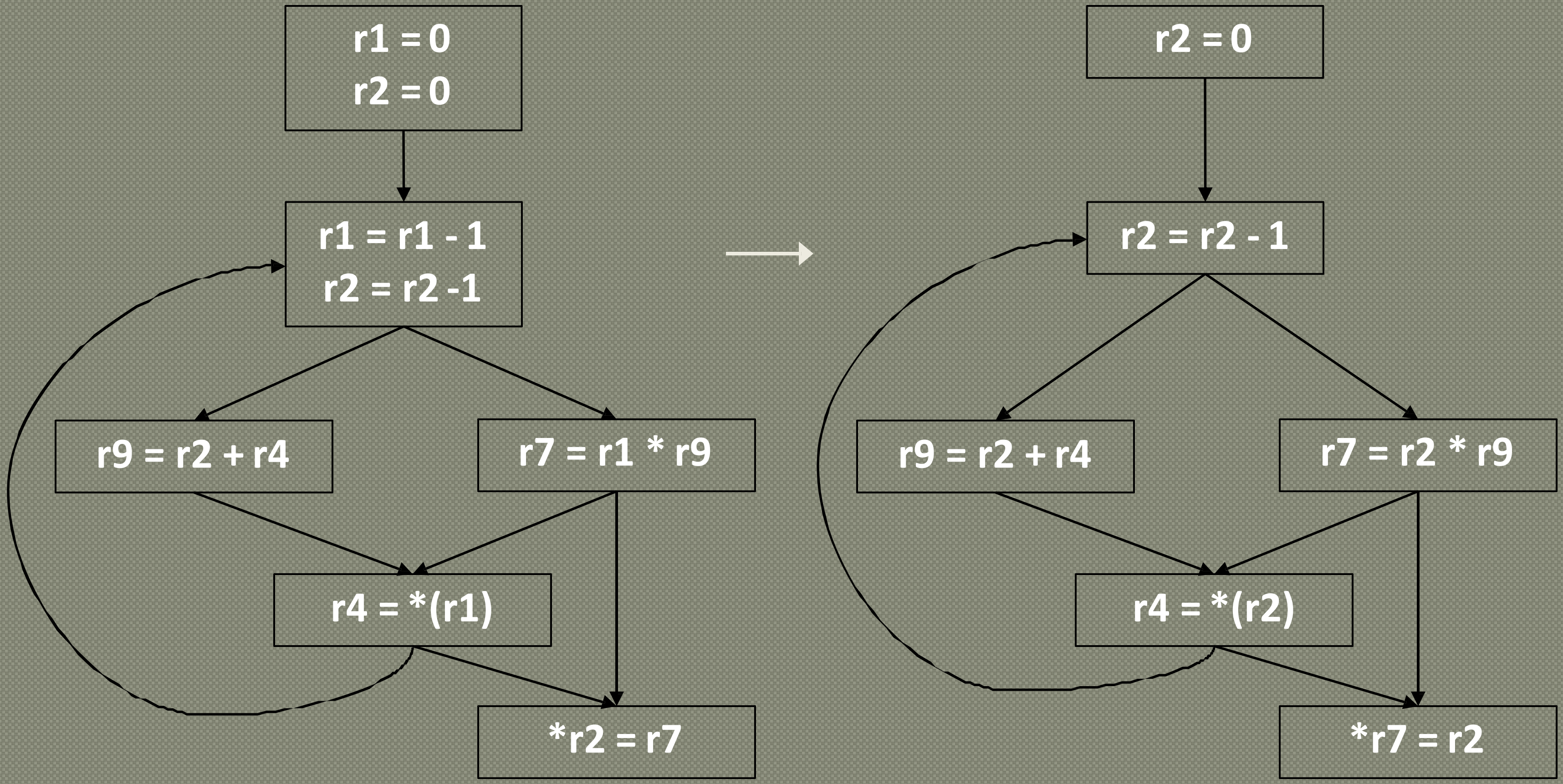


Induction Variable Elimination

- Remove unnecessary basic induction variables from the loop by substituting uses with another basic induction variable.

- Rules:
  - Find two basic induction variables, *x* and *y*
  - *x* and *y* in the same family
    - Incremented at the same place
  - Increments are equal
  - Initial values are equal
  - *x* is not live at exit of loop
  - For each BB where *x* is defined, there is no use of x between the first and the last definition of *y*

Example: Induction Variable Elimination



r1 = 0
r2 = 0

r1 = r1 - 1
r2 = r2 -1

r9 = r2 + r4

r7 = r1 * r9

r4 = *(r1)

*r2 = r7

r2 = 0

r2 = r2 - 1

r9 = r2 + r4

r7 = r2 * r9

r4 = *(r2)

*r7 = r2

Induction Variable Elimination

- Variants:

Complexity of elimination

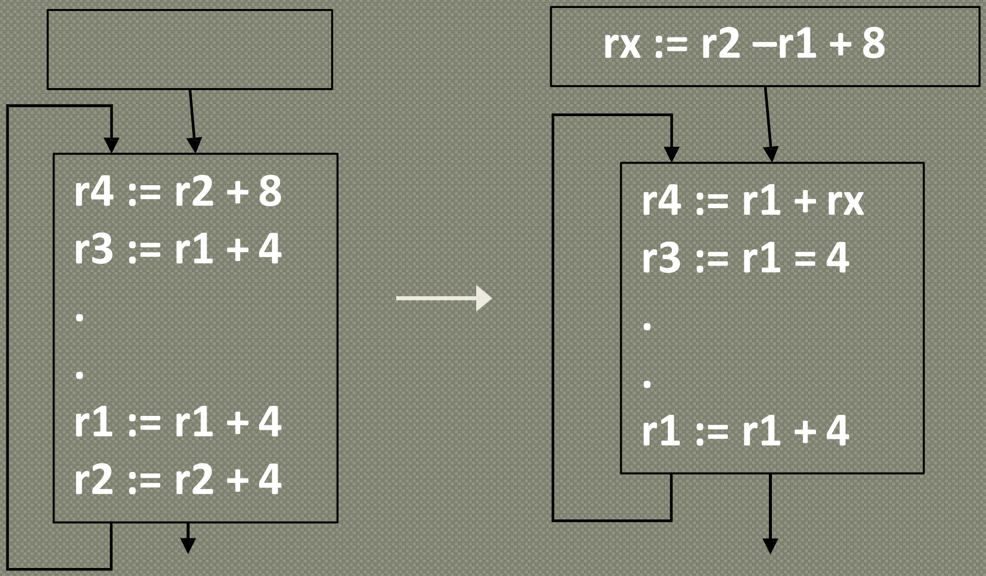
1. Trivial: induction variable that are never used except to increment themselves and not live at the exit of loop
2. Same increment, same initial value (discussed)
3. Same increment, initial values are a known constant offset from one another
4. Same increment, nothing known about the relation of initial value
5. Different increments, nothing known about the relation of initial value

- 1,2 are basically free
- 3-5 require complex pre-header operations

Example: Induction Variable Elimination

- **Case 4: Same increment, unknown initial value**

  For the induction variable that we are eliminating, look at each non-incremental use, generate the same sequence of values as before. If that can be done without adding any extra statements in the loop body, then the transformation can be done.



```
r4 := r2 + 8
r3 := r1 + 4
.
.
r1 := r1 + 4
r2 := r2 + 4
```

```
rx := r2 –r1 + 8
```

```
r4 := r1 + rx
r3 := r1 = 4
.
.
r1 := r1 + 4
```

Loop Unrolling

- Replicate the body of a loop (N-1) times, resulting in total N copies.
  - Enable overlap of operations from different iterations
  - Increase potential of instruction level parallelism (ILP)
- Variants:
  - Unroll multiple of known trip counts
  - Unroll with remainder loop
  - While loop unroll

Constant Folding

- Evaluate constant expressions at compile time
- Only possible when side-effect freeness guaranteed

c:= 1 + 3 ⇨ c:= 4

true not ⇨ false

Caveat: Floats — implementation could be different between machines!

Global Data Flow Analysis

- Collect information about the whole program.
- Distribute the information to each block in the flow graph.

- *Data flow information*: Information collected by data flow analysis.
- *Data flow equations*: A set of equations solved by data flow analysis to gather data flow information.

Data flow analysis

- **IMPORTANT!**
  - Data flow analysis should never tell us that a transformation is safe when in fact it is not.
  - When doing data flow analysis we must be
    - Conservative
      - Do not consider information that may not preserve the behavior of the program
    - Aggressive
      - Try to collect information that is as exact as possible, so we can get the greatest benefit from our optimizations.

Global Iterative Data Flow Analysis

- Global:
  - Performed on the flow graph
  - Goal = to collect information at the beginning and end of each basic block
- Iterative:
  - Construct data flow equations that describe how information flows through each basic block and solve them by iteratively converging on a solution.

Global Iterative Data Flow Analysis

- Components of data flow equations
  - Sets containing collected information
    - in set: information coming into the BB from outside (following flow of data)
    - gen set: information generated/collected within the BB
    - kill set: information that, due to action within the BB, will affect what has been collected outside the BB
    - out set: information leaving the BB
  - Functions (operations on these sets)
    - Transfer functions describe how information changes as it flows through a basic block
    - Meet functions describe how information from multiple paths is combined.

Global Iterative Data Flow Analysis

- Algorithm sketch
  - Typically, a bit vector is used to store the information.
    - For example, in reaching definitions, each bit position corresponds to one definition.
  - We use an iterative fixed-point algorithm.
  - Depending on the nature of the problem we are solving, we may need to traverse each basic block in a forward (top-down) or backward direction.
    - The order in which we "visit" each BB is not important in terms of algorithm correctness, but is important in terms of efficiency.
  - In & Out sets should be initialized in a conservative and aggressive way.

Typical problems

- Reaching definitions
  - For each use of a variable, find all definitions that reach it.
- Upward exposed uses
  - For each definition of a variable, find all uses that it reaches.
- Live variables
  - For a point p and a variable v, determine whether v is live at p.
- Available expressions
  - Find all expressions whose value is available at some point p.

Global Data Flow Analysis

- **A typical data flow equation:**

S: statement

$$out[S] = gen[S] \bigcup (in[S] - kill[S])$$

***in*[S]: Information goes into S**

***kill*[S]: Information get killed by S**

***gen*[S]: New information generated by S**

***out*[S]: Information goes out from S**

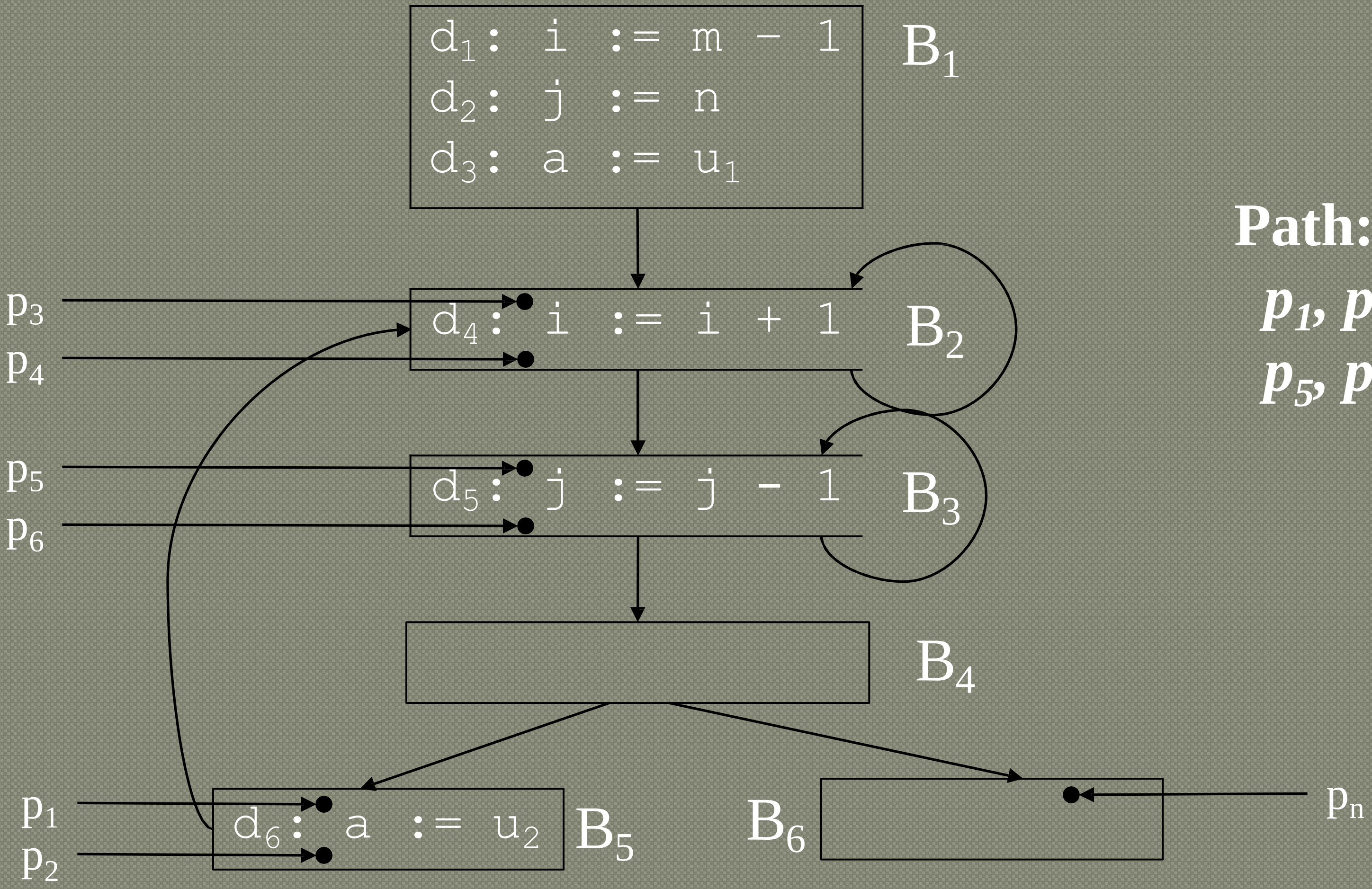
Global Data Flow Analysis

- The notion of *gen* and *kill* depends on the desired information.
- In some cases, *in* may be defined in terms of *out* - equation is solved as analysis traverses in the backward direction.
- Data flow analysis follows control flow graph.
  - Equations are set at the level of basic blocks, or even for a statement

Points and Paths

- *Point* within a basic block:
  - **A location between two consecutive statements.**
  - **A location before the first statement of the basic block.**
  - **A location after the last statement of the basic block.**
- *Path*: A path from a point p_1 to p_n is a sequence of points p_1,
- p_2, … p_n such that for each $i : 1 \leq i \leq n$,
  - **p_i is a point immediately preceding a statement and p_{i+1} is the point immediately following that statement in the same block,** <u>**or**</u>
  - **p_i is the last point of some block and p_{i+1} is first point in the successor block.**

Example: Paths and Points



Path:

$p_1, p_2, p_3, p_4,$
$p_5, p_6 \dots p_n$

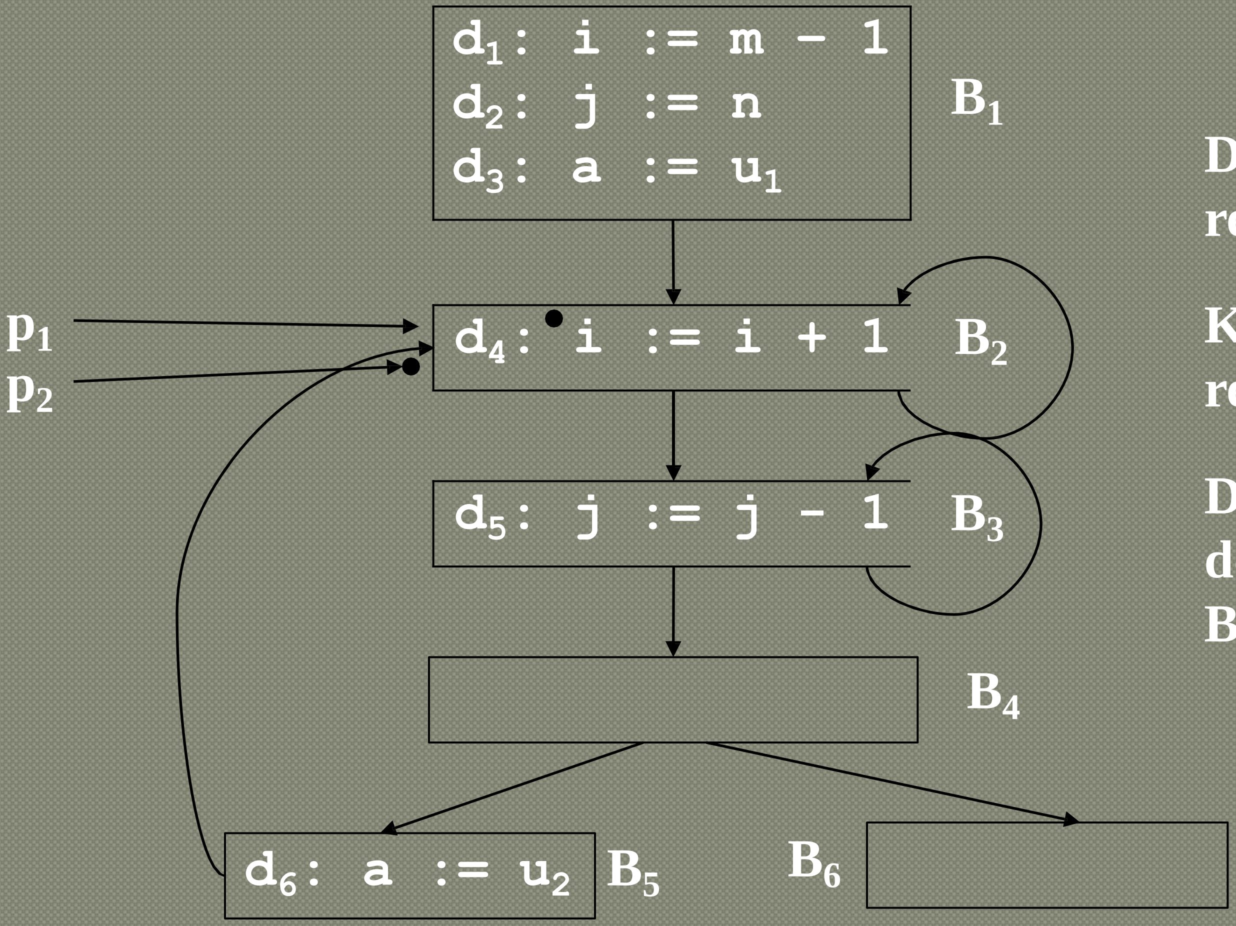
Reaching Definition

- Definition of a variable *x* is a statement that assigns or *may* assign a value to *x.*
  - *Unambiguous Definition:* The statements that certainly assigns a value to *x*
    - Assignments to *x*
    - Read a value from I/O device to *x*
  - *Ambiguous Definition:* Statements that may assign a value to *x*
    - Call to a procedure with *x* as parameter (call by ref)
    - Call to a procedure which can access *x* (*x* being in the scope of the procedure)
    - *x* is an alias for some other variable (*aliasing*)
    - Assignment through a pointer that could refer *x*

Reaching Definition

- A definition *d reaches* a point *p*
  - if there is a path from the point immediately following *d* to *p* <u>and</u>
  - *d* is not *killed* along the path (*i.e.* there is not redefinition of the same variable in the path)
- A definition of a variable is *killed* between two points when there is another definition of that variable along the path.

Example: Reaching Definition



d_1: i := m - 1

d_2: j := n

d_3: a := u_1

d_4: i := i + 1

d_5: j := j - 1

d_6: a := u_2

Definition of *i* (*d_1*) reaches *p_1*

Killed as *d_4*, does not reach *p_2*.

Definition of *i* (*d_1*) does not reach B_3, B_4, B_5 and B_6.

Reaching Definition

- Non-Conservative view: A definition *might* reach a point even if it might not.
  - Only unambiguous definition kills a earlier definition
  - All edges of flow graph are assumed to be traversed.

```
if (a == b) then a = 2
 else if (a == b) then a = 4
```

The definition “a=4” is not reachable.

Whether each path in a flow graph is taken is an undecidable problem

Data Flow analysis of a Structured Program

- Structured programs have well defined loop constructs – the resultant flow graph is always reducible.
  - Without loss of generality we only consider while-do and if-then-else control constructs

```
S → id := E | S ; S
      | if E then S else S | do S while E
E → id + id  | id
```

The non-terminals represent *regions*.

Data Flow analysis of a Structured Program

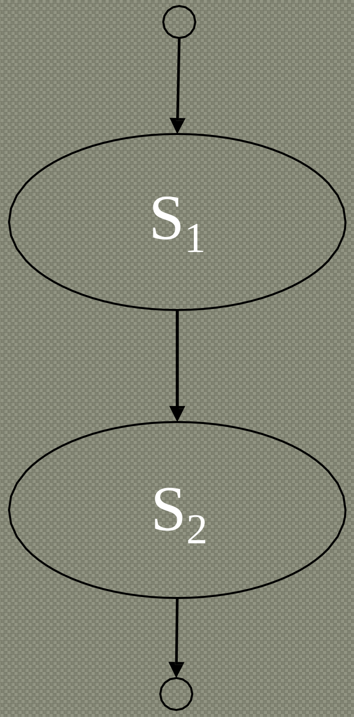
- **Region: A graph *G'= (N',E')* which is portion of the control flow graph *G*.**
  - **The set of nodes *N'* is in *G'* such that**
    - ***N'* includes a header *h***
    - ***h* dominates all node in *N'***
  - **The set of edges *E'* is in *G'* such that**
    - **All edges *a* → *b* such that *a,b* are in *N'***

Data Flow analysis of a Structured Program

- Region consisting of a statement S:
  - Control can flow to only one block outside the region
- Loop is a special case of a region that is strongly connected and includes all its back edges.
- Dummy blocks with no statements are used as technical convenience (indicated as open circles)

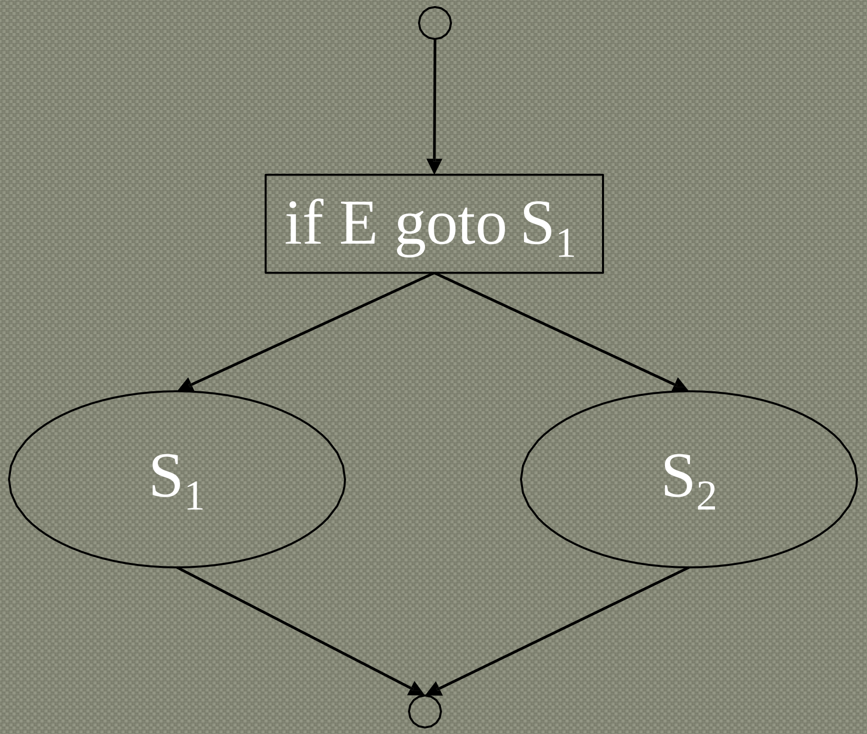
Structured Program: Composition of Regions

S → S_1 ; S_2



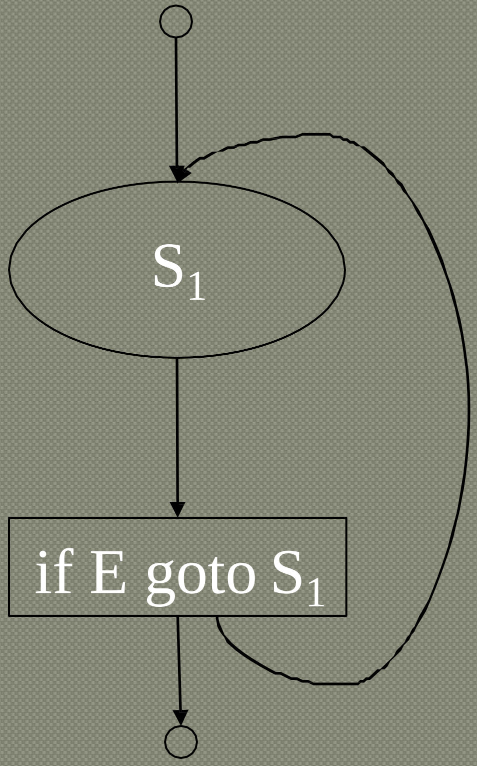
Structured Program: Composition of Regions

$S \rightarrow$ **if** E **then** S_1 **else** S_2



Structured Program: Composition of Regions

S → **do** S_1 **while** E



Data Flow Equations

- **Each region (or NT) has four attributes:**
  - ***gen*[S]: Set of definitions generated by the block S.**

    If a definition *d* is in *gen*[S], then *d* reaches the end of block S.
  - ***kill*[S]: Set of definitions killed by block S.**

    If *d* is in *kill*[S], *d* never reaches the end of block S. Every path from the beginning of S to the end S must have a definition for a (where *a* is defined by *d*).

Data Flow Equations

  - *in*[S]: The set of definition those are live at the entry point of block S.
  - *out*[S]: The set of definition those are live at the exit point of block S.
- The data flow equations are inductive or syntax directed.
  - *gen* and *kill* are synthesized attributes.
  - *in* is an inherited attribute.

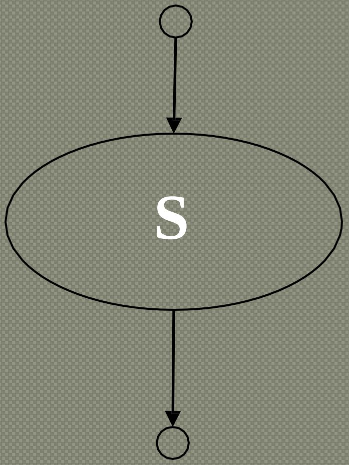
Data Flow Equations

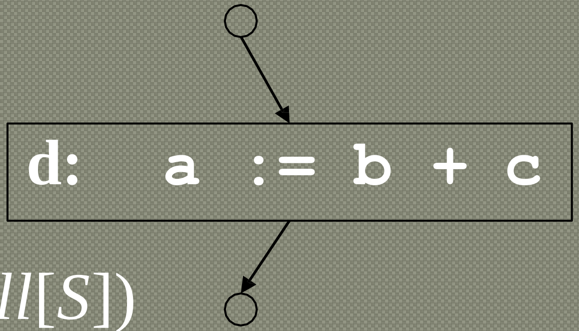
- ***gen*[S] concerns with a single basic block. It is the set of definitions in S that reaches the end of S.**
- **In contrast *out*[S] is the set of definitions (possibly defined in some other block) live at the end of S considering all paths through S.**

Data Flow Equations Single statement

$$gen[S] = \{d\}$$

$$kill[S] = D_a - \{d\}$$





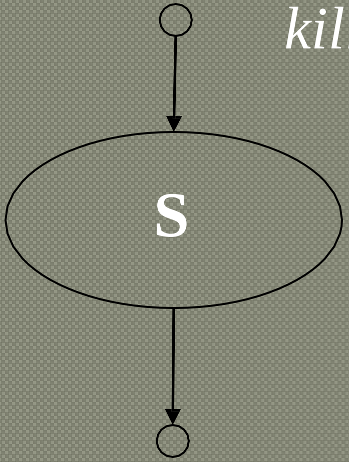
$$out[S] = gen[S] \bigcup (in[S] - kill[S])$$

D_a: The set of definitions in the program for variable a

Data Flow Equations Composition

$$gen[S] = gen[S_2] \bigcup (gen[S_1] - kill[S_2])$$

$$kill[S] = kill[S_2] \bigcup (kill[S_1] - gen[S_2])$$

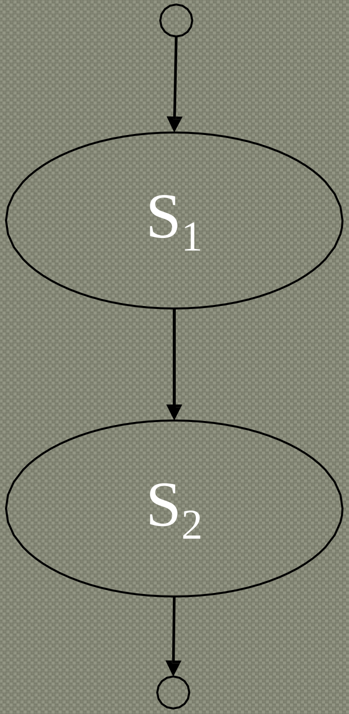


$$\longrightarrow$$

$$in[S_1] = in[S\ ]$$

$$in[S_2] = out[S_1]$$

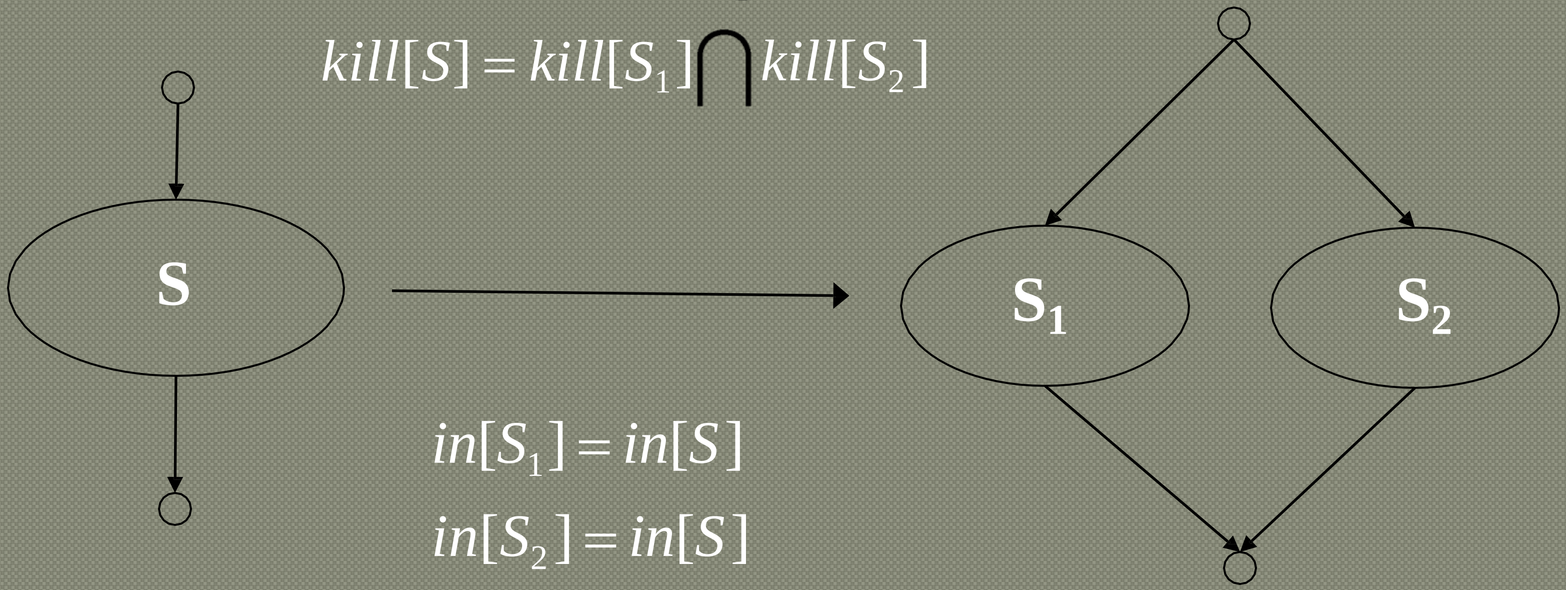
$$out[S] = out[S_2]$$



Data Flow Equations if-then-else

$$gen[S] = gen[S_1] \bigcup gen[S_2]$$

$$kill[S] = kill[S_1] \bigcap kill[S_2]$$



$$in[S_1] = in[S]$$

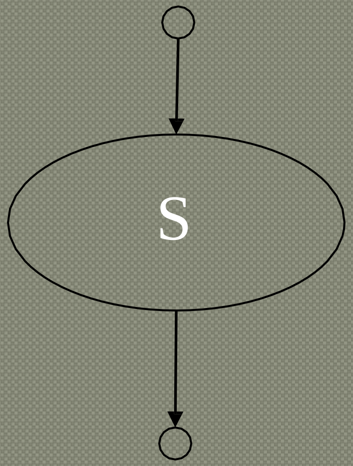
$$in[S_2] = in[S]$$

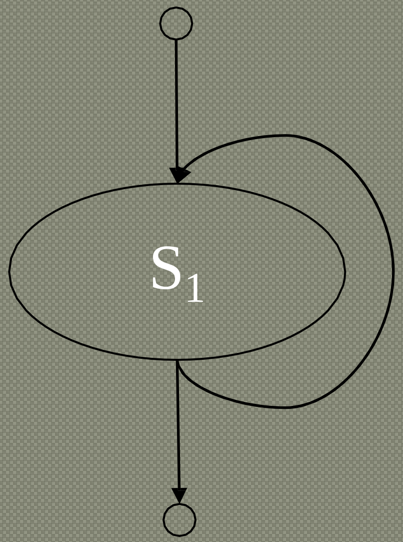
$$out[S] = out[S_1] \bigcup out[S_2]$$

Data Flow Equations
Loop

$$gen[S] = gen[S_1]$$

$$kill[S] = kill[S_1]$$





$$in[S_1] = in[S] \bigcup gen[S_1]$$

$$out[S] = out[S_1]$$

Data Flow Analysis

- The attributes are computed for each region. The equations can be solved in two phases:
  - *gen* and *kill* can be computed in a single pass of a basic block.
  - *in* and *out* are computed iteratively.
    - Initial condition for *in* for the whole program is
    - In can be computed top- down $\varnothing$
    - Finally *out* is computed

Dealing with loop

- Due to back edge, *in*[S] cannot be used as *in* [S_1]
- *in*[S_1] and *out*[S_1] are interdependent.
- The equation is solved iteratively.
- The general equations for *in* and *out*:

$$in[S] = \bigcup (out[Y]\text{: Y is a predecessor of S})$$

$$out[S] = gen[S] \bigcup (in[S] - kill[S])$$

Reaching definitions

- **What is safe?**
  - **To assume that a definition reaches a point even if it turns out not to.**
  - **The computed set of definitions reaching a point *p* will be a superset of the actual set of definitions reaching *p***
  - **Goal : make the set of reaching definitions as small as possible (i.e. as close to the actual set as possible)**

Reaching definitions

- How are the gen and kill sets defined?
  - gen[B] = {definitions that appear in B and reach the end of B}
  - kill[B] = {all definitions that never reach the end of B}
- What is the direction of the analysis?
  - forward
  - out[B] = gen[B] $\cup$ (in[B] - kill[B])

definitions

- **What is the confluence operator?**
  - **union**
  - **in[B] = ∪ out[P], over the predecessors P of B**
- **How do we initialize?**
  - **start small**
    - **Why? Because we want the resulting set to be as small as possible**
  - **for each block B initialize out[B] = gen[B]**

Computation of *gen* and *kill* sets

```
for each basic block BB do
    gen(BB) = $\emptyset$;  kill(BB) = $\emptyset$;
    for each statement (d: $x$ := $y$ op $z$) in sequential order in BB, do
        kill(BB) = kill(BB) U G[$x$];
        G[$x$] = d;
    endfor
    gen(BB) = U G[$x$]: for all id $x$
endfor
```

Computation of *in* and *out* sets

```
for all basic blocks BB     in(BB) ∅=
for all basic blocks BB   out(BB) = gen(BB)
change = true
while (change) do
    change = false
    for each basic block BB, do
        old_out = out(BB)
        in(BB) = U(out(Y)) for all predecessors Y of BB
        out(BB) = gen(BB) + (in(BB) – kill(BB))
        if (old_out != out(BB)) then change = true
    endfor
endfor
```

Live Variable (Liveness) Analysis

- **Liveness: For each point p in a program and each variable *y*, determine whether *y* can be used before being redefined, starting at p.**

- **Attributes**
  - ***use* = set of variable used in the BB prior to its definition**
  - ***def* = set of variables defined in BB prior to any use of the variable**
  - ***in* = set of variables that are live at the entry point of a BB**
  - ***out* = set of variables that are live at the exit point of a BB**

Live Variable (Liveness) Analysis

- **Data flow equations:**

$$in[B] = use[B] \bigcup (out[B] - def[B])$$

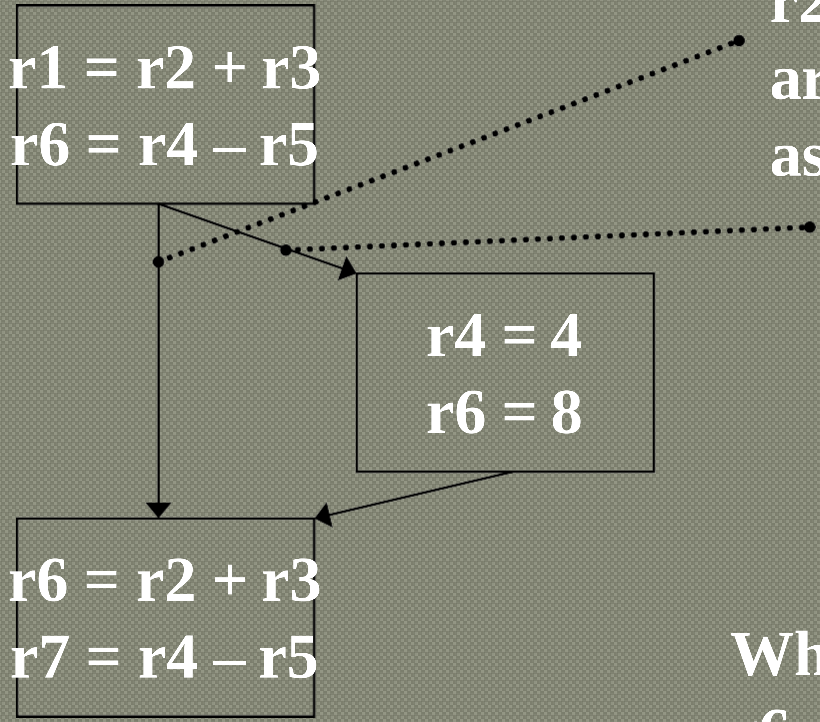
$$out[B] = \bigcup_{S=succ(B)} in[S]$$

  - **1st Equation: a var is live, *coming in* the block, if either**
    - **it is used before redefinition in B**

    or

    - **it is live coming out of B and is not redefined in B**
  - **2nd Equation: a var is live *coming out* of B, iff it is live coming in to one of its successors.**

Example: Liveness



r1 = r2 + r3
r6 = r4 – r5

r4 = 4
r6 = 8

r6 = r2 + r3
r7 = r4 – r5

r2, r3, r4, r5 are all live as they are consumed later, r6 is dead as it is redefined later

r4 is dead, as it is redefined. So is r6. r2, r3, r5 are live

What does this mean?
r6 = r4 – r5 is useless,
it produces a dead
value !!
Get rid of it!

Computation of *use* and *def* sets

```
for each basic block BB do
    def(BB) = ∅ ;    use(BB) = ∅ ;
    for each statement (x := y op z) in sequential order, do
        for each operand y, do
            if (y not in def(BB))
                use(BB) = use(BB) U {y};
        endfor
        def(BB) = def(BB) U {x};
endfor
```

def is the union of all the LHS's

use is all the ids used before defined

Computation of *in* and *out* sets

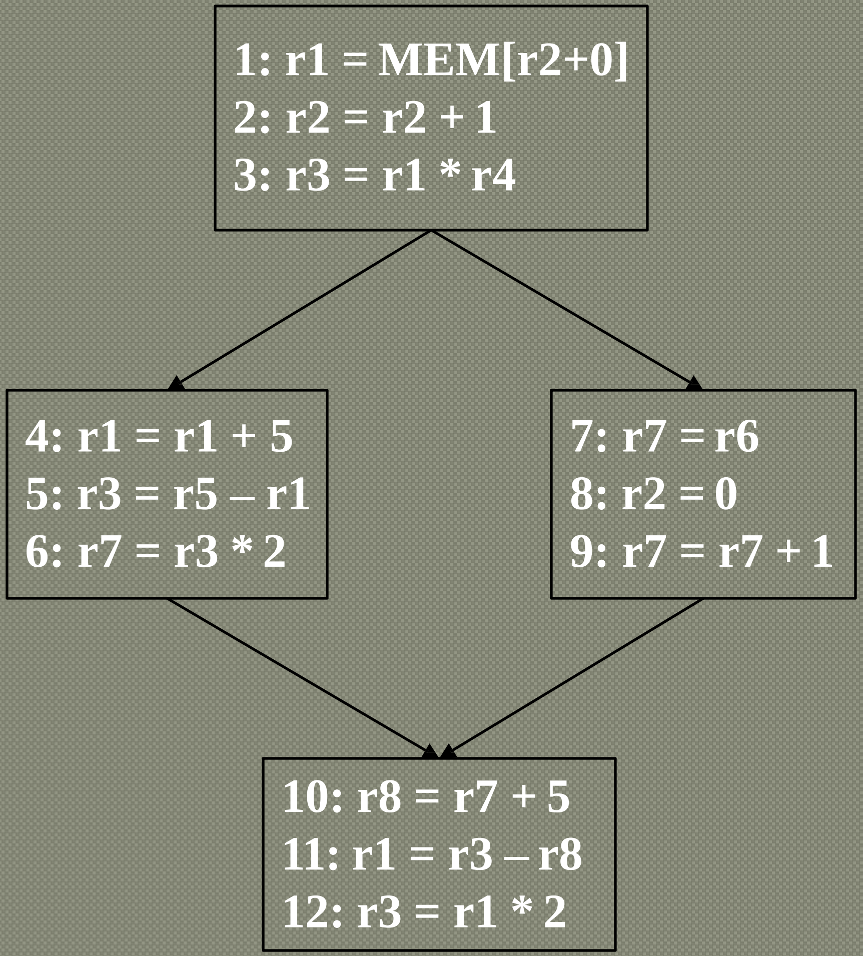
```
for all basic blocks BB
    in(BB)∅=    ;

change = true;
while (change) do
   change = false
   for each basic block BB do
      old_in = in(BB);
      out(BB) = U{in(Y): for all successors Y of BB}
      in(X) = use(X) U (out(X) – def(X))
      if (old_in != in(X)) then change = true
   endfor
endfor
```

DU/UD Chains

- **Convenient way to access/use reaching definition information.**
- **Def-Use chains (DU chains)**
  - **Given a def, what are all the possible consumers of the definition produced**
- **Use-Def chains (UD chains)**
  - **Given a use, what are all the possible producers of the definition consumed**

Example: DU/UD Chains



1: r1 = MEM[r2+0]
2: r2 = r2 + 1
3: r3 = r1 * r4

4: r1 = r1 + 5
5: r3 = r5 – r1
6: r7 = r3 * 2

7: r7 = r6
8: r2 = 0
9: r7 = r7 + 1

10: r8 = r7 + 5
11: r1 = r3 – r8
12: r3 = r1 * 2

DU Chain of r1:
(1) -> 3,4
(4) ->5

DU Chain of r3:
(3) -> 11
(5) -> 11
(12) ->

Some-things to Think About

- **Liveness and Reaching definitions are basically the same thing!**
  - **All dataflow is basically the same with a few parameters**
    - **Meaning of gen/kill (use/def)**
    - **Backward / Forward**
    - **All paths / some paths (must/may)**
      - **So far, we have looked at may analysis algorithms**
      - **How do you adjust to do must algorithms?**
- **Dataflow can be slow**
  - **How to implement it efficiently?**
  - **How to represent the info?**

Generalizing Dataflow Analysis

- **Transfer function**
  - **How information is changed by BB**

    out[BB] = gen[BB] + (in[BB] – kill[BB]) **forward analysis**

    in[BB] = gen[BB] + (out[BB] – kill[BB]) **backward analysis**
- **Meet/Confluence function**
  - **How information from multiple paths is combined**

    in[BB] **= U *out[P]* : *P* is pred of *BB*** **forward analysis**

    out[BB] **= U *in[P]* : *P* is succ of *BB*** **backward analysis**

Generalized Dataflow Algorithm

```
change = true;
while (change)
    change = false;
    for each BB
        apply meet function
        apply transfer function
        if any changes → change = true;
```

Example: Liveness by upward exposed uses

```
for each basic block BB, do
    $gen[BB]=\varnothing$
    $kill[BB]=\varnothing$

    for each operation ($x$ := $y$ op $z$) in reverse order in BB, do
            $gen[BB]=gen[BB]-\{x\}$
            $kill[BB]=kill[BB]\bigcup\{x\}$

        for each source operand of op, $y$, do
                $gen[BB]=gen[BB]\bigcup\{y\}$
                $kill[BB]=kill[BB]-\{y\}$
        endfor
    endfor
endfor
```

Beyond Upward Exposed Uses

- **Upward exposed defs**
  - ***in* = *gen* + (*out* – *kill*)**
  - ***out* = U(*in*(succ))**
  - **Walk ops reverse order**
    - ***gen* += {dest}   kill += {dest}**

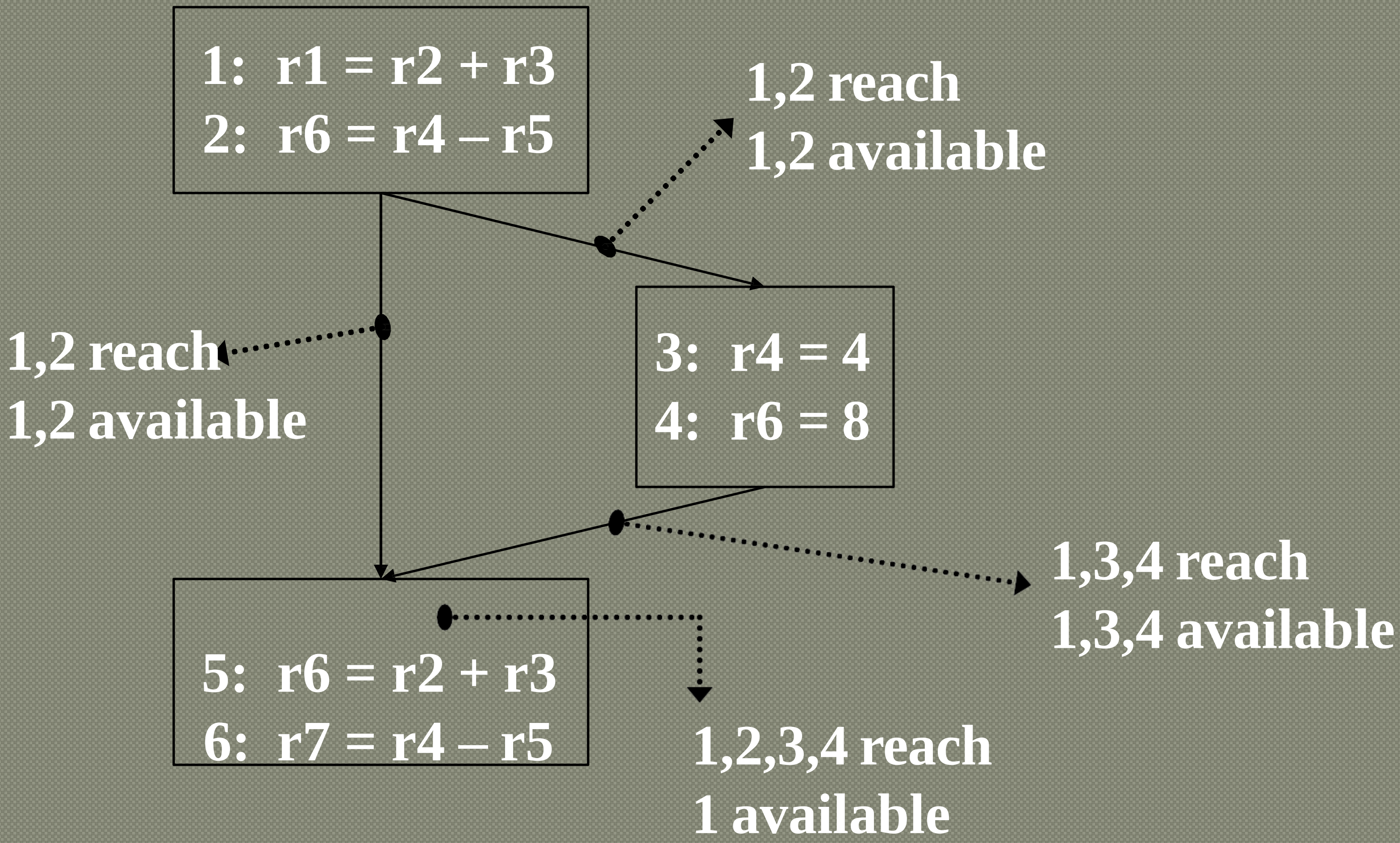
- **Downward exposed uses**
  - ***in* = U(*out*(pred))**
  - ***out* = *gen* + (*in* - *kill*)**
  - **Walk in forward order**
    - ***gen* += {src}; *kill* -= {src};**
    - ***gen* -= {dest}; *kill* += {dest};**

- **Downward exposed defs**
  - ***in* = U(*out*(pred))**
  - ***out* = *gen* + (*in* - *kill*)**
  - **Walk in forward order**
    - ***gen* += {dest}; *kill* += {dest};**

All Path Problem

- Up to this point
  - Any path problems (maybe relations)
    - Definition reaches along some path
    - Some sequence of branches in which def reaches
    - Lots of defs of the same variable may reach a point
  - Use of <u>Union operator </u>in meet function
- All-path: Definition guaranteed to reach
  - Regardless of sequence of branches taken, def reaches
  - Can always count on this
  - Only 1 def can be guaranteed to reach
  - Availability (as opposed to reaching)
    - Available definitions
    - Available expressions (could also have reaching expressions, but not that useful)

Reaching vs Available Definitions



1: r1 = r2 + r3
2: r6 = r4 – r5

1,2 reach
1,2 available

1,2 reach
1,2 available

3: r4 = 4
4: r6 = 8

1,3,4 reach
1,3,4 available

5: r6 = r2 + r3
6: r7 = r4 – r5

1,2,3,4 reach
1 available

Available Definition Analysis (Adefs)

- **A definition d is *available* at a point p if along <u>all</u> paths from d to p, d is not killed**
- **Remember, a definition of a variable is *killed* between 2 points when there is another definition of that variable along the path**
  - **r1 = r2 + r3 kills previous definitions of r1**
- **Algorithm:**
  - **Forward dataflow analysis as propagation occurs from defs downwards**
  - **Use the Intersect function as the meet operator to guarantee the all-path requirement**
  - ***gen/kill/in/out* similar to reaching defs**
    - **Initialization of *in/out* is the tricky part**

Compute Adef *gen/kill* Sets

```
for each basic block BB do
    gen(BB) =      ;    kill(BB) =      ;
    for each statement∅(d: x := y op∅z) in sequential order in BB, do
        kill(BB) = kill(BB) U G[x];
        G[x] = d;
   endfor
   gen(BB) = U G[x]: for all id x
endfor
```

Exactly the same as Reaching defs !!

Compute Adef *in/out* Sets

```
U = universal set of all definitions in the prog
in(0) = 0;   out(0) = gen(0)
for each basic block BB, (BB != 0), do
    in(BB) = 0;     out(BB) = U – kill(BB)

change = true
while (change) do
    change = false
    for each basic block BB, do
        old_out = out(BB)
        in(BB) = $\bigcap$ out(Y) : for all predecessors Y of BB
        out(BB) = GEN(X) + (IN(X) – KILL(X))
        if (old_out != out(X)) then    change = true
    endfor
endfor
```

Available Expression Analysis (Aexprs)

- **An *expression* is a RHS of an operation**
  - **Ex: in “r2 = r3 + r4” “r3 + r4” is an expression**
- **An expression e is available at a point p if along *all paths* from e to p, e is not *killed*.**
- **An expression is *killed* between two points when one of its source operands are redefined**
  - **Ex: “r1 = r2 + r3” kills all expressions involving r1**
- **Algorithm:**
  - **Forward dataflow analysis**
  - **Use the Intersect function as the meet operator to guarantee the all-path requirement**
  - **Looks exactly like adefs, except *gen/kill/in/out* are the RHS’s of operations rather than the LHS’s**

Available Expression

- **Input: A flow graph with *e_kill[B]* and *e_gen[B]***
- **Output: *in[B]* and *out[B]***
- **Method:**

```
    foreach basic block B
        $in[B_1] := \;\;;\quad out[B_1] := e\_gen[B_1];$
        $out[B] = \emptyset U - e\_kill[B];$
    change=true
    while(change)
        change=false;
        for each basic block B;
            $in[B] := \bigcap out[P]$: P is pred of B
            $old\_out := out[B];$
            $out[B] := e\_gen[B] \cup (in[B] - e\_kill[B])$
            if $(out[B] \neq old\_out[B])$ change :=true;
```

Efficient Calculation of Dataflow

- Order in which the basic blocks are visited is important (faster convergence)
- Forward analysis – DFS order
  - Visit a node only when all its predecessors have been visited
- Backward analysis – Post DFS order
  - Visit a node only when all of its successors have been visited

Representing Dataflow Information

- **Requirements – Efficiency!**
  - **Large amount of information to store**
  - **Fast access/manipulation**
- **Bitvectors**
  - **General strategy used by most compilers**
  - **Bit positions represent defs (rdefs)**
  - **Efficient set operations: union/intersect/isone**
  - **Used for *gen*, *kill*, *in*, *out* for each BB**

Optimization using Dataflow

- Classes of optimization

1. Classical (machine independent)
    - Reducing operation count (redundancy elimination)
    - Simplifying operations
2. Machine specific
    - Peephole optimizations
    - Take advantage of specialized hardware features
3. Instruction Level Parallelism (ILP) enhancing
    - Increasing parallelism
    - Possibly increase instructions

Types of Classical Optimizations

- **Operation-level – One operation in isolation**
  - **Constant folding, strength reduction**
  - **Dead code elimination (global, but 1 op at a time)**
- **Local – Pairs of operations in same BB**
  - **May or may not use dataflow analysis**
- **Global – Again pairs of operations**
  - **Pairs of operations in different BBs**
- **Loop – Body of a loop**

Constant Folding

- **Simplify operation based on values of target operand**
  - **Constant propagation creates opportunities for this**
- **All constant operands**
  - **Evaluate the op, replace with a move**
    - **r1 = 3 * 4 → r1 = 12**
    - **r1 = 3 / 0 → ??? Don’t evaluate excepting ops !, what about FP?**
  - **Evaluate conditional branch, replace with BRU or noop**
    - **if (1 < 2) goto BB2 → goto BB2**
    - **if (1 > 2) goto BB2 → convert to a noop Dead code**
- **Algebraic identities**
  - **r1 = r2 + 0, r2 – 0, r2 | 0, r2 ^ 0, r2 << 0, r2 >> 0 → r1 = r2**
  - **r1 = 0 * r2, 0 / r2, 0 & r2 → r1 = 0**
  - **r1 = r2 * 1, r2 / 1 → r1 = r2**

Strength Reduction

- **Replace expensive ops with cheaper ones**
  - **Constant propagation creates opportunities for this**
- **Power of 2 constants**
  - **Mult by power of 2: r1 = r2 * 8 → r1 = r2 << 3**
  - **Div by power of 2: r1 = r2 / 4 → r1 = r2 >> 2**
  - **Rem by power of 2: r1 = r2 % 16 → r1 = r2 & 15**
- **More exotic**
  - **Replace multiply by constant by sequence of shift and adds/subs**
    - **r1 = r2 * 6**
      - **r100 = r2 << 2; r101 = r2 << 1; r1 = r100 + r101**
    - **r1 = r2 * 7**
      - **r100 = r2 << 3; r1 = r100 – r2**

Dead Code Elimination

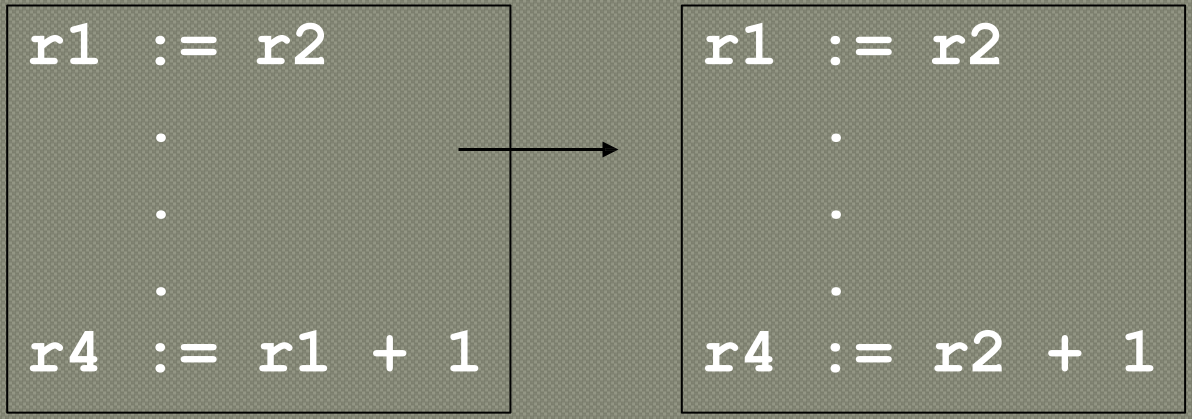
- **Remove statement d: *x* := *y* op *z* whose result is never consumed.**
- **Rules:**
  - **DU chain for d is empty**
  - ***y* and *z* are not live at d**

Constant Propagation

- **Forward propagation of moves/assignment of the form d: rx := L where L is literal**
  - **Replacement of “rx” with “L” wherever possible.**
  - **d must be available at point of replacement.**

Forward Copy Propagation

- Forward propagation of RHS of assignment or mov's.



```
r1  := r2
     .
     .
     .
r4  := r1 + 1
```

```
r1  := r2
     .
     .
     .
r4  := r2 + 1
```

  - Reduce chain of dependency
  - Possibly create dead code

Forward Copy Propagation

- **Rules:**

  Statement d_S is source of copy propagation

  Statement d_T is target of copy propagation

  - **d_S is a mov statement**
  - **src(d_S) is a register**
  - **d_T uses dest(d_S)**
  - **d_S is available definition at d_T**
  - **src(d_S) is a available expression at d_T**

Backward Copy Propagation

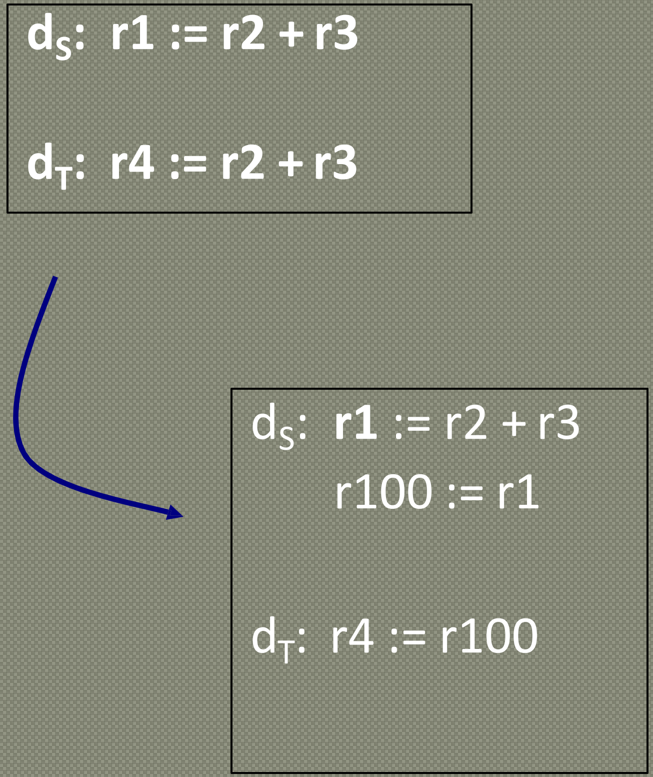
- **Backward propagation of LHS of an assignment.**

  d_T: r1 := r2 + r3 → r4 := r2 + r3
  r5 := r1 + r6 → r5 := r4 + r6
  d_S: r4 := r1 → Dead Code

- **Rules:**
  - **d_T and d_S are in the same basic block**
  - **dest(d_T) is register**
  - **dest(d_T) is not live in *out*[B]**
  - **dest(d_S) is a register**
  - **d_S uses dest(d_T)**
  - **dest(d_S) not used between d_T and d_S**
  - **dest(d_S) not defined between d_1 and d_S**
  - **There is no use of dest(d_T) after the first definition of dest(d_S)**

Local Common Sub-Expression Elimination

- **Benefits:**
  - Reduced computation
  - Generates mov statements, which can get copy propagated
- **Rules:**
  - d_S and d_T has the same expression
  - src(d_S) == src(d_T) for all sources
  - For all sources *x*, *x* is not redefined between d_S and d_T



d_S: r1 := r2 + r3

d_T: r4 := r2 + r3

d_S: **r1** := r2 + r3
r100 := r1

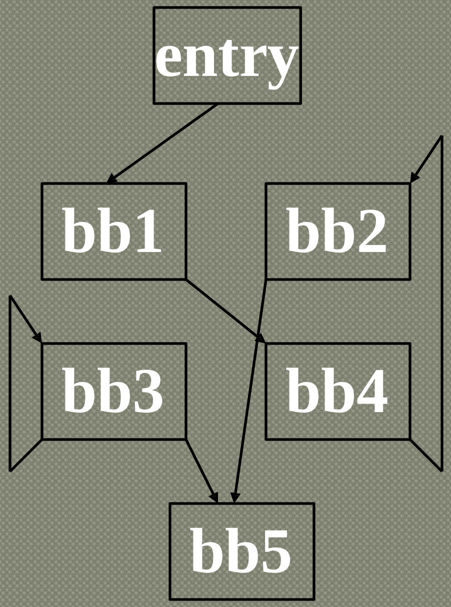
d_T: r4 := r100

Global Common Sub-Expression Elimination

- **Rules:**
  - **d_S and d_T has the same expression**
  - **src(d_S) == src(d_T) for all sources of d_S and d_T**
  - **Expression of d_S is available at d_T**

Unreachable Code Elimination

```
Mark initial BB visited
to_visit = initial BB
while (to_visit not empty)
    current = to_visit.pop()
    for each successor block of current
        Mark successor as visited;
        to_visit += successor
    endfor
endwhile
Eliminate all unvisited blocks
```



Which BB(s) can be deleted?

Unit-5

Code generation: Machine dependent code generation, object code forms, generic code generation algorithm, register allocation and assignment using DAG representation of Block

CODE GENERATION

- The final phase in our compiler model is the code generator. It takes as input an intermediate representation of the source program and produces as output an equivalent target program.
- The requirements traditionally imposed on a code generator are severe. The output code must be correct and of high quality, meaning that it should make effective use of the resources of the target machine. Moreover, the code generator itself should run efficiently.

ISSUES IN THE DESIGN OF A CODE GENERATOR

While the details are dependent on the target language and the operating system, issues such as memory management, instruction selection, register allocation, and evaluation order are inherent in almost all code generation problems .

INPUT TO THE CODE GENERATOR

- The input to the code generator consists of the intermediate representation of the source program produced by the front end, together with information in the symbol table that is used to determine the run time addresses of the data objects denoted by the names in the intermediate representation.
- There are several choices for the intermediate language, including: linear representations such as postfix notation, three address representations such as quadruples, virtual machine representations such as syntax trees and dags.

INPUT TO THE CODE GENERATOR

- We assume that prior to code generation the front end has scanned, parsed, and translated the source program into a reasonably detailed intermediate representation, so the values of names appearing in the intermediate language can be represented by quantities that the target machine can directly manipulate (bits, integers, reals, pointers, etc.). We also assume that the necessary type checking has take place, so type conversion operators have been inserted wherever necessary and obvious semantic errors (e.g., attempting to index an array by a floating point number) have already been detected. The code generation phase can therefore proceed on the assumption that its input is free of errors. In some compilers, this kind of semantic checking is done together with code generation .

TARGET PROGRAMS

- The output of the code generator is the target program. The output may take on a variety of forms: absolute machine language, relocatable machine language, or assembly language.
- Producing an absolute machine language program as output has the advantage that it can be placed in a location in memory and immediately executed. A small program can be compiled and executed quickly. A number of “student-job” compilers, such as WATFIV and PL/C, produce absolute code.

TARGET PROGRAMS

- Producing a relocatable machine language program as output allows subprograms to be compiled separately. A set of relocatable object modules can be linked together and loaded for execution by a linking loader. Although we must pay the added expense of linking and loading if we produce relocatable object modules, we gain a great deal of flexibility in being able to compile subroutines separately and to call other previously compiled programs from an object module. If the target machine does not handle relocation automatically, the compiler must provide explicit relocation information to the loader to link the separately compiled program segments

TARGET PROGRAMS

- Producing an assembly language program as output makes the process of code generation somewhat easier .We can generate symbolic instructions and use the macro facilities of the assembler to help generate code .The price paid is the assembly step after code generation.
- Because producing assembly code does not duplicate the entire task of the assembler, this choice is another reasonable alternative, especially for a machine with a small memory, where a compiler must uses several passes.

MEMORY MANAGEMENT

- Mapping names in the source program to addresses of data
- objects in run time memory is done cooperatively by the front end and the code generator. We assume that a name in a three-address statement refers to a symbol table entry for the name.

MEMORY MANAGEMENT

If machine code is being generated, labels in three address statements have to be converted to addresses of instructions. This process is analogous to the "back patching". Suppose that labels refer to quadruple numbers in a quadruple array. As we scan each quadruple in turn we can deduce the location of the first machine instruction generated for that quadruple, simply by maintaining a count of the number of words used for the instructions generated so far. This count can be kept in the quadruple array (in an extra field), so if a reference such as j: *goto i* is encountered, and i is less than j, the current quadruple number, we may simply generate a jump instruction with the target address equal to the machine location of the first instruction in the code for quadruple i. If, however, the jump is forward, so i exceeds j, we must store on a list for quadruple i the location of the first machine instruction generated for quadruple j. Then we process quadruple i, we fill in the proper machine location for all instructions that are forward jumps to i.

INSTRUCTION SELECTION

- The nature of the instruction set of the target machine determines the difficulty of instruction selection. The uniformity and completeness of the instruction set are important factors. If the target machine does not support each data type in a uniform manner, then each exception to the general rule requires special handling.
- Instruction speeds and machine idioms are other important factors. If we do not care about the efficiency of the target program, instruction selection is straightforward. For each type of three- address statement we can design a code skeleton that outlines the target code to be generated for that construct.

INSTRUCTION SELECTION

- For example, every three address statement of the form x := y + z, where x, y, and z are statically allocated, can be translated into the code sequence
-
- MOV y, R0 /* load y into register R0 */
- ADD z, R0 /* add z to R0 */
- MOV R0, x /* store R0 into x */

INSTRUCTION SELECTION

- Unfortunately, this kind of statement – by - statement code generation often produces poor code. For example, the sequence of statements
-   a := b + c
-   d := a + e
- would be translated into
-   MOV b, R0
-   ADD   c,   R0
-   MOV R0, a
-   MOV a, R0
-   ADD   e,   R0
-   MOV   R0, d

INSTRUCTION SELECTION

- Here the fourth statement is redundant, and so is the third if ‘a’ is not subsequently used.
- The quality of the generated code is determined by its speed and size.
- A target machine with a rich instruction set may provide several ways of implementing a given operation. Since the cost differences between different implementations may be significant, a naive translation of the intermediate code may lead to correct, but unacceptably inefficient target code.

INSTRUCTION SELECTION

- For example if the target machine has an "increment" instruction (INC), then the three address statement a := a+1 may be implemented more efficiently by the single instruction INC a, rather than by a more obvious sequence that loads a into a register, add one to the register, and then stores the result back into a.
- MOV a, R0
- ADD #1,R0
- MOV R0, a

INSTRUCTION SELECTION

Instruction speeds are needed to design good code sequence but unfortunately, accurate timing information is often difficult to obtain. Deciding which machine code sequence is best for a given three address construct may also require knowledge about the context in which that construct appears.

REGISTER ALLOCATION

Instructions involving register operands are usually shorter and faster than those involving operands in memory. Therefore, efficient utilization of register is particularly important in generating good code. The use of registers is often subdivided into two subproblems:

REGISTER ALLOCATION

1. **During register allocation, we select the set of variables that will reside in registers at a point in the program.**
2. **During a subsequent register assignment phase, we pick the specific register that a variable will reside in.**

REGISTER ALLOCATION

- Finding an optimal assignment of registers to variables is difficult, even with single register values. Mathematically, the problem is NP-complete. The problem is further complicated because the hardware and/or the operating system of the target machine may require that certain register usage conventions be observed.
- Certain machines require register pairs (an even and next odd numbered register) for some operands and results. For example, in the IBM System/370 machines integer multiplication and integer division involve register pairs.

REGISTER ALLOCATION

- The multiplication instruction is of the form
- M x, y
- where x, is the multiplicand, is the even register of an even/odd register pair.
- The multiplicand value is taken from the odd register pair. The multiplier y is a single register. The product occupies the entire even/odd register pair.
- The division instruction is of the form
- D x, y

REGISTER ALLOCATION

- where the 64-bit dividend occupies an even/odd register pair whose even register is x; y represents the divisor. After division, the even register holds the remainder and the odd register the quotient.
- Now consider the two three address code sequences

(a)and (b) in which the only difference is the operator in the second statement. The shortest assembly sequence for (a) and (b) are given in(c).

- Ri stands for register i. L, ST and A stand for load, store and add respectively. The

REGISTER ALLOCATION

- t := a + b
- t := t * c
- t := t / d

(a)

t := a + b

t := t + c

t := t / d

(b)

- Two three address code sequences

REGISTER ALLOCATION

```
•  L     R1, a                      L        R0, a
•  A     R1, b                      A        R0, b
•  M     R0, c                      A        R0, c
•  D     R0, d                      SRDA     R0, 32
•  ST    R1, t                      D        R0, d
•                                   ST       R1, t
•     (a)                                (b)
•
```

Optimal machine code sequence

CHOICE OF EVALUATION ORDER

- The order in which computations are performed can affect the efficiency of the target code. Some computation orders require fewer registers to hold intermediate results than others. Picking a best order is another difficult, NP-complete problem. Initially, we shall avoid the problem by generating code for the three -address statements in the order in which they have been produced by the intermediate code generator.

APPROCHES TO CODE GENERATION

- The most important criterion for a code generator is that it produce correct code. Correctness takes on special significance because of the number of special cases that code generator must face. Given the premium on correctness, designing a code generator so it can be easily implemented, tested, and maintained is an important design goal.