ELECTROMAGNETIC WAVES AND TRANSMISSION LINES(AECB13)

Course code:AECB13

II. B.Tech II semester

Regulation: IARE R-18

BY

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CO's	Course outcomes
CO1	Understand coulomb's law and gauss's law to different charge distributions, it's applications and applications of Laplace's and Poisson's equations.
CO2	Evaluate the physical interpretation of Maxwell's equations and applications for various fields.

COs	Course Outcomes
CO3	Understand the behavior of electromagnetic waves incident on the interface between two different media.
CO4	Understand the significance of transmission lines and concept of attenuation, loading, and analyze the loading technique to the transmission lines.
CO5	Formulate and analyze the smith chart to estimate impedance, VSWR, reflection coefficient, OC and SC lines.

Prerequisites for EMTL

1. vector algebra.
2. Coordinate Systems.
3. Vector calculus.

Prerequisites for EMTL

COOR DIN ATES YS TEM S

- *RECTANGULAR or Cartesian*
  - *CYLINDRICAL*
  - *SPHERICAL*

Choice is based on symmetry of problem

Examples:

Sheets - RECTANGULAR

Wires/Cables - CYLINDRICAL

Spheres - SPHERICAL

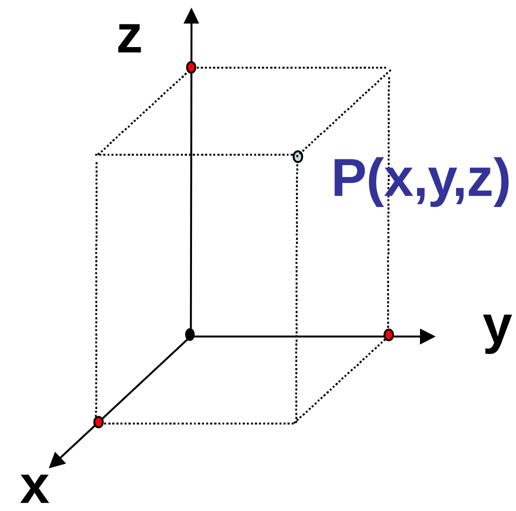
Cartesian Coordinates Or Recatangular Coordinates

Coordinate system variables are P (x, y, z)

$$-\infty < x < \infty$$

$$-\infty < y < \infty$$

$$-\infty < z < \infty$$



A vector A in Cartesian coordinates can be written as

$$(A_x, A_y, A_z) \quad \text{or} \quad A_x a_x + A_y a_y + A_z a_z$$

where a_x,a_y and a_z are unit vectors along x, y and z-directions.

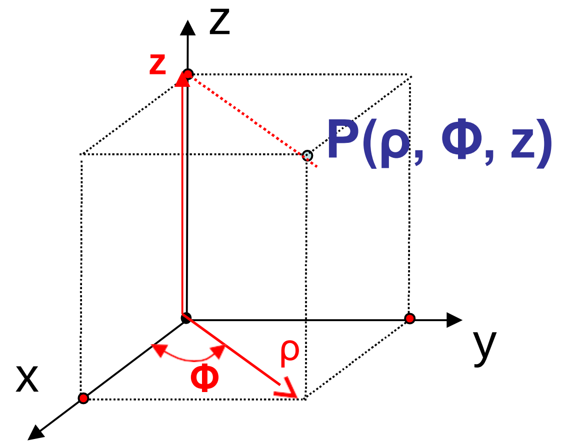
Cylindrical Coordinates

P (ρ, Φ, z)

$$0 \leq \rho < \infty$$

$$0 \leq \phi < 2\pi$$

$$-\infty < z < \infty$$



A vector A in Cylindrical coordinates can be written as

$$(A_\rho, A_\phi, A_z) \quad \text{or} \quad A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

where a_ρ,a_Φ and a_z are unit vectors along ρ, Φ and z-directions.

x= ρ cos Φ, y=ρ sin Φ,   z=z

$$\rho = \sqrt{x^2 + y^2}, \phi = \tan^{-1}\frac{y}{x}, z = z$$

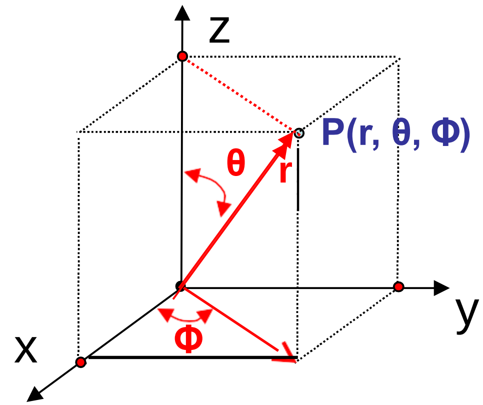
Spherical Coordinates

P (r, θ, Φ)

$$0 \le r < \infty$$

$$0 \le \theta \le \pi$$

$$0 \le \phi < 2\pi$$



A vector A in Spherical coordinates can be written as

$$(A_r, A_\theta, A_\phi) \quad \text{or} \quad A_r a_r + A_\theta a_\theta + A_\phi a_\phi$$

where a_r, a_θ, and a_Φ are unit vectors along r, θ, and Φ-directions.

x=r sin θ cos Φ, y=r sin θ sin Φ, Z=r cos θ

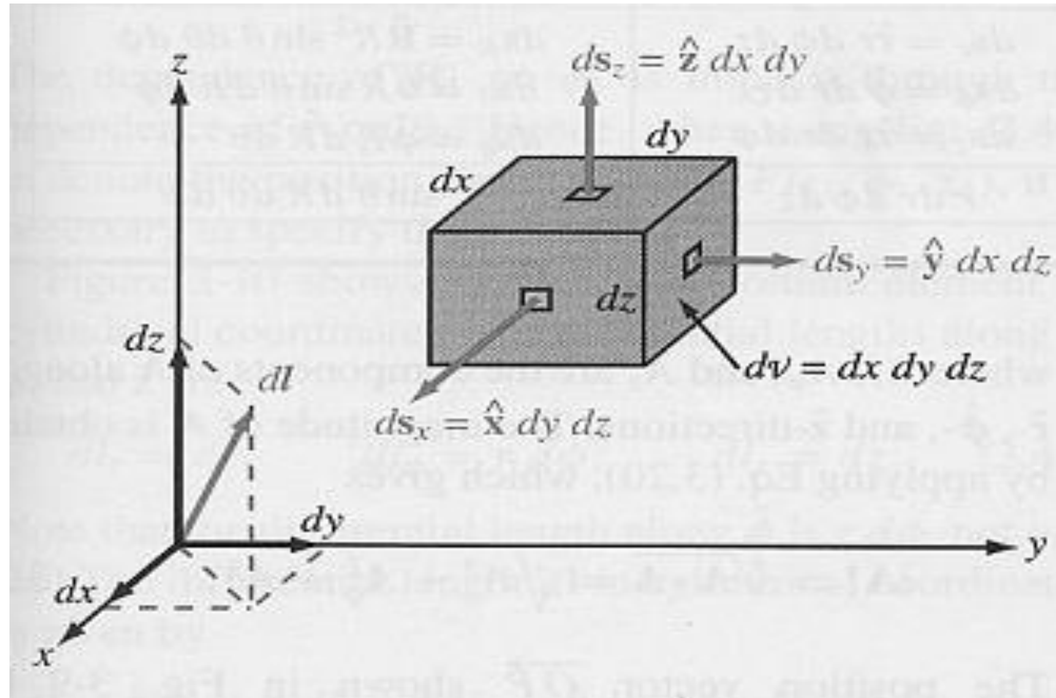
$$r = \sqrt{x^2 + y^2 + z^2}, \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \phi = \tan^{-1}\frac{y}{x}$$

Differential Length, Area and Volume

Cartesian Coordinates

Differential displacement

$$dl = dx a_x + dy a_y + dz a_z$$



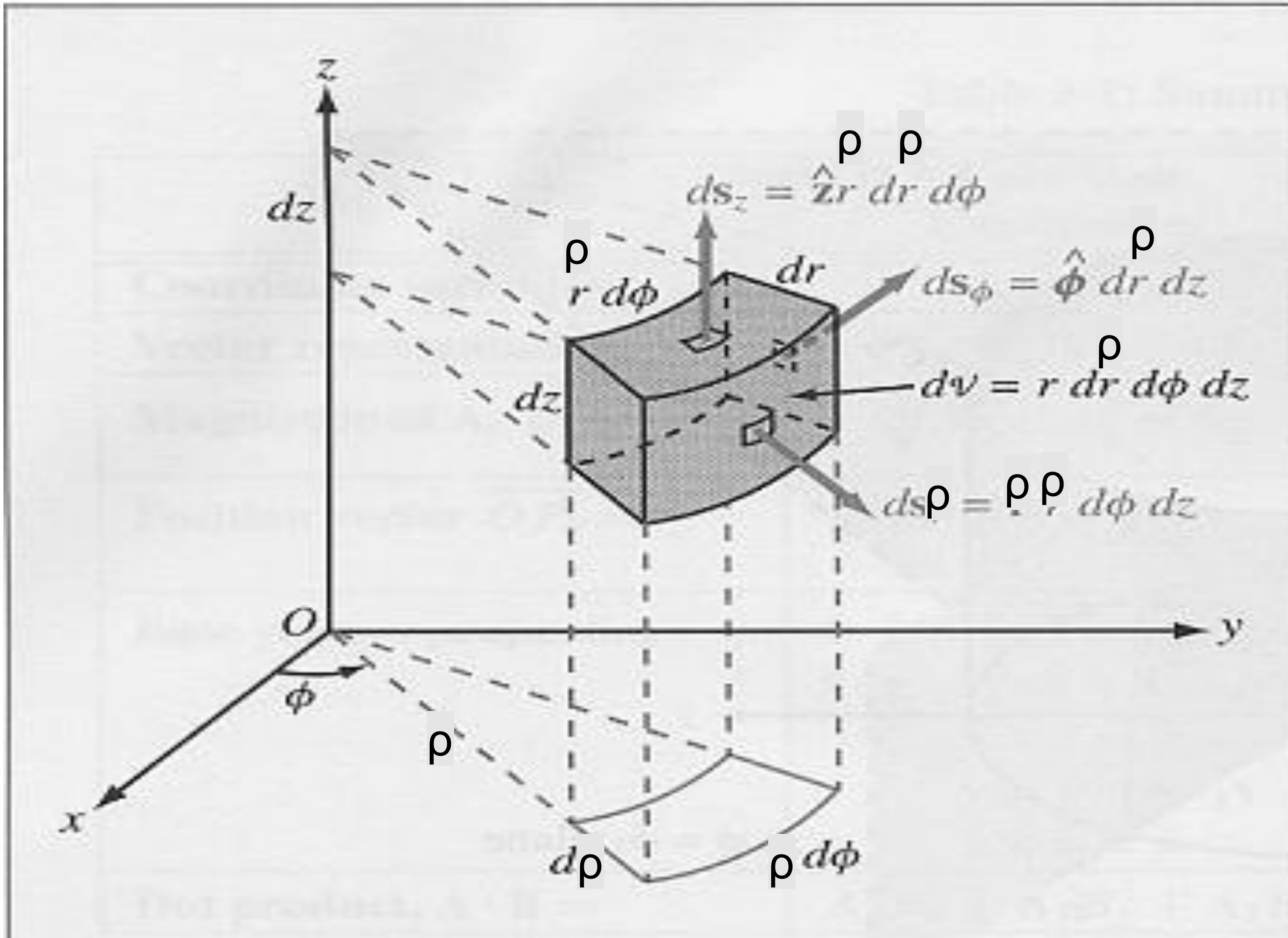
Differential area

$$dS = dydz a_x = dxdz a_y = dxdy a_z$$

Differential Volume

$$dV = dxdydz$$

Cylindrical Coordinates



Differential Length, Area and Volume

Cylindrical Coordinates

Differential displacement

$$dl = d\rho a_\rho + \rho d\phi a_\phi + dz a_z$$

Differential area

$$dS = \rho d\phi dz a_\rho = d\rho dz a_\phi = \rho d\rho d\phi a_z$$

Differential Volume

$$dV = \rho d\rho d\phi dz$$

Differential Length, Area and Volume

Spherical Coordinates

Differential displacement

$$dl = dr a_r + r d\theta a_\theta + r\sin\theta d\phi a_\phi$$

Differential area

$$dS = r^2 \sin\theta d\theta d\phi a_r = r\sin\theta dr d\phi a_\theta = r dr d\theta a_\phi$$

Differential Volume

$$dV = r^2 \sin\theta dr d\theta d\phi$$

Line, Surface and Volume Integrals

Line Integral

$$\oint_L A.dl$$

Surface Integral

$$\psi = \int_S A.dS$$

Volume Integral

$$\int_V p_v \, dv$$

The Del Operator

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

Gradient of a scalar function is a vector quantity.

$$\nabla f$$

- Divergence of a vector is a scalar quantity.

$$\nabla . A$$

Curl of a vector is a vector quantity.

$$\nabla \times A$$

The Laplacian of a scalar A

$$\nabla^2 A$$

Del Operator

Cartesian Coordinates

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

Cylindrical Coordinates

$$\nabla = \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z$$

Spherical Coordinates

$$\nabla = \frac{\partial}{\partial r} a_r + \frac{1}{r} \frac{\partial}{\partial \theta} a_\theta + \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} a_\phi$$

Gradient of a Scalar

The gradient of a scalar field V is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

$$\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z$$

$$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} a_\phi$$

Divergence of a Vector

The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P.

$$divA = \nabla .A = \lim_{\Delta v \to 0} \frac{\oint_S A.dS}{\Delta v}$$

$$\nabla .A = \frac{\partial A}{\partial x} + \frac{\partial A}{\partial y} + \frac{\partial A}{\partial z}$$

$$\nabla .A = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Curl of a Vector

The curl of A is an axial vector whose magnitude is the maximum circulation of A per unit area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

$$curl A = \nabla \times A = \left(\lim_{\Delta s \to 0} \frac{\oint_L A.dl}{\Delta S} \right)_{\max} a_n$$

Where ΔS is the area bounded by the curve L and a_n is the unit vector normal to the surface ΔS

Curl of a Vector

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Cartesian Coordinates

$$\nabla \times A = \frac{1}{\rho}\begin{vmatrix} \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

Cylindrical Coordinates

$$\nabla \times A = \frac{1}{r^2 \sin\theta}\begin{vmatrix} a_r & r a_\theta & r \sin\theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin\theta A_\phi \end{vmatrix}$$

Spherical Coordinates

Divergence or Gauss' Theorem

The divergence theorem states that the total outward flux of a vector field A through the closed surface S is the same as the volume integral of the divergence of A.

$$\oint A.dS = \int_V \nabla .A dv$$

Stokes's Theorem

Stokes's theorem states that the circulation of a vector field A around a closed path L is equal to the surface integral of the curl of A over the open surface S bounded by L, provided A and $\nabla \times A$ ***are continuous on S***

$$\oint_L A.dl = \int_S (\nabla \times A).dS$$

ELECTROMAGNETIC THEORY AND TRANSMISSION LINES

ELECTROSTATICS-I

OBJECTIVES

COULOMB’S LAW

ELECTRIC FIELD INTENSITY

ELECTRIC FLUX DENSITY

DIFFERENT CONTINUOUS CHARGE DISTRIBUTIONS

GAUSS’S LAW

APPLICATIONS OF GAUSS’S LAW

ELECTRIC POTENTIAL

RELATION B/W E&V

ENERGY DENSITY

COULOMB'S LAW:

- Coulomb's law is an experimental law formulated in 1785 by Charles Augustine dc Coulomb.
- It deals with the force, a point charge on another point charge. The polarity of charges may be positive or negative ,like charges repel while unlike charges attract.
- Charges are generally measured in coulomb(C)
- One coulomb is approximately equivalent to $6*10^{18}$ electrons
- One electron charge(e)=$-1.6019*10^{-19}$ C

Coulomb's law states that the force 'F' between two point charges Q1 and Q2 is

1.Along the line joining them

2.Directly proportional to the product $Q_1 Q_2$ of the charges

3.Inversely proportional to the square of the distance 'R' between them.

$$F = \frac{K Q_1 Q_2}{R^2} \text{----------------(1)}$$

where K is proportionality constant and $K = \frac{1}{4\pi\epsilon_0}$ -----------(2)

where ϵ_0 is permittivity of free space and is given by

$$\epsilon_0 = 8.854*10^{-12} \cong \frac{10^{-9}}{36\pi} \text{ F/m}$$

$$K = \frac{1}{4\pi\epsilon_0} \cong 9*10^9 \text{ m/F}$$

If point charges Q_1 & Q_2 are located at points having position vectors

r_1 & r_2, then the force F_{12} on Q_2 due to Q_1 is given by

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} a_{R_{12}} \quad \text{------------------}(4)$$

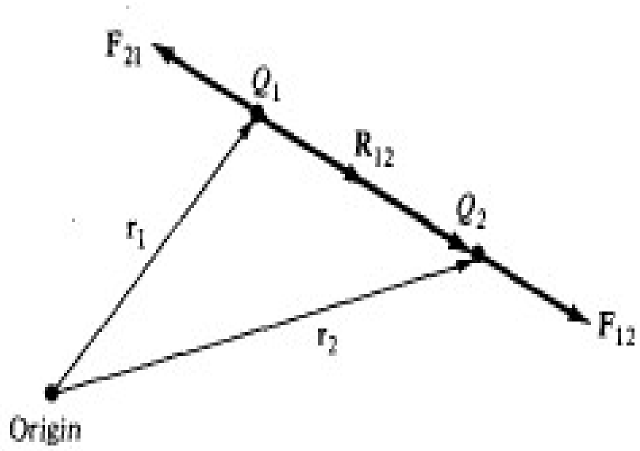


Figure 4.1 Coulomb vector force on point changes Q_1 and Q_2.

Substitute equation(2) in equation(1),

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \quad \text{-------------}(3)$$

where $R_{12} = r_2 - r_1$-------------(a)

If there are ‘N’ point charges Q_1, Q_2,------Q_N located respectively, at points with position vectors r_1, r_2, -------- r_N ,the resultant force ‘F’ on a charge ‘Q’ located at point ‘r’ is the forces exerted on ‘Q’ by each of the charges Q_1, Q_2,------Q_N.

$$F = \frac{QQ_1(r - r_1)}{4\pi\epsilon_0|r - r_1|^3} + \frac{QQ_2(r - r_2)}{4\pi\epsilon_0|r - r_2|^3} + \text{-----------------} + \frac{QQ_N(r - r_N)}{4\pi\epsilon_0|r - r_N|^3}$$

(or)

$$F = \frac{Q}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{Q(r - r_K)}{|r - r_K|^3} \quad \text{--------------(6)}$$

ELECTRIC FIELD INTENSITY(E):

ELECTRIC FIELDS DUE TO CONTINUOUS CHARGE DISTRIBUTIONS:

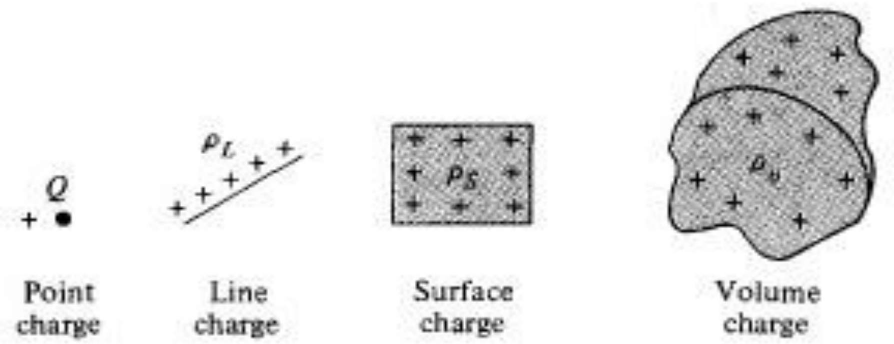


Figure 4.5 Various charge distributions and charge elements.

By replacing Q in eq(8) with charge element $dQ=\rho_l dl$, $\rho_S ds$ &$\rho_v dv$,

$$E = \int \frac{\rho_l dl}{4\pi\epsilon_0 R^2}\, a_R$$ (line charge) --------(10)

$$E = \int \frac{\rho_s ds}{4\pi\epsilon_0 R^2}\, a_R$$ (surface charge) ------------(11)

$$E = \int \frac{\rho_v dv}{4\pi\epsilon_0 R^2}\, a_R$$ (volume charge) ---------(12)

A LINE CHARGE:

Line charge with uniform charge density ρ_L extending from A to B along the z-axis.

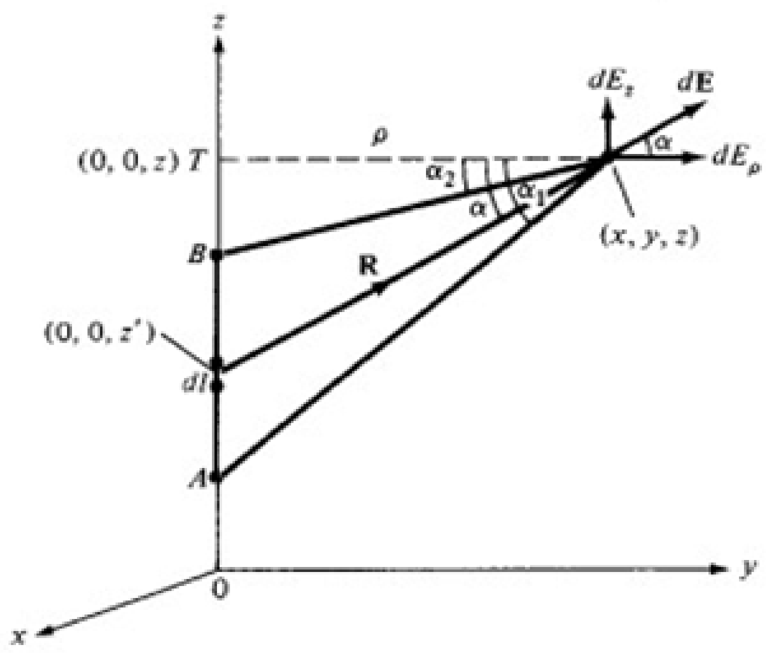


Figure 4.6 Evaluation of the **E** field due to a line charge.

charge element dQ associated with element dl = dz of line is,

A SURFACE CHARGE:

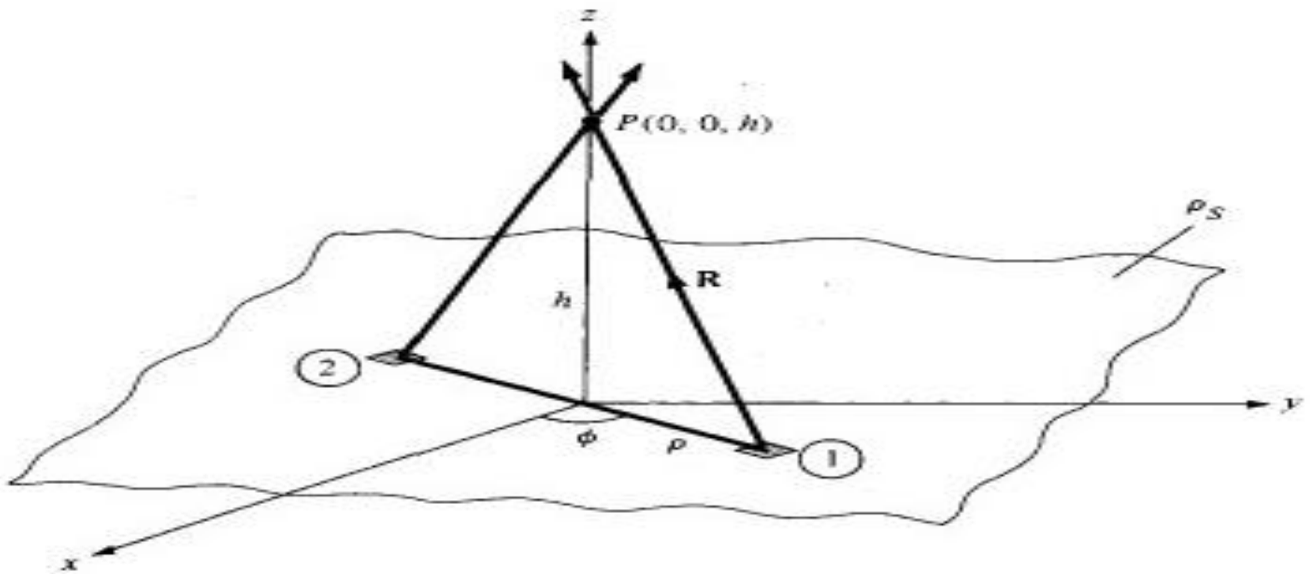


Figure 4.7 Evaluation of the **E** field due to an infinite sheet of charge.

The total charge is,

A VOLUME CHARGE:

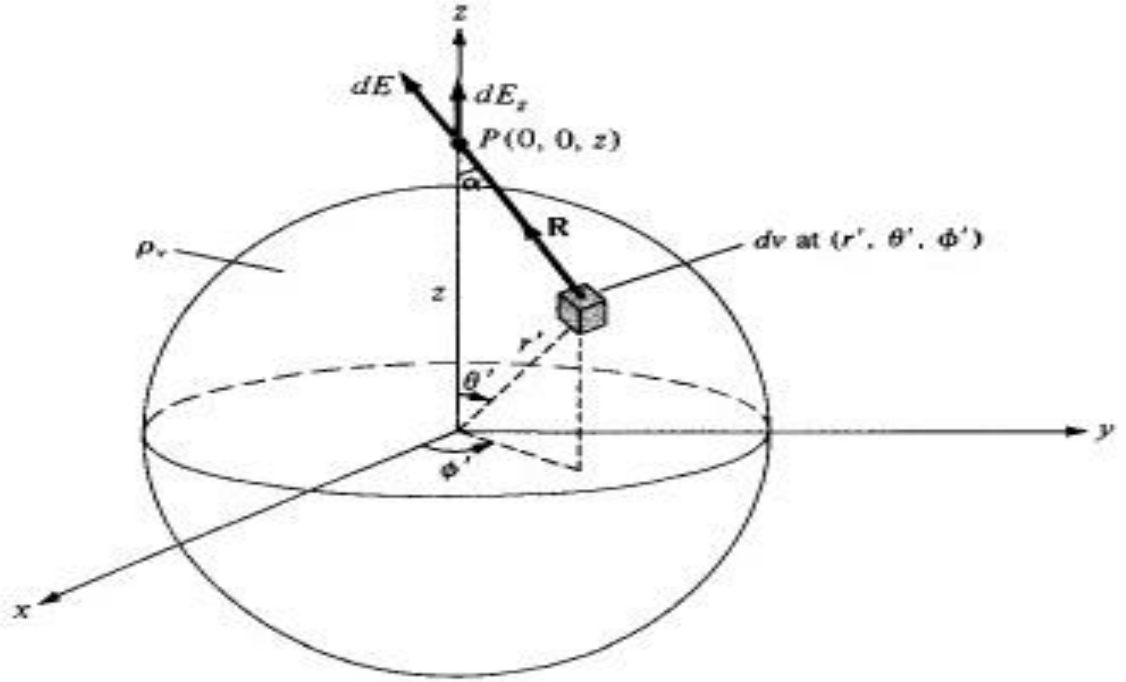


Figure 4.8 Evaluation of the **E** field due to a volume charge distribution.

The total charge in a sphere of radius a is,

ELECTRIC FLUX DENSITY:

GAUSS'S LAW:

"Gauss law states that total electric flux 'ψ' through any closed surface is equal to the charge enclosed by that surface".

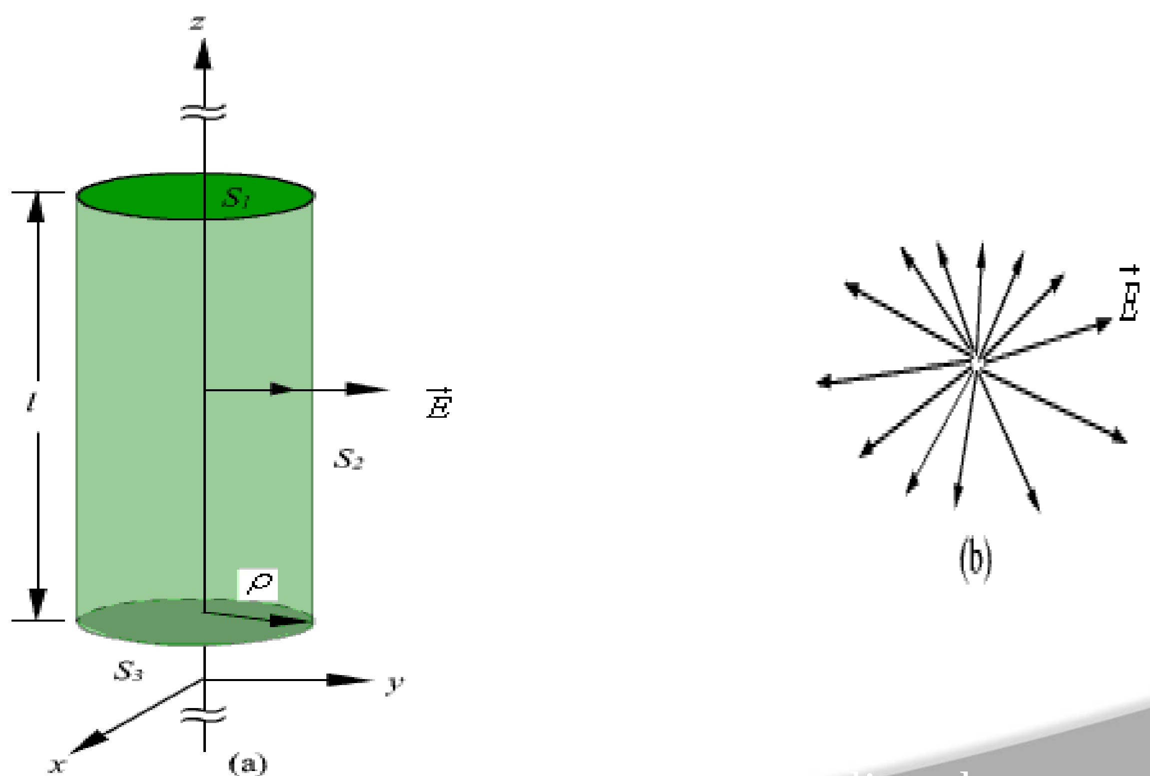
$$Q = \oint \boldsymbol{D}.\boldsymbol{ds} = \int \boldsymbol{\rho_v}\mathbf{d}\boldsymbol{v}$$

$\boldsymbol{\rho_v} = \boldsymbol{\nabla}.\mathbf{D} \quad \Rightarrow$

Gauss law is an alternative statement of coulomb's law

APPLICATIONS OF GAUSS'S LAW:

An infinite line charge:



...finite line charge

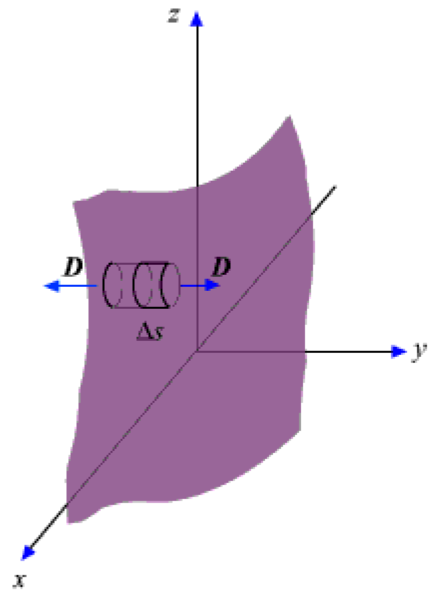
If we consider a close cylindrical surface, using Gauss's theorem

AN INFINITE SHEET OF CHARGE:

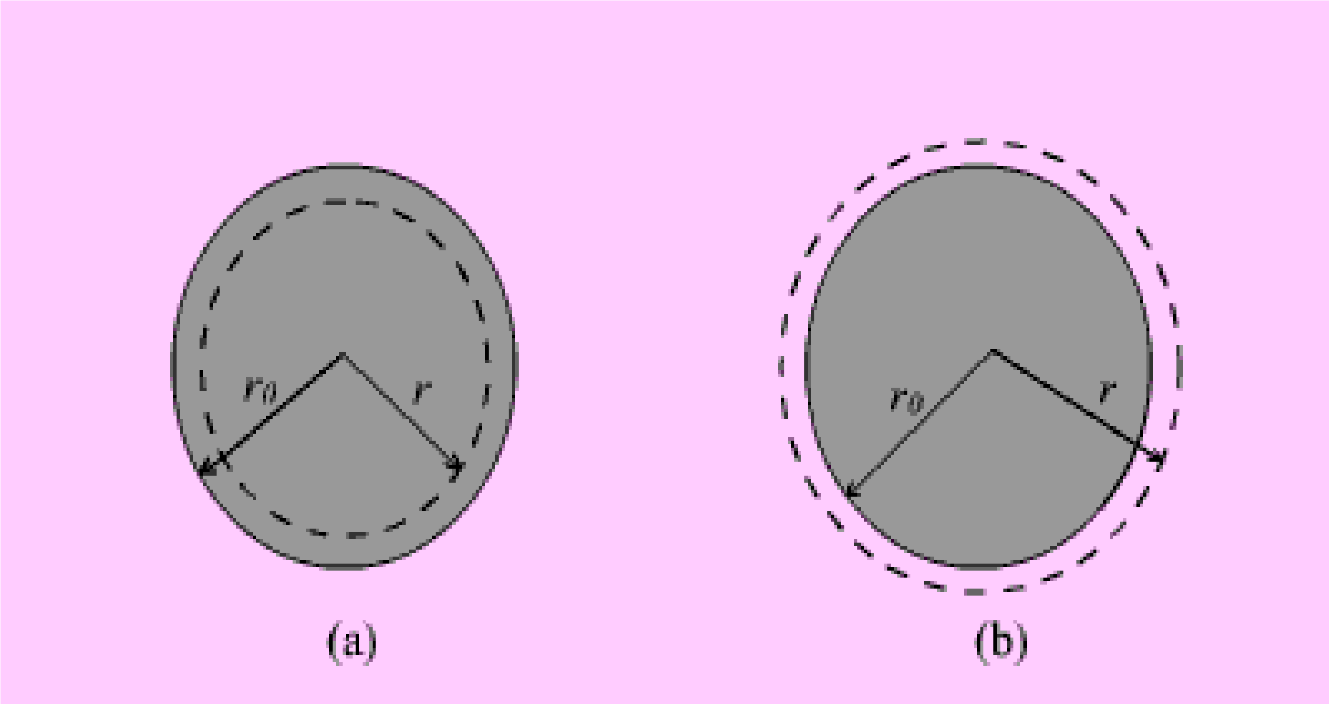
top

botto

Δs



UNIFORMLY CHARGED SPHERE:



For the region r≤ r_0 ,the total enclosed charge,

$$\mathbf{D} = \frac{r}{3}\rho_v a_r \text{--------(3)} \qquad 0 \le r \le r_0$$

For the region r≥ r_0 ; the total enclosed charge,

$Q_{enc} = \int \rho_v dv = \rho_v \int dv$

$Q_{enc} = \rho_v \int_{r=0}^{r_0} dr \int_{\emptyset=0}^{2\pi} d\emptyset \int_{\theta=0}^{\pi} r^2 \sin\theta \mathrm{dr}\, \mathrm{d}\theta$

$Q_{enc} = \rho_v \frac{4}{3} \pi {r_0}^3$ -------(4)

&$\psi = \oint D dS = \mathrm{D} \oint dS$

$\psi = \mathrm{D}4\pi r^2$-------------(4)

Hence, $\psi = Q_{enc}, \rho_v \frac{4}{3} \pi {r_0}^3 = \mathrm{D}4\pi r^2$ ------------(5)

$\mathbf{D} = \frac{{r_0}^3}{3r^2} \rho_0 a_r$ r≥ r_0 -----------(6)

$$D = \begin{cases} \dfrac{r}{3}\rho_v a_r & 0 \le r \le r_0 \\ \dfrac{{r_0}^3}{3r^2}\,\rho_0\, a_r & r \ge r_0 \end{cases} \qquad \text{---------}(7)$$

ELECTRIC POTENTIAL:

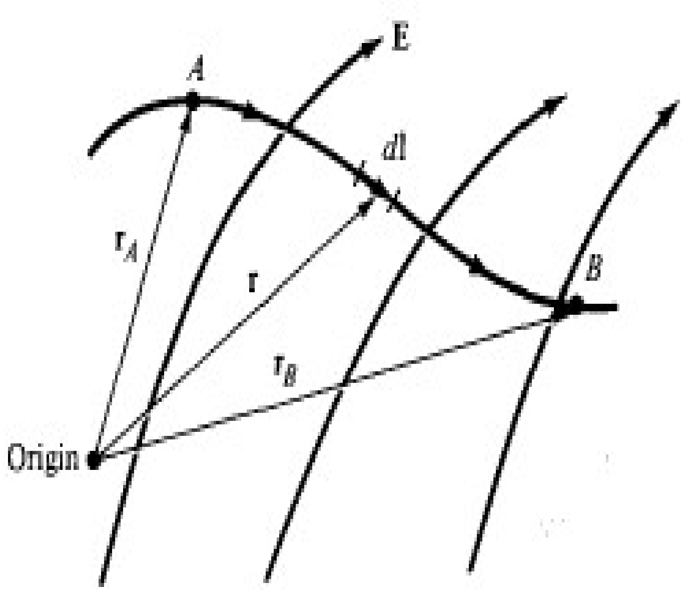


Figure 4.18 Displacement of point charge Q in an electrostatic field $\mathbf{E}$.

The potential difference b/w two points,

$$V_{AB} = \frac{W}{Q} = - \int_A^B E.dl$$

If the E field is due to a point charge Q located at the origin,

$$E = \frac{Q}{4\pi\, \varepsilon_0 r^2} a_r$$

$$V_{AB} = - \int_{r_A}^{r_B} \frac{Q}{4\pi\, \varepsilon_0 r^2}\ a_r.dr\, a_r$$

$$V_{AB} = \frac{Q}{4\pi\, \varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad \text{(or)} \qquad V_{AB} = V_B - V_A$$

If we assume the potential at infinity is zero. If V_A =0, as r→ ∞&$r_B = r$,

$$V = \frac{Q}{4\pi\, \varepsilon_0 r}$$

The potential at any point is the potential difference between that point & a chosen point at which the potential is zero.

$$\boldsymbol{V_{AB} = V_B - V_A = \frac{W}{Q} = - \int_A^B E.dl}$$

Assume zero potential at infinity , the potential at a distance 'r' from the point charge is the work done per unit charge by an external agent in transferring a test charge from infinity to that point.

$$V = -\int_{-\infty}^{r} E.dl$$

If the point charge is not located at the origin but at a point where position vector is r' The potential V(r),

$$V(r) = \frac{Q}{4\pi\,\varepsilon_0|r-r\prime|}$$

For 'n' point charges Q_1, Q_2,------Q_N. Located at points with position vectors r_1, r_2,------r_n, the potential at r is

$$V(r) = \frac{Q_1}{4\pi\,\varepsilon_0|r-r_1|} + \frac{Q_2}{4\pi\,\varepsilon_0|r-r_2|} + \text{------------} + \frac{Q_n}{4\pi\,\varepsilon_0|r-r_n|} \text{(or)}$$

$$V(r) = \frac{1}{4\pi\,\varepsilon_0}\sum_{k=1}^{n}\frac{Q_k}{|r-r_k|}$$

For continuous charge distributions ,with charge element $\rho_L dl$, $\rho_s dS$&$\rho_V dV$

V(r) = $\frac{1}{4\pi\,\varepsilon_0}\int_L \frac{\rho_L(r')dl\prime}{|r-r\prime|}$ (Line charge)

V(r) = $\frac{1}{4\pi\,\varepsilon_0}\int_S \frac{\rho_S(r')dS\prime}{|r-r\prime|}$(Surface charge)

V(r) = $\frac{1}{4\pi\,\varepsilon_0}\int_v \frac{\rho_v(r')dv\prime}{|r-r\prime|}$(Volume charge)

If E is known ,

V = - $\int E.dl$ +C

RELATIONSHIP BETWEEN E AND V –MAXWELL'S EQUATION:

The potential difference between points A & B is independent of path taken.

V_{BA} = - V_{AB}

V_{BA}+ $V_{AB} = \oint E.dl$ =0 -------(1)

Applying Stokes's theorem to eq(1),

$\oint_L E.dl = \int_S (\nabla \times E).dS = 0$ (or)

$\nabla \times E$=0

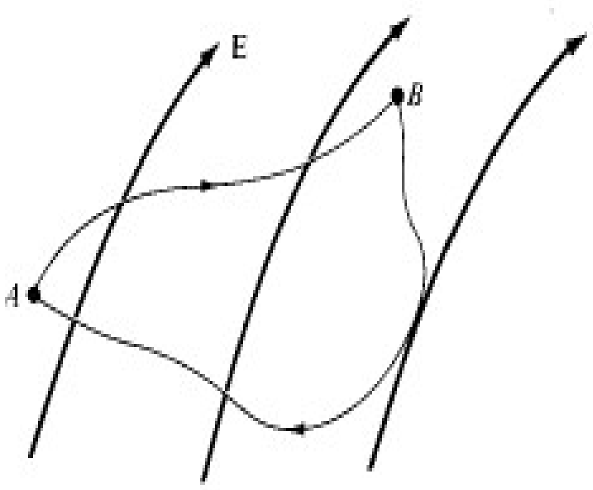


Figure 4.19 Conservative nature of an electrostatic field.

Conservative vectors:

"The vectors whose line integral does not depend on the path of integration are known as conservative vectors."

$$V = -\int E.dl$$

$$dV = -E.dl = -E_x dx - E_y dy - E_z dz$$

$$dV = \frac{\partial V}{\partial x}\, dx + \frac{\partial V}{\partial y}\, dy + \frac{\partial V}{\partial z}\, dz \quad then,$$

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad \& E_z = -\frac{\partial V}{\partial z}$$

$$\boldsymbol{E = -\nabla V}$$

ENERGY DENSITY IN ELECTROSTATIC FIELDS:

- Consider the three point charges Q_1, Q_2, Q_3 are placed in an empty space.
- No work is required to transfer Q_1 from infinity to P_1 because the space is initially charge free and there is no electric field.

$$W = -Q \int_A^B E.dl \quad \text{i.e. } W=0$$

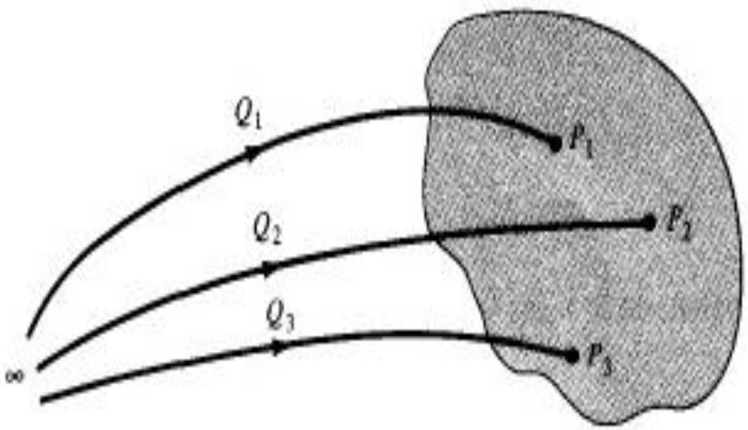


Figure 4.22 Assembling of charges.

- The work done in transferring Q_2 from infinity to P_2 is equal to the product of Q_2 and the potential V_{21} at P_2 due to Q_1.
- The work done in positioning Q_3 at P_3 is equal to $Q_3(V_{32} + V_{31})$.

The total work done in positioning the three charges,

$$W_E = W_1 + W_2 + W_3$$

$$W_E = 0 + Q_2V_{21} + Q_3(V_{32} + V_{31})$$

If the charges are positioned in reverse order,

$$W_E = 0 + Q_2V_{23} + Q_1(V_{12} + V_{13})$$

$$2W_E = Q_1(V_{12} + V_{13}) + Q_2(V_{21} + V_{23}) + Q_3(V_{32} + V_{31})$$

$$2\,W_E = Q_1V_1 + Q_2V_2 + Q_3V_3$$

$$2\,W_E = 1/2(\,Q_1V_1 + Q_2V_2 + Q_3V_3)$$

If there are 'n' point charges,

$$W_E = \frac{1}{2}\sum_{K=1}^{n} Q_K V_K$$

Instead of point charges, the region has a continuous charge distributions,

$$W_E = \frac{1}{2}\int_L \rho_L \text{Vdl} \qquad \text{(Line charge)}$$

$$W_E = \frac{1}{2}\int_S \rho_S \text{VdS} \qquad \text{(Surface charge)}$$

$$W_E = \frac{1}{2}\int_V \rho_v \text{Vdv} \qquad \text{(Volume charge)}$$

Since $\rho_v = \nabla . D$,

$$W_E = \frac{1}{2}\int_V (\nabla . D)\text{Vdv}$$

$\nabla . VA = \text{A}.\nabla\text{V} + \text{V}(\nabla . A)$ (or)

$\text{V}(\nabla . A) = \nabla . VA - \text{A}.\nabla\text{V}$ Vector identity

$W_E = \frac{1}{2}\int_V(\nabla . VD)$ dV - $\frac{1}{2}\int_V(D.\nabla V)$dV

By applying divergence theorem to the first term on the right hand side of the above equation,

$$W_E = \frac{1}{2}\oint_S VD.dS - \frac{1}{2}\int_V(D.\nabla V)\text{dV}$$

$$W_E = -\frac{1}{2}\int_V(D.\nabla V)\text{dV}$$

$W_E = \frac{1}{2}\int_V(D.E)$dV   ($\because$ E = -∇V & D = $\varepsilon_0 E$)

$$W_E = \frac{1}{2}\int_V(\varepsilon_0 E^2)\text{ dV}$$

$W_E = \frac{1}{2}\varepsilon_0 E^2$ Which is the electrostatic energy density

ELECTROSTATICS-II

OBJECTIVES

CONVECTION CURRENT

CONDUCTION CURRENT

CONTINUITY EQUATION

RELAXATION TIME

POISSON’S AND LAPLACE’S EQUATION

POLARIZATION

DIELECTRIC CONSTANT

BOUNDARY CONDITIONS

RESISTANCE & CAPACITANCE

INTRODUCTION:

Materials may be classified in terms of their conductivity (σ) as **conductors and non-conductors** or technically as **metals and insulators (or dielectrics)**

A material with high conductivity ($\sigma >> 1$) is termed as metal.

Ex: copper & aluminum.

A material with low conductivity ($\sigma << 1$) is termed as an insulator.

Ex: glass & rubber

A material whose conductivity lies between metals and insulators are called as semiconductors.

Ex: Si & Ge

At T=0 K, some conductors exhibit infinite conductivity and are called superconductors .

Ex: Lead & aluminium

CONVECTION CURRENT:

The current through a given area is the electric charge passing through the area per unit time.

i.e. $I = \frac{dQ}{dt}$ ---------(1)

If current $'\Delta I'$ flows through a planar surface '$\Delta S'$, the current density is

$J = \frac{\Delta I}{\Delta S}$ or

$$\Delta I = J\Delta S \quad -----(2)$$

Total current flowing through a surface is 'S'

$I = \int J.ds$---------(3)

Consider a filament of the above fig. If there is a flow of charge of density,

The current density at a given point is the current through a unit normal area at that point.

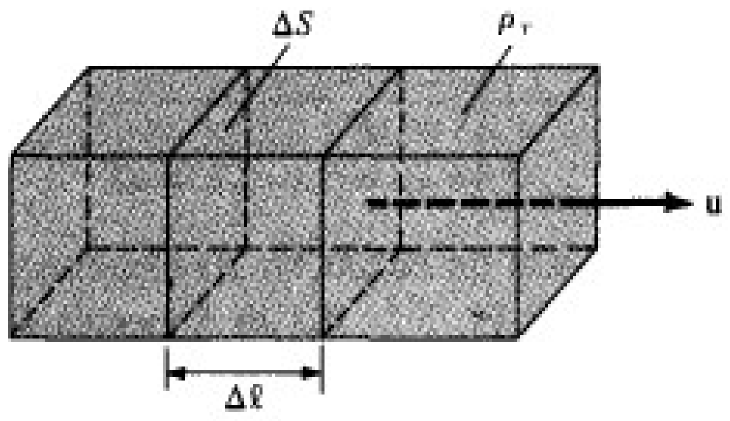


Figure 5.1 Current in a filament.

The ‘y’ directed current density ‘J_y’ is given by

$$J_y = \frac{\Delta I}{\Delta s} = \rho_v u_y \quad \text{--------}(5)$$

In general $J = \rho_v u$

Where ‘I’ is the convection current(A) and

‘J’ is convection current density(A/m^2)

CONDUCTION CURRENT:

when an electric field 'E' is applied, the force on an electron with

charge -e is

$$F = - eE\text{--------}(1) \quad (\because\ F = QE \quad \text{or} \quad E = \frac{F}{Q})$$

If an electron with mass 'm' is moving in an electric field 'E' with an average drift velocity 'u' , according to Newton's law the average change in momentum of the free electron must match the applied force.

$$\frac{mu}{\tau} = - eE$$

$$\text{or} \quad u = - \frac{e\tau}{m} E \text{---------}(2)$$

from the above eq' the drift velocity of the electron is directly proportional to the applied field.

If there are ‘n’ electrons per unit volume , the electric charge density is given

J = $\boldsymbol{\sigma E}$ is the point form of ohm’s law

CONTINUITY EQUATION:

The principle of charge conservation, the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the surface of the volume.

current I_{out} coming out of the closed surface is

$$I_{out} = \oint J.\mathrm{ds} = - \frac{dQ_{in}}{dt} \quad \text{-------(1)}$$

where Q_{in} is the total charge enclosed by the closed surface and

J is conduction current density.

By the divergence theorem ,

$$\nabla . J = - \frac{\partial \rho_v}{\partial t}$$

The above equation is termed as continuity of current equation

or continuity equation

RELAXATION TIME:

The Maxwell's first equation is given by $\boldsymbol{\rho_v = \nabla .D}$

we know that $J = \sigma E$

$\rho_v = \nabla.\varepsilon E \quad (\because D = \varepsilon E)$

$\frac{\rho_v}{\varepsilon} = \nabla.E$ (from the Gauss law)

$\frac{\sigma\rho_v}{\varepsilon} = \nabla.\sigma E \Rightarrow \frac{\sigma\rho_v}{\varepsilon} = \nabla.J$

From the continuity equation $\boldsymbol{\nabla.J = -\frac{\partial\rho_v}{\partial t}}$

$\frac{\partial\rho_v}{\partial t} = -\frac{\sigma\rho_v}{\varepsilon} \Rightarrow \frac{\partial\rho_v}{\rho_v} = -\frac{\sigma}{\varepsilon}\partial t$

$\rho_v = \rho_{v0}\, e^{\frac{-t}{T_r}}$

where $\boldsymbol{T_r = \frac{\varepsilon}{\sigma}}$ **is known as relaxation time or rearrangement time**

POISSON'S AND LAPLACE'S EQUATION:

The Maxwell's first equation is given by $\rho_v = \nabla.\mathbf{D}$

$\rho_v = \nabla.\varepsilon E$ $(\because D = \varepsilon E)$

$\rho_v = \varepsilon \nabla.E$

$\rho_v = \varepsilon \nabla.(-\nabla V)$ $(\because E = -\nabla V)$

$\frac{\rho_v}{\varepsilon} = -\nabla^2 V$ or

$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$ which is known as Poisson's law

A special case of the above equation occurs when $\rho_v = 0$ (i.e. for a free charge region)

$\nabla^2 V = 0$ which is known as Laplace's equation

Laplace's equation in Cartesian coordinate system as

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

POLARIZATION:

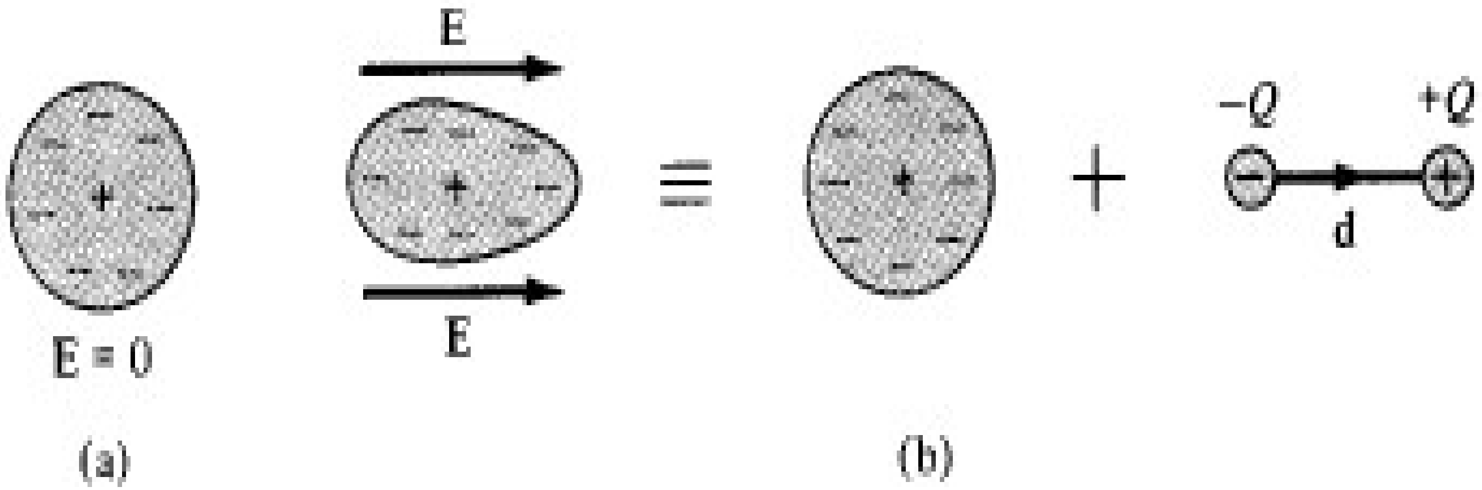


Figure 5.6 Polarization of a nonpolar atom or molecule.

- When an electric field E is applied, the positive charge is displaced from its equilibrium position in the direction of E by the force F+ = QE while the negative charge is displaced in the opposite direction by the force F_ = QE.
- A dipole results from the displacement of the charges and the dielectric is said to be polarized.
- In the polarized state, the electron cloud is distorted by the applied electric field E. This distorted charge distribution is equivalent, by the principle of superposition, to the original distribution plus a dipole whose moment is

$$P = Qd \quad \text{-----(1)}$$

where d is the distance vector from -Q to +Q of the dipole.

Dipole: An electric dipole is formed when two point charges of equal magnitude but opposite sign are separated by a small distance.

If there are N dipoles in a volume Δv of the dielectric, the total dipole moment due to the electric field is

$Q_1 d_1 + Q_2 d_2 +$ -----------$+ Q_N d_N = \sum_{k=1}^{N} Q_k d_k$ --------(2)

Polarization P: The dipole moment per unit volume of the dielectric(in C/m^2)

$$P = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} Q_k d_k}{\Delta v} \text{ -----------(3)}$$

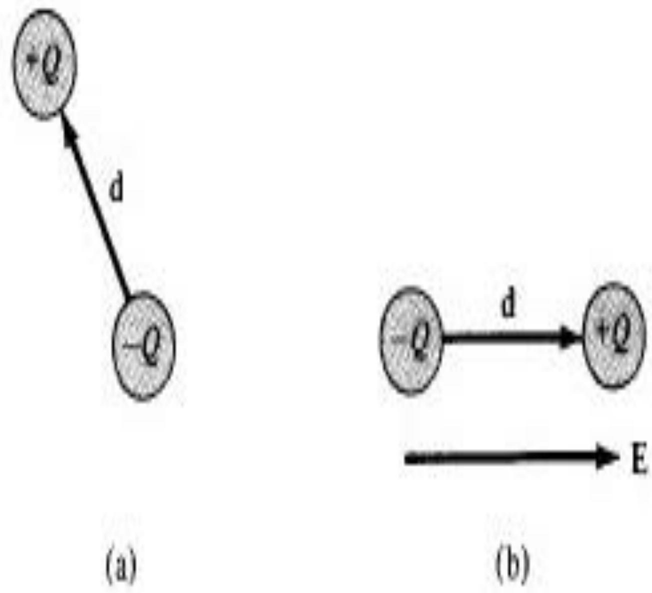


Figure 5.7 Polarization of a polar molecule: **(a)** permanent dipole (**E** = 0), **(b)** alignment of permanent dipole (**E** ≠ 0).

$$P = X_e \varepsilon_0 E$$

Where X_e is known as the electric Susceptibility of the material

DIELECTRIC CONSTANT & STRENGTH:

We know that **$P = X_e \varepsilon_0 E$ -------(1)**

$\mathbf{D} = \varepsilon_0 E + P$ ---------(2)

$D = \varepsilon_0 (1+X_e) E$

$D = \varepsilon_0 \varepsilon_r E$**--------(3)**

or $D = \varepsilon E$ -------(4)

where $\varepsilon = \varepsilon_0 \varepsilon_r$--------(5)

$\varepsilon_r = 1+X_e = \frac{\varepsilon}{\varepsilon_0}$ which is known as dielectric constant or relative permittivity

BOUNDARY CONDITIONS:

If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called boundary conditions.

we will consider the boundary conditions at an interface separating

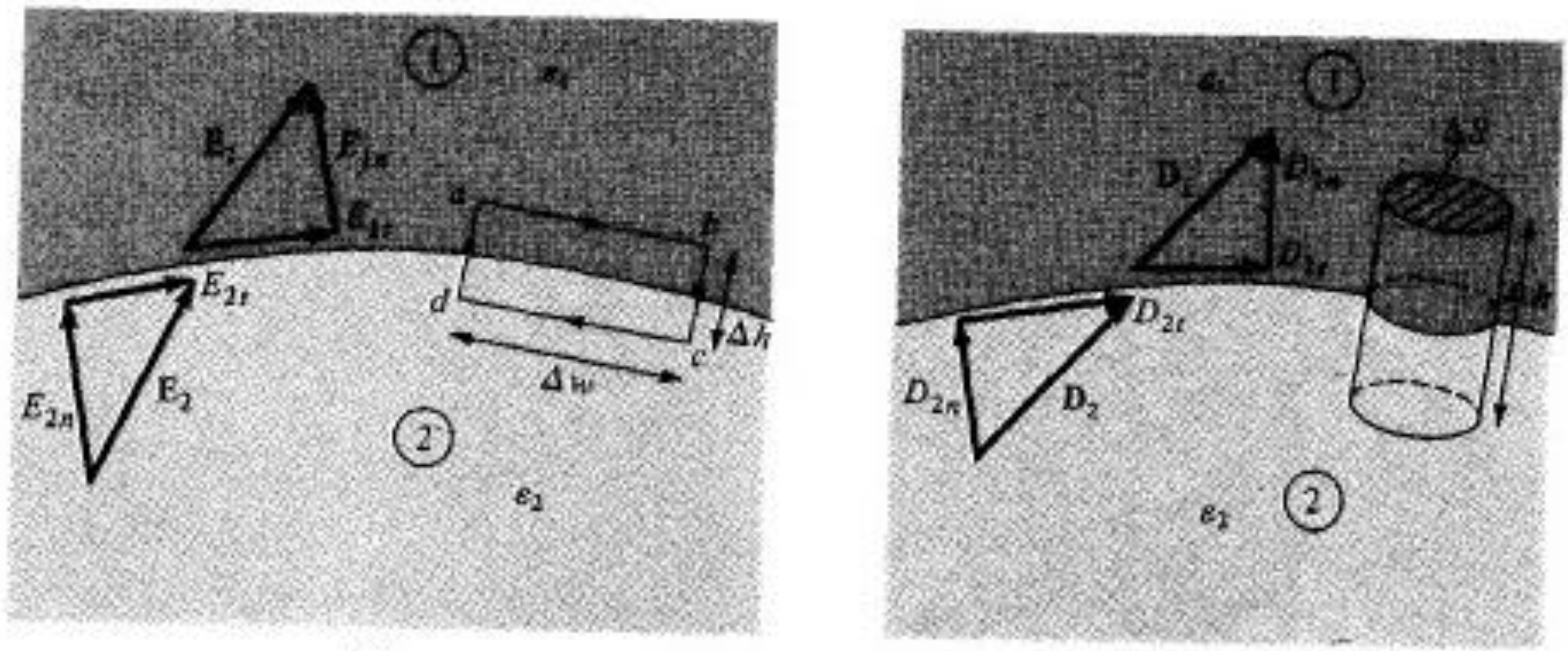
- dielectric (ε_{r1}) and dielectric (ε_{r2})
- conductor and dielectric
- conductor and free space

To determine the boundary conditions, we need to use Maxwell's equations:

$$\oint E.\mathrm{dl} = 0$$

$$\oint D.ds = Q_{enc}$$

DIELECTRIC – DIELECTRIC BOUNDARY CONDITIONS:



(a) (b)

Figure 5.10 Dielectric–dielectric boundary.

Fig a):determining $E_{1t}= E_{2t}$ b) determining $D_{1n} = D_{2n}$

We apply eq(3) to the closed path abcda of fig(a).

assuming that the path is very small with respect to the variation of E. We obtain as

$$0 = E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{1n}\frac{\Delta h}{2} + E_{2n}\frac{\Delta h}{2} \text{ -------(4)}$$

where $E_t = |E_t|$ and $E_n = |E_n|$

$$0 = (E_{1t} - E_{2t})\,\Delta w$$

As $\Delta h \to 0$,

$E_{1t} = E_{2t}$ ---------(5)

The tangential components of E are the same on the two sides of the boundary

In other words, E, undergoes no change on the boundary and it is said to be continuous across the boundary.

Since D = εE = $D_t + D_n$, eq. (5) can be written as

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

Where ρ_s is the free charge density placed deliberately at the boundary.

If no free charges exist at the interface $\rho_s = 0$,

$D_{1n} = D_{2n}$ ----------(9)

Thus the normal component of D is continuous across the interface; that is,

D_n undergoes no change at the boundary.

Since D = εE

$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$-----------(10)

the above equation tells us that normal component of E is discontinuous at the boundary.

The equations (5), (6), (9) & (10) are collectively called as boundary conditions.

they must be satisfied by an electric field at the boundary separating two different dielectrics.

we can use the boundary conditions to determine the "refraction" of the electric field across the interface. Consider D_1 or E_1, and D_2 or E_2 making angles θ_1 & θ_2 with the normal to the interface.

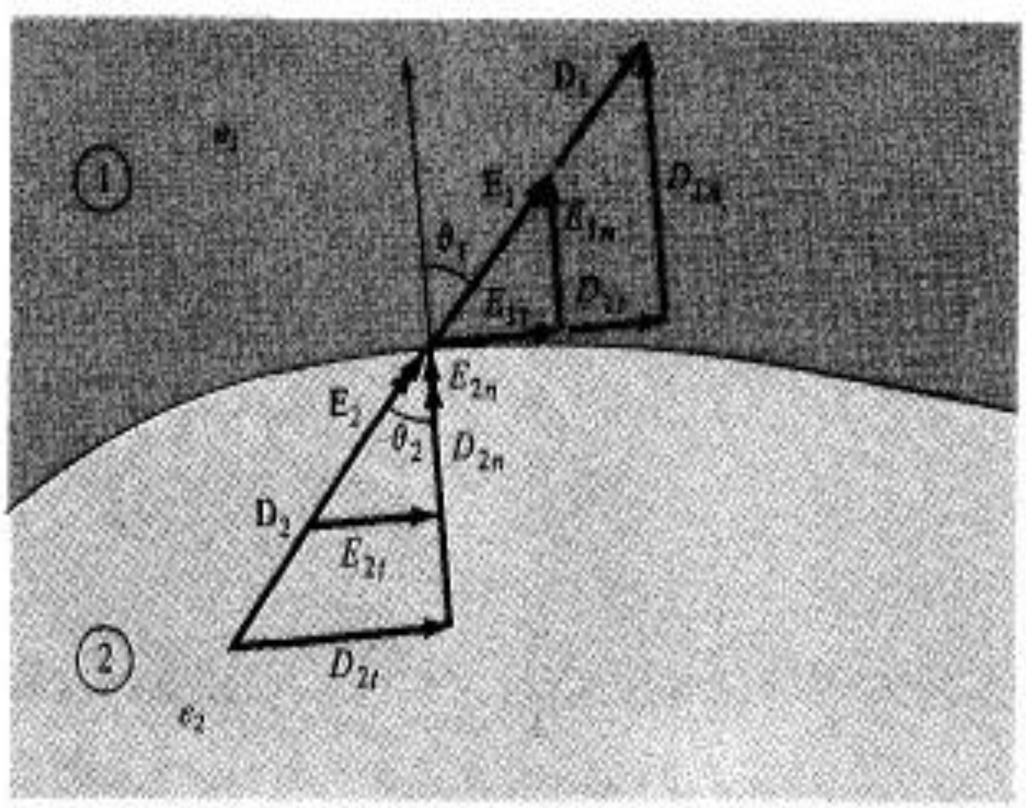


Figure 5.11 Refraction of **D** or **E** at a dielectric–dielectric boundary.

$$E_1 \sin\theta_1 = E_{1t} = E_{2t} = E_2 \sin\theta_2$$

or $\quad E_1 \sin\theta_1 = E_2 \sin\theta_2$ -----------(11)

$$\varepsilon_1 E_1 \cos\theta_1 = D_{1n} = D_{2n} = \varepsilon_2 E_2 \cos\theta_2$$

or $\quad \varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$ ----------(12)

$$\frac{E_1 \sin\theta_1}{\varepsilon_1 E_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{\varepsilon_2 E_2 \cos\theta_2}$$

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\boldsymbol{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$ ----------(13) $(\because\ \varepsilon_1 = \varepsilon_0\varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_0\varepsilon_{r2})$

Which is known as law of refraction of the electric field at a boundary free of charge

CONDUCTOR – DIELECTRIC BOUNDARY CONDITIONS:

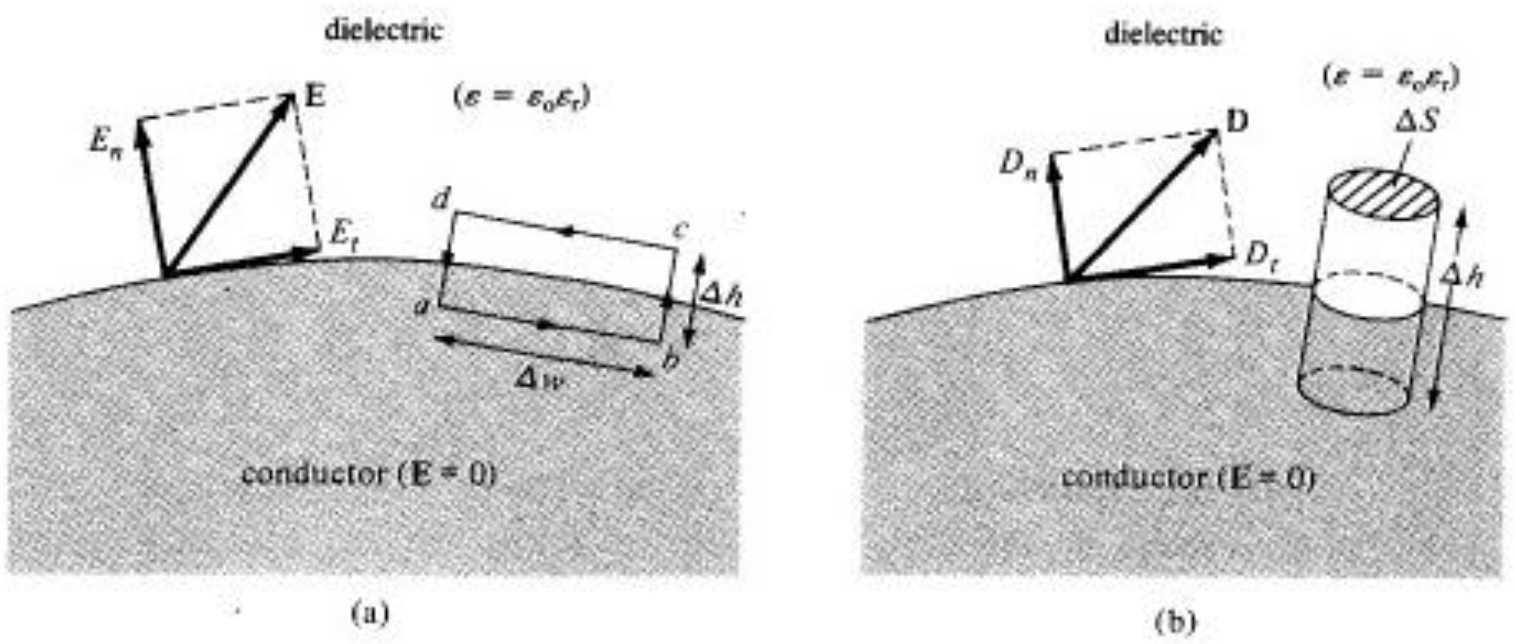


Figure 5.12 Conductor–dielectric boundary.

- The conductor is assumed to be perfect (i.e., $\sigma \rightarrow \infty$ or $\rho_c \rightarrow 0$).
- Although such a conductor is not practically realizable, we may regard conductors such as copper and silver as though they were perfect conductors.

To determine the boundary conditions for a conductor-dielectric interface, we follow the same procedure used for dielectric-dielectric interface except that we incorporate the fact that E = 0 inside the conductor.

we know that

$\oint E.\text{dl} = 0$ ------(1)

Applying eq (1) to the closed path abcda of Figure (a) gives

$$0 = 0* \Delta\text{w} + 0 * \frac{\Delta h}{2} + E_n \frac{\Delta h}{2} - E_t\Delta\text{w} - E_n \frac{\Delta h}{2} - 0 * \frac{\Delta h}{2}$$

as $\Delta\text{h} \rightarrow 0$, $E_t = 0$ -----(2)

we know that

$\oint D.ds = Q_{enc}$ -------(3)

Similarly, by applying eq. (3) to the cylindrical Gaussian surface of Figure

$$D_n = \rho_s$$

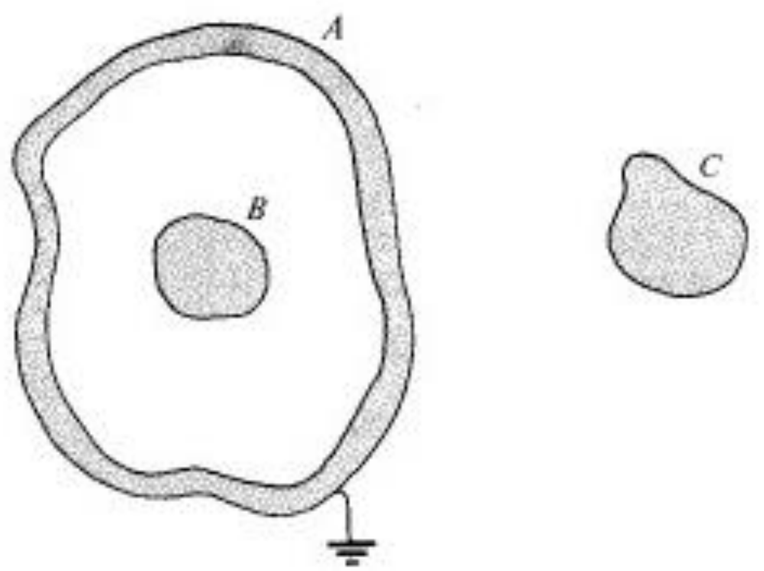


Figure 5.13 Electrostatic screening.

Thus under static conditions, the following conclusions can be made about a perfect conductor:

1. No electric field may exist within a conductor; that is,

$\rho_v = 0, \quad E = 0$ ------(6)

2. Since E= -Δv = 0, there can be no potential difference between any two points in the conductor; that is, a conductor is an equipotential body.

3. The electric field E can be external to the conductor and normal to its surface; that is

$D_t = \varepsilon_0 \varepsilon_r E_t = 0, \quad D_n = \varepsilon_0 \varepsilon_r E_n = \rho_s$ ----------(7)

An important application of the fact that E = 0 inside a conductor is in electrostatic screening or shielding.

CONDUCTOR – FREE SPACE BOUNDARY CONDITIONS:

The boundary conditions at the interface between a conductor and free space can be obtained from eq (7) by replacing ε_r by 1 (because free space as a special dielectric for which $\varepsilon_r = 1$).

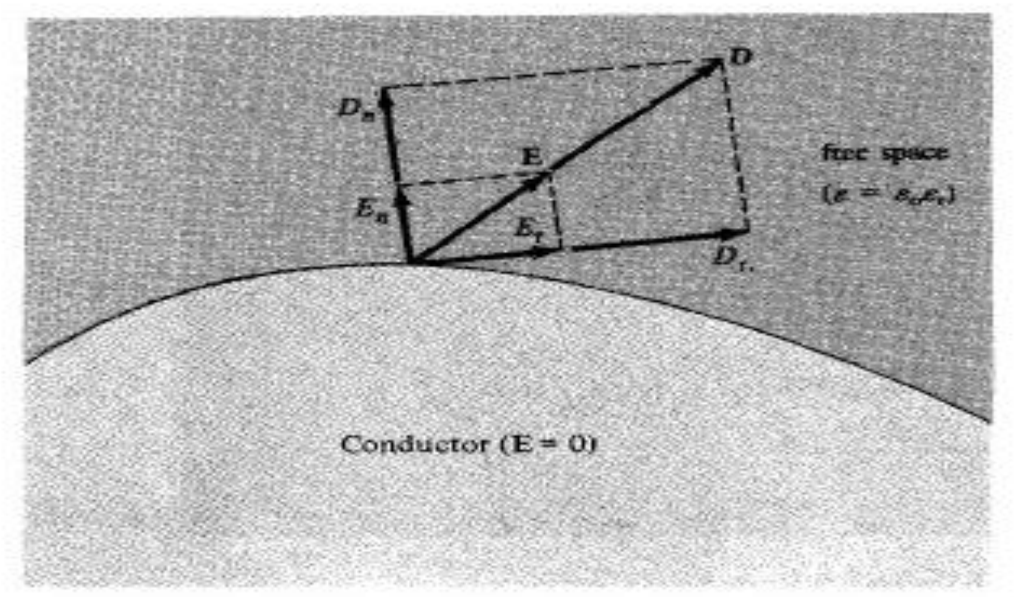


Figure 5.14 Conductor–free space boundary.

The electric field E to be external to the conductor and normal to its surface. Thus the boundary conditions are

$D_t = \varepsilon_0 E_t = 0, \quad D_n = \varepsilon_0 E_n = \rho_s$

RESISTANCE AND CAPACITANCE:

$$R = \frac{V}{I} = \frac{\int E.dl}{\oint \sigma E.ds}$$

$$(\because\ I = \oint J.ds \quad , J = \sigma E)$$

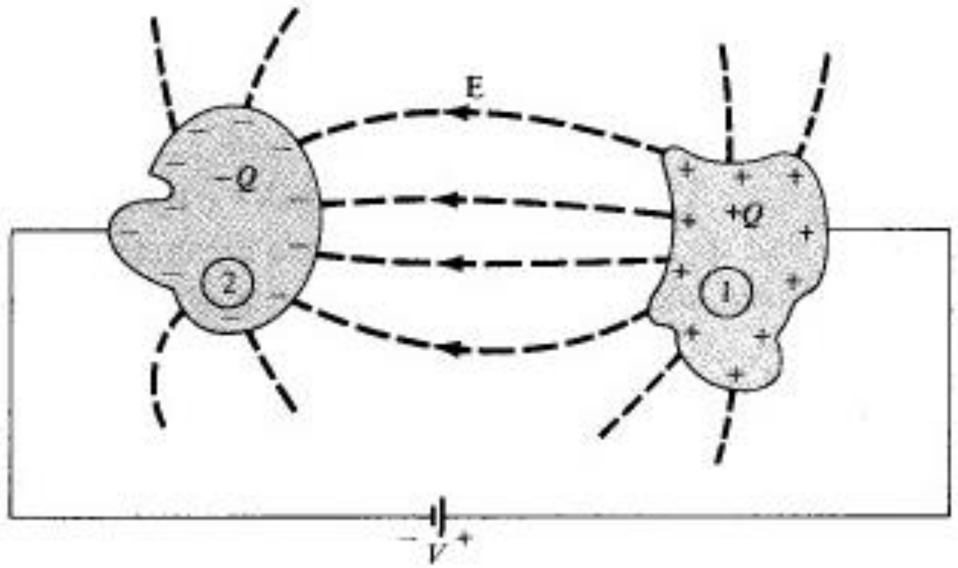


Figure 6.12 A two-conductor capacitor.

Consider the two-conductor capacitor of figure shown. The conductors are

PARALLEL PLATE CAPACITOR:

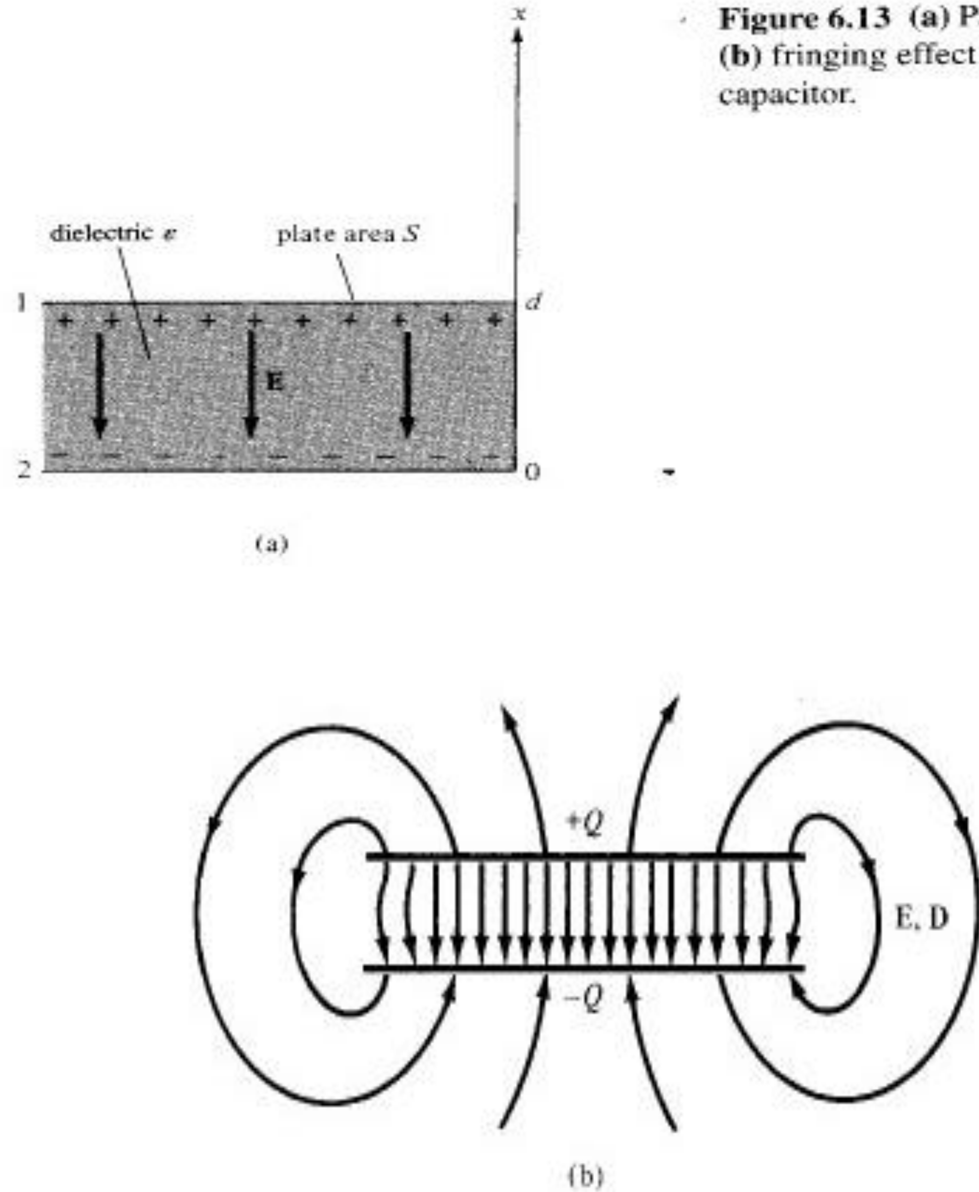


Figure 6.13 **(a)** Parallel-plate capacitor, **(b)** fringing effect due to a parallel-plate capacitor.

Consider the parallel-plate capacitor of fig shown, Suppose that each of the

From surface charge distribution,

D = $\frac{\rho_s}{2} a_z$ D = -$\rho_s a_x$ or

E = $\frac{\rho_s}{\varepsilon}(-a_x)$

E = - $\frac{Q}{\varepsilon S} a_x$ ----------(2)

V = - $\int_2^1 E . dl$ = - $\int_0^d (-\frac{Q}{\varepsilon S} a_x)$ dx a_x (or) V = $\frac{Qd}{\varepsilon S}$

For a parallel plate capacitor

C = $\frac{Q}{V} = \frac{\varepsilon S}{d}$ -------(3)

$\varepsilon_r = \frac{C}{C_0}$ ---------(4)

We know that $W_E = \int \varepsilon_0 E^2$dv --------(5)

the energy stored in a capacitor, $\boldsymbol{W_E}$ **= ½ C**$\boldsymbol{V^2}$**= ½ QV =** $\boldsymbol{\frac{Q^2}{2C}}$

COAXIAL CAPACITOR:

A coaxial capacitor is essentially a coaxial cable or coaxial cylindrical capacitor.

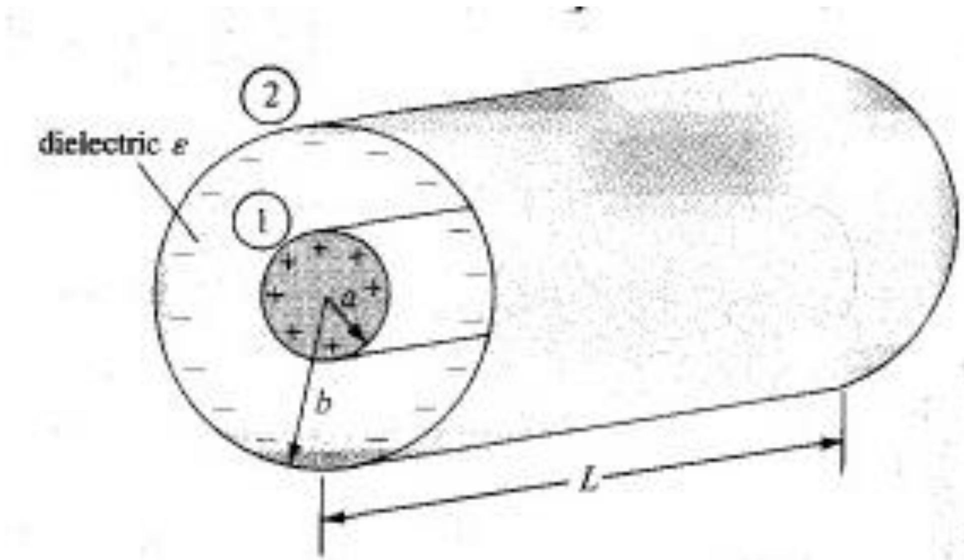


Figure 6.14 Coaxial capacitor.

- Consider length L of two coaxial conductors of inner radius a and outer radius b (b > a) .
- Let the space between the conductors be filled with a homogeneous dielectric with permittivity ε.

We assume that conductors 1 and 2, respectively, carry +Q and -Q uniformly distributed on them.

By applying Gauss's law to an arbitrary Gaussian cylindrical surface of radius ρ (a <ρ< b),

$$Q = \oint D.ds$$

$$= \varepsilon \oint E.ds = \varepsilon E_\rho 2\pi\rho L \quad (\because D = \varepsilon E)$$

Hence $E = \frac{Q}{\varepsilon\, 2\pi\rho L} a_\rho$ --------(1)

Neglecting flux fringing at the cylinder ends,

$$V = -\int_2^1 E.dl = -\int_b^a \left(\frac{Q}{\varepsilon\, 2\pi\rho L} a_\rho\right).d\rho\, a_\rho$$

$$V = \frac{Q}{2\pi\varepsilon L} \ln\frac{b}{a}$$

The capacitance of a coaxial cylinder is given by

$$\mathbf{C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{ln\frac{b}{a}}}$$

SPHERICAL CAPACITOR:

- A spherical capacitor is the case of two concentric spherical conductors.
- Consider the inner sphere of radius a and outer sphere of radius b (b> a) separated by a dielectric medium with permittivity ε .

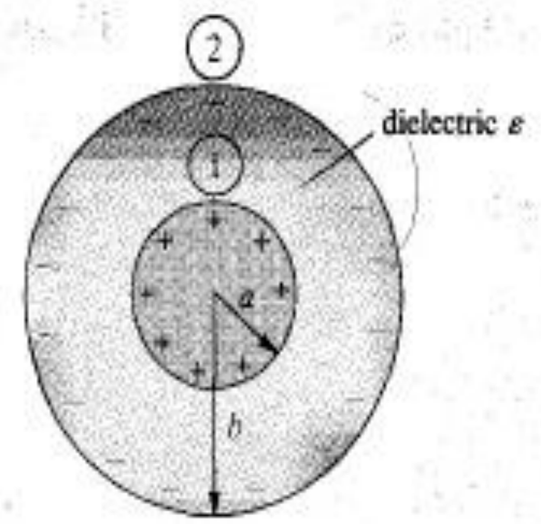


Figure 6.15 Spherical capacitor.

- We assume charges +Q and -Q on the inner and outer spheres
-

The capacitance of the spherical capacitor is

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}} \text{ let } b \rightarrow \infty, \quad C = 4\pi\varepsilon a$$

which is the capacitance of a spherical capacitor whose outer plate is infinitely large.

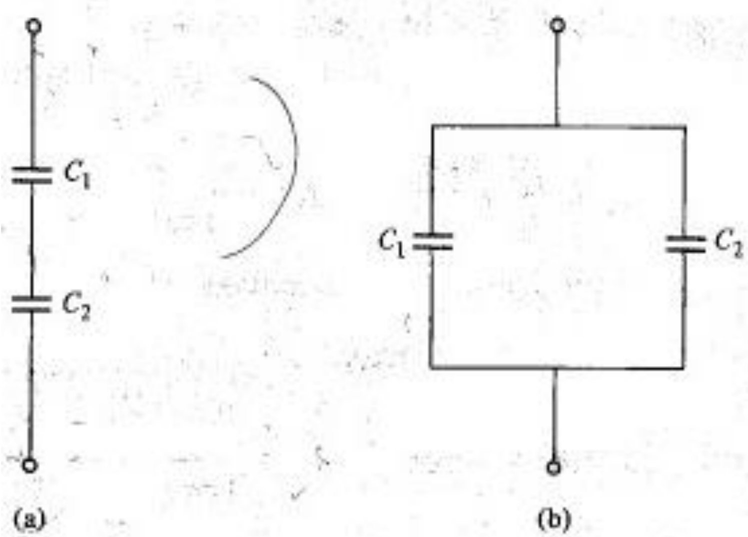


Figure 6.16 Capacitors in **(a)** series, and **(b)** parallel.

Two capacitors with capacitance C1 and C2 are in parallel, the total capacitance is

$$C = C_1 + C_2$$

Two capacitors with capacitance C1 and C2 are in series, the total capacitance is,

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$R = \frac{V}{I} = \frac{\int E.dl}{\oint \sigma E.ds}$$

$$C = \frac{Q}{V} = \frac{\varepsilon \oint E.ds}{\int E.dl}$$

$RC = \frac{\varepsilon}{\sigma}$ which is the relaxation time, T_r

For a parallel plate capacitor,

MAGNETOSTATICS

OBJECTIVES

AMPERE'S LAW OF FORCE

MAGNETIC FLUX DENSITY

LORENTZ FORCE

BIOT-SAVART 'S LAW

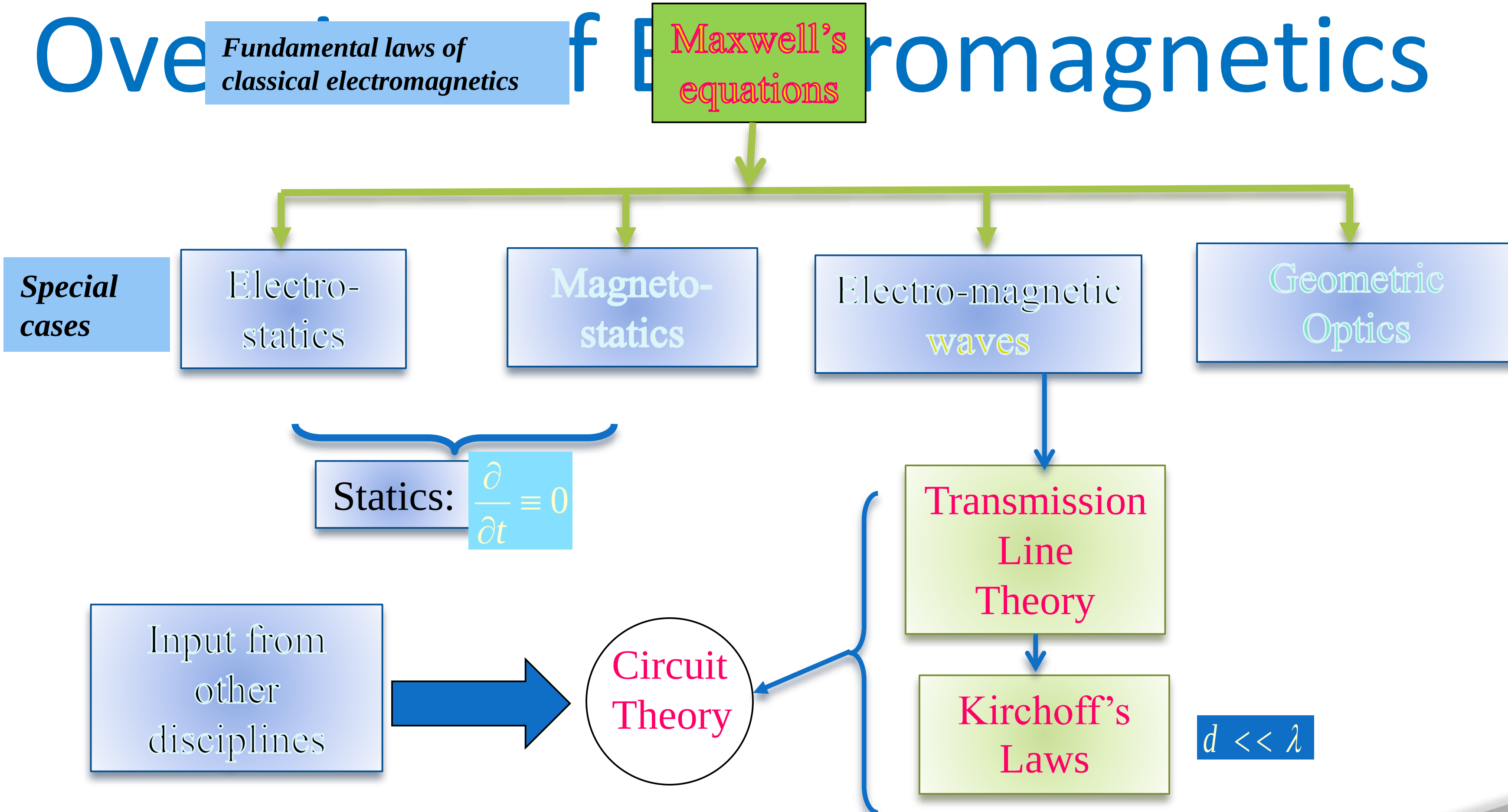
APPLICATIONS OF AMPERE'S LAW IN INTEGRAL FORM

VECTOR MAGNETIC POTENTIAL

MAGNETIC DIPOLE

MAGNETIC FLUX

Ove f E romagnetics



INTRODUCTION:

- An electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic (or magneto static) field is produced.
- A magneto static field is produced by a constant current flow (or direct current). This current flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in vacuum tubes, or conduction currents as in current-carrying wires.
- In this chapter, we consider magnetic fields in free space due to direct current. Magneto static fields in material space.

TABLE 7.1 Analogy between Electric and Magnetic Fields*

Term	Electric	Magnetic
Basic laws	$\mathbf{F} = \dfrac{Q_1 Q_2}{4\pi\varepsilon r^2}\mathbf{a}_r$	$d\mathbf{B} = \dfrac{\mu_o I\, d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2}$
	$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$	$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$
Force law	$\mathbf{F} = Q\mathbf{E}$	$\mathbf{F} = Q\mathbf{u} \times \mathbf{B}$
Source element	dQ	$Q\mathbf{u} = I\, d\mathbf{l}$
Field intensity	$E = \dfrac{V}{\ell}$ (V/m)	$H = \dfrac{I}{\ell}$ (A/m)
Flux density	$\mathbf{D} = \dfrac{\Psi}{S}$ (C/m^2)	$\mathbf{B} = \dfrac{\Psi}{S}$ (Wb/m^2)
Relationship between fields	$\mathbf{D} = \varepsilon\mathbf{E}$	$\mathbf{B} = \mu\mathbf{H}$
Potentials	$\mathbf{E} = -\nabla V$	$\mathbf{H} = -\nabla V_m \; (\mathbf{J} = 0)$
	$V = \displaystyle\int \frac{\rho_L dl}{4\pi\varepsilon r}$	$\mathbf{A} = \displaystyle\int \frac{\mu I\, d\mathbf{l}}{4\pi R}$
Flux	$\Psi = \int \mathbf{D} \cdot d\mathbf{S}$	$\Psi = \int \mathbf{B} \cdot d\mathbf{S}$
	$\Psi = Q = CV$	$\Psi = LI$
	$I = C\dfrac{dV}{dt}$	$V = L\dfrac{dI}{dt}$
Energy density	$w_E = \dfrac{1}{2}\mathbf{D} \cdot \mathbf{E}$	$w_m = \dfrac{1}{2}\mathbf{B} \cdot \mathbf{H}$
Poisson's equation	$\nabla^2 V = -\dfrac{\rho_v}{\varepsilon}$	$\nabla^2 \mathbf{A} = -\mu\mathbf{J}$

BIOT-SAVART'S LAW:

"Biot-Savart's law states that the magnetic field intensity dH produced at a point P, by the differential current element Idl is proportional to the product Idl and the sine of the angle α between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element."

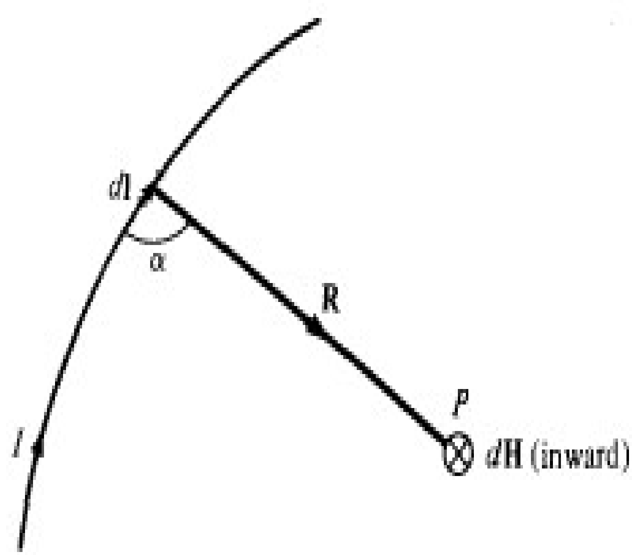


Figure 7.1 magnetic field $d\mathbf{H}$ at P due to current element $I\,d\mathbf{l}$.

That is,

$$\mathrm{dH} \propto \frac{Idl\sin\alpha}{R^2} \qquad \text{------(1)}$$

$$\mathrm{dH} = \frac{KIdl\sin\alpha}{R^2} \qquad \text{------(2)}$$

where k is the constant of proportionality. In SI units, k = l/4π, so eq. (2) becomes

$$\mathrm{dH} = \frac{KIdl\sin\alpha}{4\pi R^2} \qquad \text{-------(3)}$$

$$\mathrm{dH} = \frac{Idl \times a_R}{4\pi R^2} = \frac{Idl \times \mathbf{R}}{4\pi R^3} \qquad \text{-----(4)}$$

Right Hand Rule and Right Hand Screw Rule:

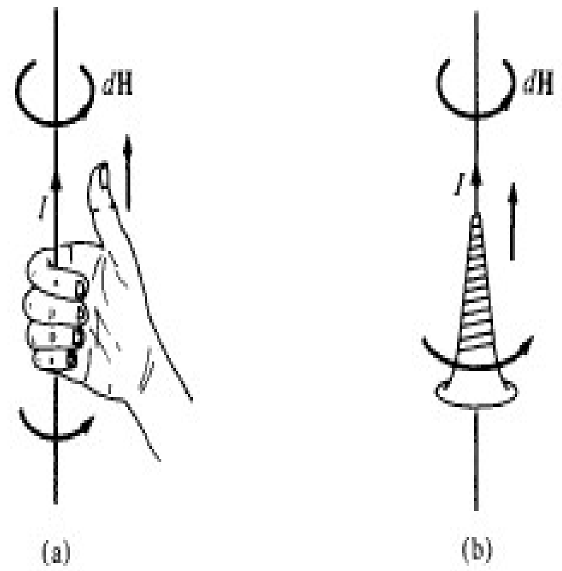


Figure 7.2 Determining the direction of $d\mathbf{H}$ using (a) the right-hand rule, or (b) the right-handed screw rule.

The direct ... ht-hand
thumb poi ... rcling the
wire in the direction of dH.

Different charge configurations and we can have different current distributions:

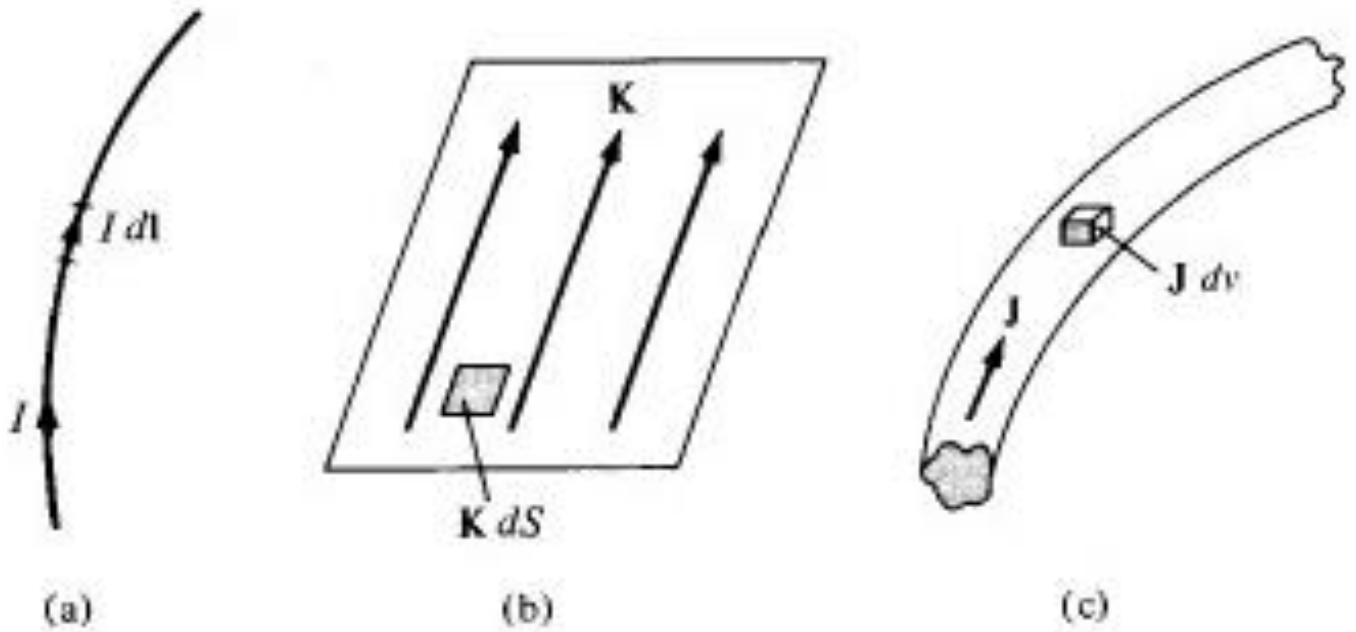


Figure 7.4 Current distributions: **(a)** line current, **(b)** surface current, **(c)** volume current.

If we define K as the surface current density and J as the volume current density ,the source elements are related as

$$\mathbf{Idl} \equiv \mathbf{Kds} \equiv \mathbf{Jdv} \quad \text{------(5)}$$

Thus in terms of the distributed current sources, the Biot-Savart law becomes

$$\mathrm{H} = \int_L \frac{Idl \times a_R}{4\pi R^2} \qquad \text{(line current)} \qquad \text{-------(6)}$$

$$\mathrm{H} = \int_S \frac{Kds \times a_R}{4\pi R^2} \qquad \text{(surface current)} \qquad \text{-------(7)}$$

$$\mathrm{H} = \int_v \frac{Jdv \times a_R}{4\pi R^2} \qquad \text{(volume current)} \qquad \text{-------(8)}$$

EXAMPLE:

The field due to a straight current carrying filamentary conductor of finite length AB.

We assume that the conductor is along the z-axis with its upper and lower ends respectively subtending angles α_2 and α_1 at P, the point at which H is to be determined.

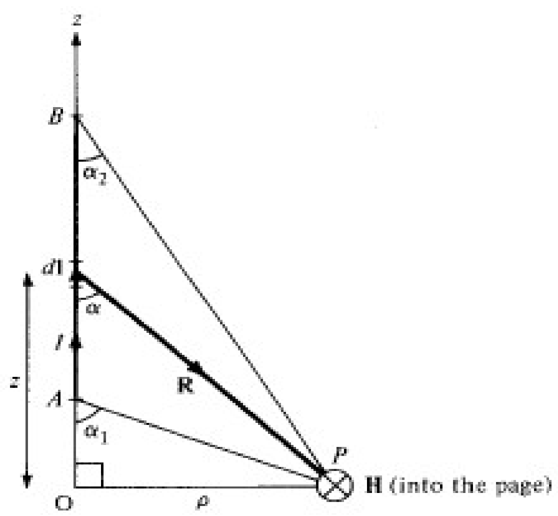


Figure 7.5 Field at point P due to a straight filamentary conductor.

If we consider the contribution dH at P due to an element dl at (0, 0, z),

$$dH = \frac{I dl \times \mathbf{R}}{4\pi R^3} \quad \text{---------(9)}$$

But dl = dz a_z and R = $\rho a_\rho - z\, a_z$

$$dl \times R = \rho dz a_\emptyset \quad \text{----------(10)}$$

$$H = \int \frac{I\rho dz a_\emptyset}{4\pi(\rho^2 + Z^2)^{\frac{3}{2}}} \quad \text{----------(11)}$$

Letting z = ρ cot α, dz = - $\rho\ cosec^2\ \alpha$ dα

$$H = -\frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2 cosec^2\alpha\, d\alpha}{\rho^3 cosec^3\alpha}$$

$$H = -\frac{I}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \sin\alpha \, d\alpha$$

(or) $$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, a_\phi \quad \text{--------(12)}$$

As a special case, when the conductor is semi-infinite (with respect to P) so that point A is now at O (0, 0, 0) while B is at (0, 0,∞); $\alpha_1 = 90°$, $\alpha_2 = 0°$, and eq. (12) becomes

$$\mathbf{H} = \frac{\mathbf{I}}{\mathbf{4\pi\rho}}\, \boldsymbol{a}_\phi \quad \text{--------(13)}$$

Another special case is when the conductor is infinite in length. For this case, point A is at (0, 0, -∞) while B is at (0, 0, ∞); $\alpha_1 = 180°$, $\alpha_2 = 0°$, so eq. (12) reduces to

$$\mathbf{H} = \frac{\mathbf{I}}{\mathbf{2\pi\rho}}\, \boldsymbol{a}_\phi \quad \text{--------(14)}$$

AMPERE'S CIRCUIT LAW: MAXWELL'S EQUATION:

Ampere's circuit law states that the line integral of the tangential component of H around a dosed path is the same as the net current I_{enc} enclosed by the path.

In other words, the circulation of H equals I_{enc}. that is,

$$\oint H.dl = I_{enc} \quad \text{--------(1)}$$

By applying Stoke's theorem to the left-hand side of eq. (1), we obtain

$$I_{enc} = \oint_L H.dl = \int_S (\nabla \times H).dS \quad \text{----------(2)}$$

But $$I_{enc} = \int_S J.dS \quad \text{----------(3)}$$

Comparing the surface integrals in eqs. (2) and (3) clearly reveals that

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} \quad \text{----------(4)}$$

∇X H = J $\neq$ 0; that is, magneto static field is not conservative.

APPLICATIONS OF AMPERE'S LAW:

We will consider an infinite line current, an infinite current sheet, and an infinitely long coaxial transmission line.

Infinite Line Current:

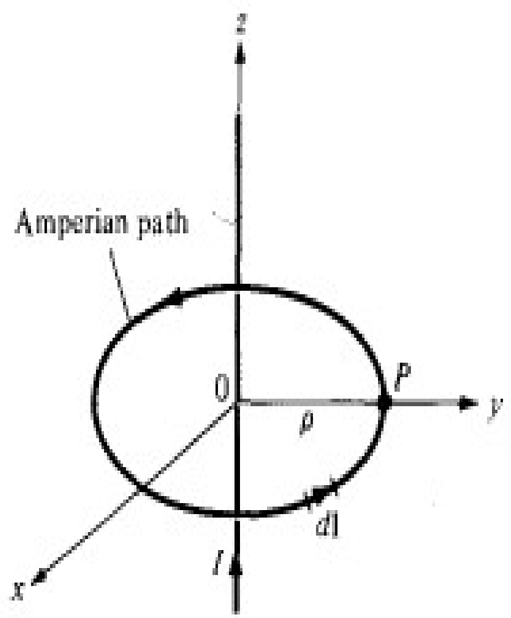


Figure 7.10 Ampere's law applied to an infinite filamentary line current.

- Consider an infinitely long filamentary current I along the z-axis as shown in the above figure.
- To determine H at an observation point P, we allow a closed path pass through P. This path, on which Ampere's law is to be applied,is known as an Amperian path .
- We choose a concentric circle as the Amperian path, which shows that H is constant provided p is constant. Since this path encloses the whole current I
- According to Ampere's law,

$$I = \int H_{\emptyset} a_{\emptyset} . \rho d\emptyset a_{\emptyset} = H_{\emptyset} \int \rho d\emptyset = H_{\emptyset}\ 2\pi\rho$$

Or $\quad H = \frac{NI}{2\pi\rho} \quad$ for $\rho_0 - a < \rho < \rho_0 + a$

An approximate value of $\quad H_{approx} = \frac{NI}{2\pi\rho_0} = \frac{NI}{l}$

INFINITE SHEET OF CURRENT:

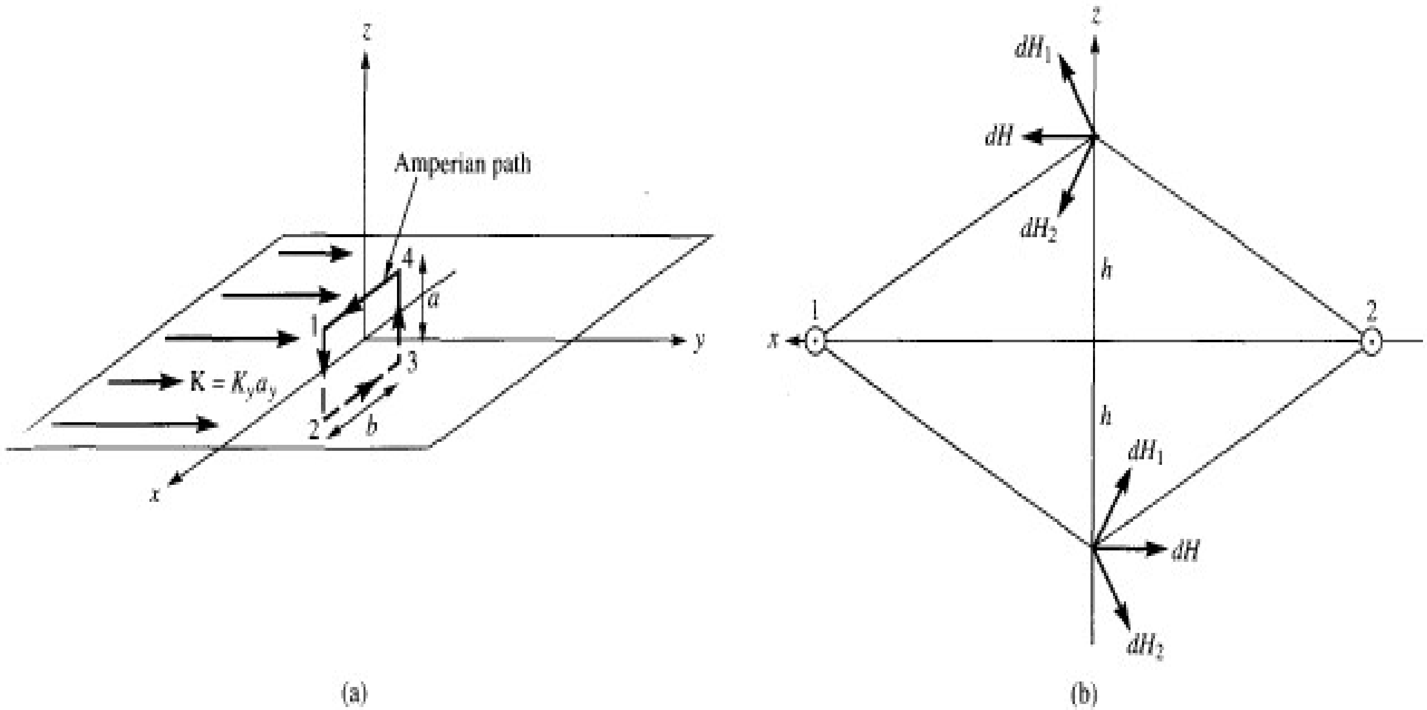


Figure 7.11 Application of Ampere's law to an infinite sheet: (**a**) closed path 1-2-3-4-1, (**b**) symmetrical pair of current filaments with current along $\mathbf{a}_y$.

- Consider an infinite current sheet in the z = 0 plane. If the sheet has a uniform current Density K = $K_y a_y$ A/m as shown in above Figure.
- By applying Ampere's law to the rectangular closed path (Amperian path) gives $\oint H.dl = I_{enc} = K_y b$ ----------(1)
- The resultant dH has only an x-component. Also, H on one side of the sheet is the negative of that on the other side.
- Due to the infinite extent of the sheet, the sheet can be regarded as consisting of such filamentary pairs so that the characteristics of H for a pair are the same for the infinite current sheets, that is,

$$H = \begin{cases} H_0 a_x & z > 0 \\ -H_0 a_x & z < 0 \end{cases} \quad \text{-------------(2)}$$

Evaluating the line integral of H in eq. (2) along the closed path in Figure (a) gives

$$\oint H.dl = (\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1) H.dl$$

$$= 0(-a) + (-H_0)(-b) + 0(a) + H_0 b$$

$$\oint H.dl = 2H_0 b \qquad \text{-------------(3)}$$

From eqs. (1) and (3), we obtain

$$H_0 = ½ K_y$$

Substituting H_0 in eq. (2) gives

$$H = \begin{cases} ½ K_y a_x & z>0 \\ -½ K_{ya_x} & z<0 \end{cases}$$

In general, for an infinite sheet of current density K A/m,

$$\mathbf{H} = ½\, \mathbf{K} \times \boldsymbol{a_n}$$

where a_n is a unit normal vector directed from the current sheet to the point of interest.

INFINITELY LONG COAXIAL TRANSMISSION LINE:

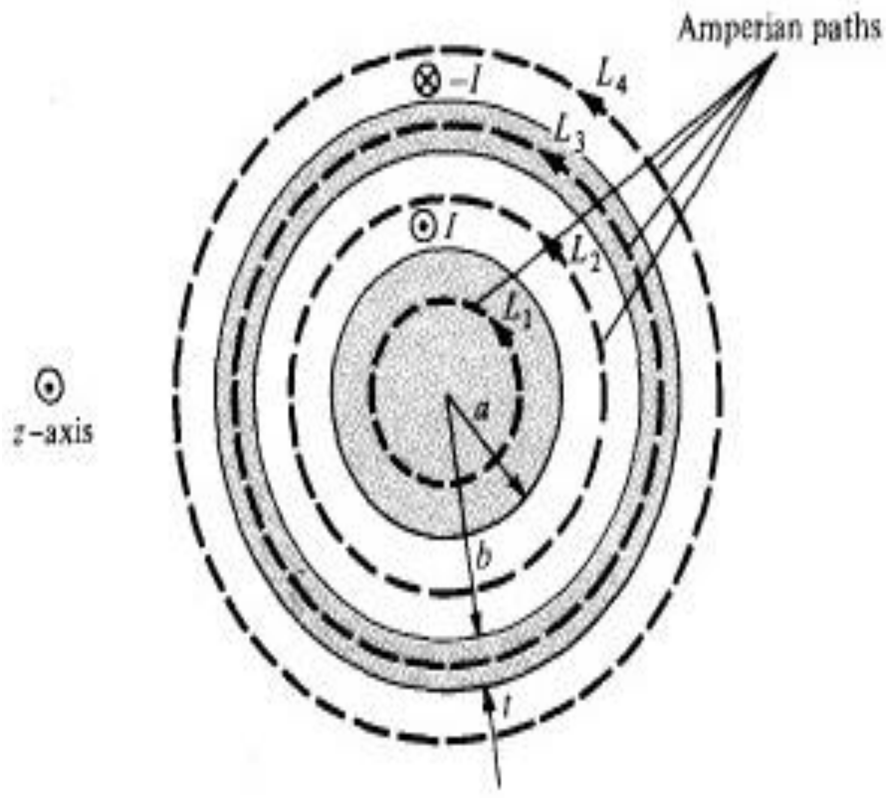


Figure 7.12 Cross section of the transmission line; the positive *z*-direction is out of the page.

- Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis.
- The cross section of the line is shown in above Figure. where the z-axis is out of the page.
- The inner conductor has radius a and carries current I while the outer conductor has inner radius b and thickness t and carries return current -I.
- To determine H everywhere assuming that current is uniformly distributed in both conductors. Since the current distribution is symmetrical, we apply Ampere's law along the Amperian path for each of the four possible regions:
- $0 \leq \rho \leq a$, $a \leq \rho \leq b$, $b \leq \rho \leq b+t$ & $\rho \geq b+t$

For region $0 \leq \rho \leq a$, we apply ampere's law to path L_1 is,

$$I_{enc} = \oint_{L_1} H.dl = \int J.dS \quad \text{------------(1)}$$

Since the current is uniformly distributed over the cross section,

$$\mathrm{J} = \frac{I}{\pi a^2} a_z , \qquad \mathrm{dS} = \rho d\rho \, d\emptyset \, a_z$$

$$I_{enc} = \int J.dS = \frac{I}{\pi a^2} \iint \rho d\rho \, d\emptyset = \frac{I}{\pi a^2} \pi \rho^2 = \frac{I\rho^2}{a^2}$$

Hence, eq(1) becomes,

$$H_\emptyset \int dl = H_\emptyset \, 2\pi\rho = \frac{I\rho^2}{a^2}$$

Or $$\boldsymbol{H_\emptyset = \frac{I\rho}{2\pi a^2}} \quad \text{--------(I)}$$

For region a$\leq$ $\boldsymbol{\rho}$ $\leq \boldsymbol{b}$, we use the $\boldsymbol{L_2}$ is amperian path,

$$I_{enc} = \oint_{L_1} H.dl = \mathrm{I}$$

$$H_{\emptyset}\, 2\pi\rho = \mathrm{I}$$

or $$\boldsymbol{H_{\emptyset} = \frac{I}{2\pi\rho}} \quad \text{-----(II)}$$

Since the whole current I is enclosed by $\boldsymbol{L_2}$.

For region b$\leq$ $\boldsymbol{\rho}$ $\leq \boldsymbol{b + t}$, we use the $\boldsymbol{L_3}$ is amperian path,

$$I_{enc} = \oint_{L_3} H.dl = H_{\emptyset}\, 2\pi\rho$$

Where $I_{enc} = \mathrm{I} + \int J.dS$

J is the current density (current per unit area) of the outer conductor and is along $-a_z$ that is,

$$J = \frac{I}{\pi[\left((b+t)^2\right)-b^2]}\, a_z$$

Thus, $I_{enc} = I - \frac{I}{\pi[\left((b+t)^2\right)-b^2]}\int_{\emptyset=0}^{2\pi} d\emptyset \int_{\rho=b}^{\rho} \rho d\rho = I\left[1 - \frac{\rho^2-b^2}{t^2+2bt}\right]$

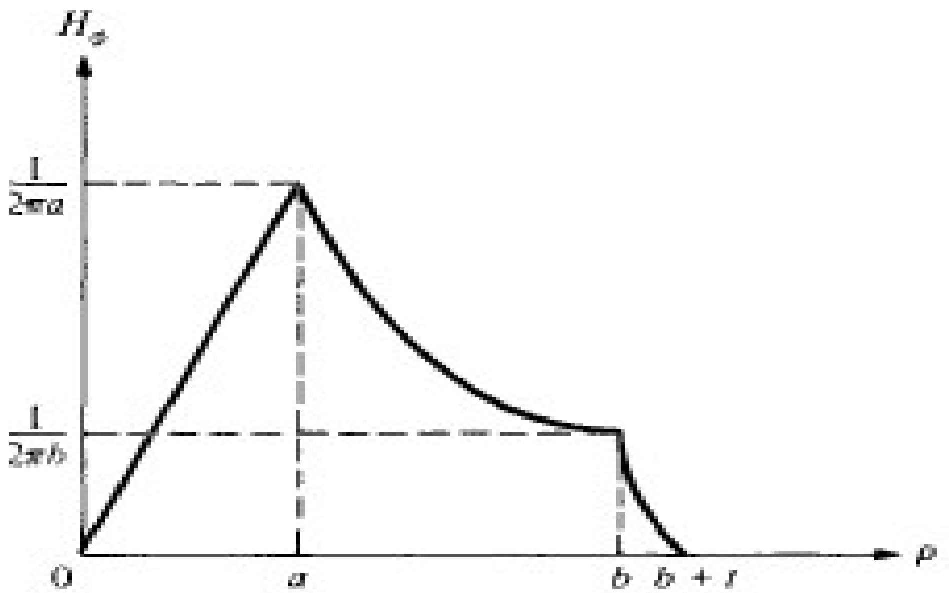
$$\boldsymbol{H_\emptyset = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2-b^2}{t^2+2bt}\right]} \quad \textbf{---------(III)}$$

For region $\rho \geq$ b+t we use the L_4 is amperian path,

$$\oint_{L_4} H.dl = I - I = 0$$

Or $\boldsymbol{H_\emptyset = 0}$ **------------(IV)**

$$\mathbf{H} = \begin{cases} \dfrac{I\rho}{2\pi a^2}\, a_\phi\,; & 0 \le \rho \le a, \\ \dfrac{I}{2\pi\rho}\, a_\phi; & a \le \rho \le b, \\ \dfrac{I}{2\pi\rho}\left[1 - \dfrac{\rho^2 - b^2}{t^2 + 2bt}\right] a_\phi; & b \le \rho \le b + t \\ 0; & \rho \ge b + t \end{cases}$$



MAGNETIC FLUX DENSITY—MAXWELL'S EQUATION:

The magnetic flux density B is similar to the electric flux density D.As D = εE in free space, the magnetic flux density B is related to the magnetic field intensity H according to

$$B = \mu_0 \; H \qquad \text{----------(1)}$$

where μ_0 is a constant known as the permeability of free space.

The constant is in henrys/meter (H/m) and has the value of

$$\mu_0 = 4\pi \text{ X } 10^{-7} \text{ H/m} \qquad \text{----------(2)}$$

The magnetic flux through a surface S is given by,

$$\psi = \int_S B.\,dS = Q \qquad \text{-----------(1)}$$

The magnetic flux line is the path to which B is tangential at every point in a magnetic field. It is the line along which the needle of a magnetic compass will orient itself if placed in the magnetic field.

For example, the magnetic flux lines due to a straight long wire are shown in below Figure.

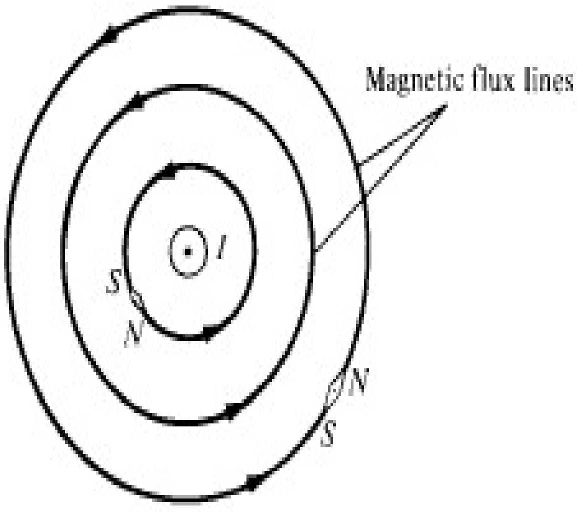


Figure 7.16 Magnetic flux lines due to a straight wire with current coming out of the page.

In an electrostatic field, the flux passing through a closed surface is the same as the charge enclosed; that is, $\psi = \oint D.\, dS = \mathrm{Q}.$

Thus it is possible to have an isolated electric charge as shown in below Figure (a), which also reveals that electric flux lines are not necessarily closed.

Unlike electric flux lines, magnetic flux lines always close upon themselves as in Figure(b). This is due to the fact that it is not possible to have isolated magnetic charges.

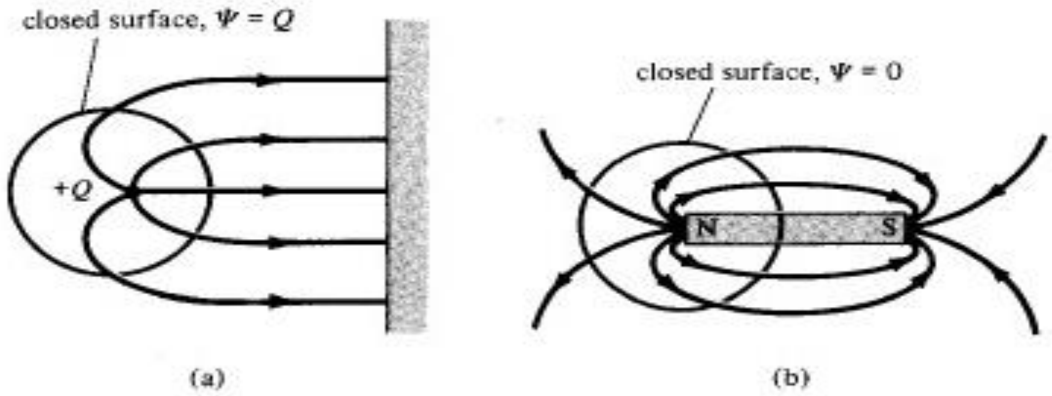


Figure 7.17 Flux leaving a closed surface due to: **(a)** isolated electric charge $\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S} = Q$. **(b)** magnetic charge, $\Psi = \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$.

An isolated magnetic charge does not exist.

Thus the total flux through a closed surface in a magnetic field must be zero;

that is, $\oint B.dS = 0$

This equation is referred to as the law of conservation of magnetic flux or Gauss's law for Magneto static fields just as

$\oint D.dS = \mathrm{Q}$ is Gauss's law for electrostatic fields.

By applying the divergence theorem to eq. (7.33), we obtain

$$\oint_S B.dS = \int_V \nabla.B dv = 0$$

$$\boldsymbol{\nabla.B = 0}$$

This equation is the fourth Maxwell's equation to be derived.

MAXWELL'S EQUATIONS FOR STATIC EM FIELDS:

Having derived Maxwell's four equations for static electromagnetic fields, we may take a moment to put them together as in Table.

TABLE 7.2 Maxwell's Equations for Static EM Fields

Differential (or Point) Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of magnetic monopole
$\nabla \times \mathbf{E} = 0$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = 0$	Conservativeness of electrostatic field
$\nabla \times \mathbf{H} = \mathbf{J}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$	Ampere's law

MAGNETIC SCALAR AND VECTOR POTENTIALS:

The electric potential V to the electric field intensity E ($E = -\nabla V$).

To define V_m and A involves recalling two important identities:

$$\nabla \times (\nabla V) = 0 \qquad \& \qquad \nabla . (\nabla \times A) = 0$$

which must always hold for any scalar field V and vector field A.

Just as $E = -\nabla V$,

we define the magnetic scalar potential V_m (in amperes) as related to H according to

$$H = -\nabla V_m \quad (\text{If } J = 0)$$

$$J = \nabla \times H = \nabla \times (-\nabla V_m) = 0$$

Thus the magnetic scalar potential V_m is only defined in a region where J = 0

V_m satisfies Laplace's equation just as V does for electrostatic fields;

Hence, $\nabla^2 V_m = 0 \quad (J = 0)$

We know that for a magneto static field, $\nabla \cdot B = 0$.

We can define the vector magnetic potential A (in Wb/m) such that

$$B = \nabla \times A$$

Just as we defined, $V = \int \frac{dQ}{4\pi\varepsilon_0 r}$

$$A = \int_L \frac{\mu_0 I dl}{4\pi R} \qquad \text{for line current}$$

$$A = \int_S \frac{\mu_0 K dS}{4\pi R} \qquad \text{for surface current}$$

$$A = \int_v \frac{\mu_0 J dv}{4\pi R} \qquad \text{for volume current}$$

FORCES DUE TO MAGNETIC FIELDS:

There are at least three ways in which force due to magnetic fields can be experienced.

The force can be

(a) due to a moving charged particle in a B field,

(b) on a current element in an external B field, or

(c) between two current elements.

A FORCE ON A CHARGED PARTICLE:

The electric force F_e on a stationary or moving electric charge Q in an electric field is given by Coulomb's experimental law and is related to the electric field intensity E as

$$F_e = QE$$

This shows that if Q is positive, F_e and E have the same direction.

A magnetic field can exert force only on a moving charge. From experiments, it is found that the magnetic force F_m experienced by a charge Q moving with a velocity u in a magnetic field B is

$$F_m = Qu \times B$$

This clearly shows that F_m is perpendicular to both u and B.

F_e is independent of the velocity of the charge and can perform work on the charge and change its kinetic energy.

F_m cannot perform work because it is at right angles to the direction of motion of the charge

$$(F_m \bullet \mathrm{dl} = 0)$$

The magnitude of F_m is generally small compared to F_e except at high velocities.

For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by, $\mathrm{F} = F_e + F_m$

Or $$\mathbf{F = Q\,(E + u \times B)}$$

This is known as the Lorentz force equation.

If the mass of the charged particle moving in E and B fields is m, by Newton's second law of motion.

$$F = m\frac{du}{dt} = Q\,(E + u \times B)$$

The solution to this equation is important in determining the motion of charged particles in E and B fields. We should bear in mind that in such fields, energy transfer can be only by means of the electric field.

TABLE 8.1 Force on a Charged Particle

State of Particle	E Field	B Field	Combined E and B Fields
Stationary	$Q\mathbf{E}$	—	$Q\mathbf{E}$
Moving	$Q\mathbf{E}$	$Q\mathbf{u} \times \mathbf{B}$	$Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$

FORCE ON A CURRENT ELEMENT:

To determine the force on a current element I dl of a current-carrying conductor due to the magnetic field B,

$$J = \rho_v u \qquad \text{----------(1)}$$

we recall the relationship between current elements:

$$Idl = KdS = Jdv \qquad \text{------------(2)}$$

Combining eqs. (1) and (2) yields,

$$Idl = \rho_v u \, dv = dQ \, u$$

Alternatively, $$I \, dl = \frac{dQ}{dt} dl = \frac{dl}{dt} dQ = dQ \, u$$

Hence, $$I \, dl = dQ \, u \qquad \text{-------------(3)}$$

Thus the force on a current element Idl in a magnetic field B is found from by merely replacing Qu by Idl;

i.e. $$dF = Idl \times B \qquad \text{----------(4)}$$

If the current I is through a closed path L or circuit, the force on the circuit is given by

$$F = \oint_L Idl \times B \qquad \text{------------(5)}$$

If instead of the line current element Idl, we have surface current elements KdS or a volume current element Jdv,

$$dF = KdS \times B \qquad \text{or} \qquad dF = Jdv \times B$$

$$F = \oint_S KdS \times B \qquad \text{or} \qquad F = \oint_v Jdv \times B$$

The magnetic field B is defined as the force per unit current element.

FORCE BETWEEN TWO CURRENT ELEMENTS:

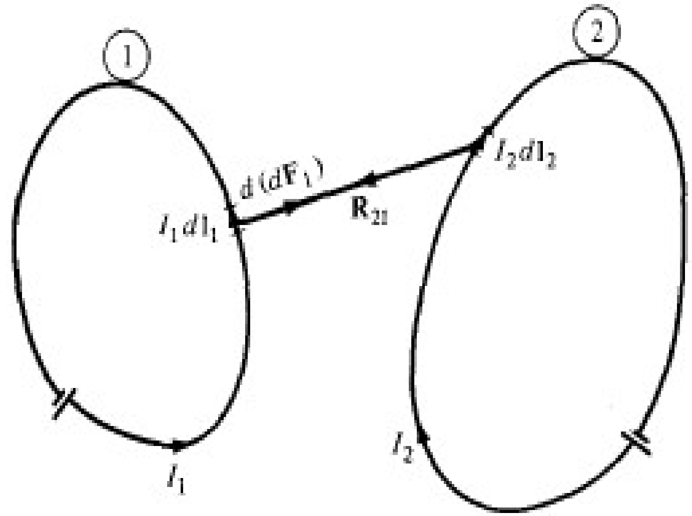


Figure 8.1 Force between two current loops.

Let us now consider the force between two elements $I_1 dl_1$ and $I_2 dl_2$. According to Biot-Savart's law, both current elements produce magnetic fields. So we may find the force $\mathrm{d}(\mathrm{d}F_1)$ on element $I_1 dl_1$ due to the field $\mathrm{d}B_2$ produced by element $I_2 dl_2$.

$$\mathrm{d}(\mathrm{d}F_1) = I_1 dl_1 \times \mathrm{d}B_2 \qquad \text{---------------(1)}$$

But from Biot-Savart's law,

$$dB_2 = \frac{\mu_{0 I_2 dl_2 \times a_{R_{21}}}}{4\pi R_{21}{}^2}$$

Hence,

$$d(dF_1) = \frac{I_1 dl_1 \times \mu_{0 I_2 dl_2 \times a_{R_{21}}}}{4\pi R_{21}{}^2} \qquad \text{------------(2)}$$

The total force F, on current loop 1 due to current loop 2.

$$F_1 = \frac{\mu_{0 I_1 I_2}}{4\pi} \oint_{l_1} \oint_{l_2} \frac{dl_1 \times dl_2 \times a_{R_{21}}}{R_{21}{}^2} \qquad \text{------------(3)}$$

The force F_2 on loop 2 due to the magnetic field B_1 from loop 1 is obtained from eq. (3) by interchanging subscripts 1 and 2.

It can be shown that $F_2 = —F_1$ thus F_1 , and F_2 obey Newton's third law that action and reaction are equal and opposite.

CLASSIFICATION OF MAGNETIC MATERIALS:

The magnetic susceptibility χ_m or the relative permeability μ_r to classify materials in terms of their magnetic property or behavior.

A material is said to be nonmagnetic if $\chi_m = 0$ (or $\mu_r = 1$); it is magnetic otherwise.

Free space, air, and materials with $\chi_m = 0$ (or $\mu_r \approx = 1$) are regarded as nonmagnetic.

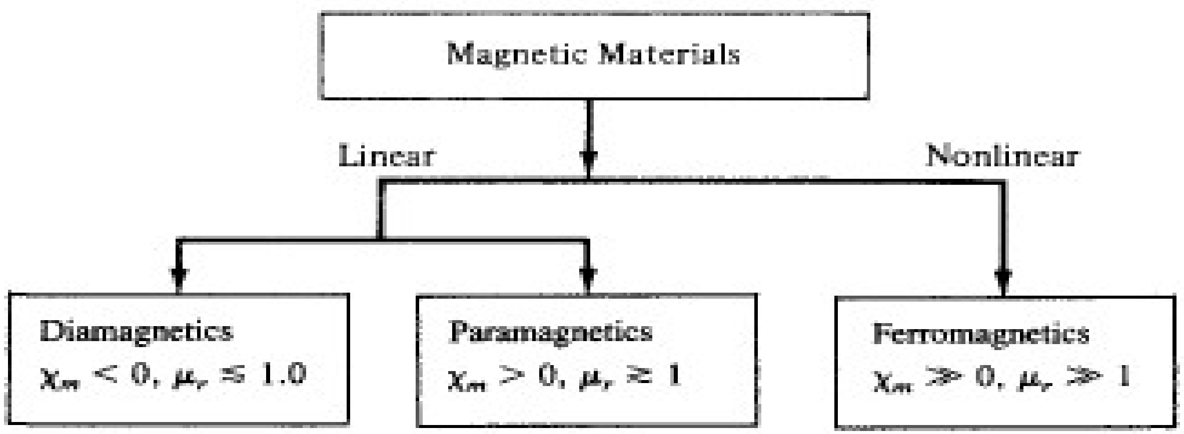


Figure 8.13 Classification of magnetic materials.

MAGNETIC BOUNDARY CONDITIONS:

We define magnetic boundary conditions as the conditions that H (or B) field must satisfy at the boundary between two different media.

We make use of Gauss's law for magnetic fields ,

$$\oint B.dS = 0 \qquad \text{------------(1)}$$

and Ampere's circuit law,

$$\oint H.dl = \mathrm{I} \qquad \text{----------(2)}$$

Consider the boundary between two magnetic media 1 and 2, characterized, respectively, by μ_1 and μ_2 as shown in below Figure.

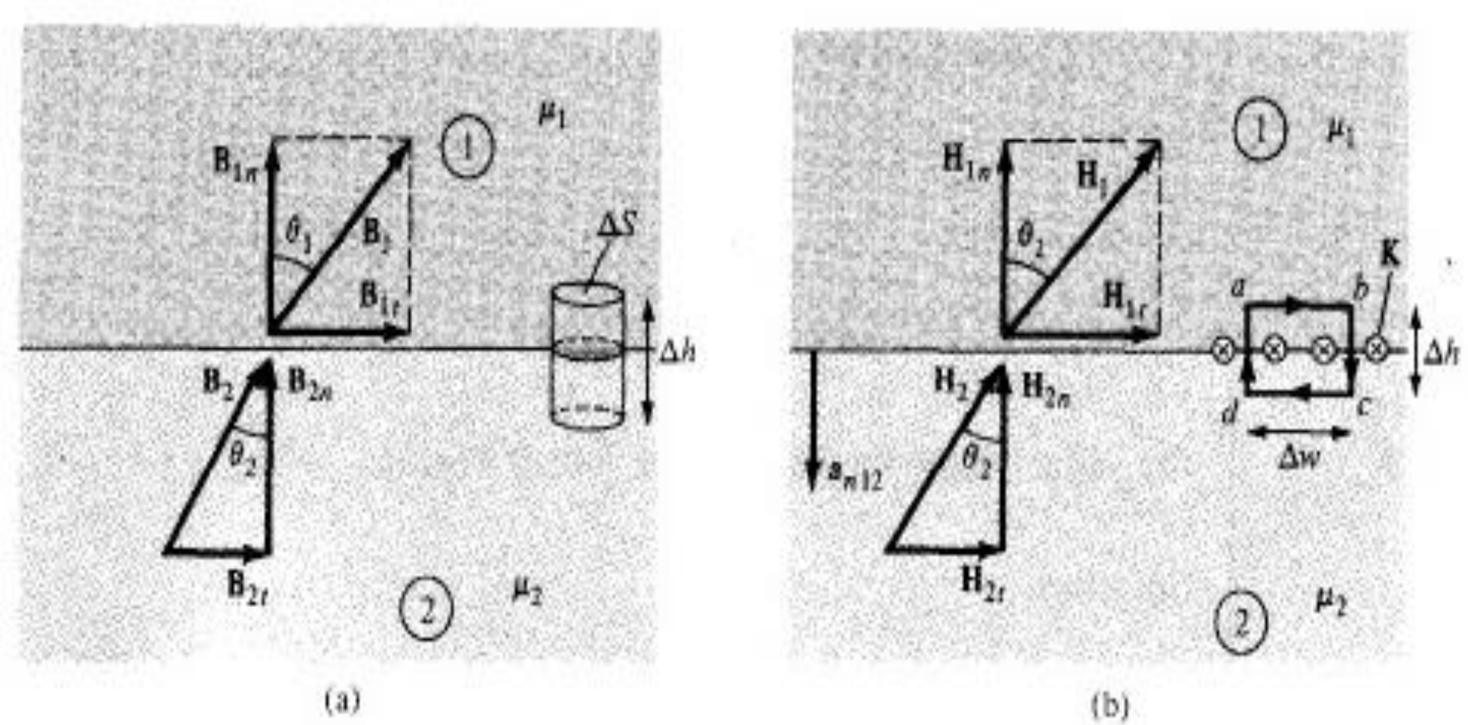


Figure 8.16 Boundary conditions between two magnetic media: **(a)** for **B**, **(b)** for **H**.

Applying eq. (1) to the pillbox (Gaussian surface) of Figure (a) and allowing $\Delta h \rightarrow 0$,

$$B_{1n}\,\Delta S - B_{2n}\,\Delta S = 0 \quad \text{-----------(3)}$$

$$\boldsymbol{B_{1n} = B_{2n}} \quad \text{or} \quad \boldsymbol{\mu_1 H_{1n} = \mu_2 H_{2n}} \quad \text{------------(4)}$$

Equation (4) shows that the normal component of B is continuous at the boundary. It also shows that the normal component of H is discontinuous at the boundary;

Similarly, we apply eq.(2) to the closed path abcda of figure(b) where surface current K on the boundary is assumed normal to the path. We obtain

$$K.\Delta w = H_{1t}\,\Delta w + H_{1n}\frac{\Delta h}{2} + H_{2n}\frac{\Delta h}{2} - H_{2t}\,\Delta w - H_{2n}\frac{\Delta h}{2} - H_{1n}\frac{\Delta h}{2} \quad \text{--------(5)}$$

As $\Delta h \rightarrow 0$, eq. (5) leads to

$$H_{1t} - H_{2t} = \mathrm{K} \qquad \text{-----------(6)}$$

This shows that the tangential component of H is also discontinuous. Equation (6) may be written in terms of B as

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = \mathbf{K} \qquad \text{-------------(7)}$$

In the general case, eq. (6) becomes

$$(H_1 - H_2) \times a_{n12} = \mathrm{K} \qquad \text{-------------(8)}$$

where a_{n12} is a unit vector normal to the interface and is directed from medium 1 to medium 2.

If the boundary is free of current or the media are not conductors (for K is free current density), K = 0 and eq. (6) becomes,

$$H_{1t} = H_{2t} \qquad \text{or} \qquad \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2} \qquad \text{---------(9)}$$

Thus the tangential component of H is continuous while that of B is discontinuous at the boundary. If the fields make an angle θ with the normal to the interface, eq. (4) results in

$$B_1 \cos\theta_1 = B_{1n} = B_{2n} = B_2 \cos\theta_2 \qquad \text{------------(10)}$$

while eq. (9) produces

$$\frac{\boldsymbol{B_1}}{\boldsymbol{\mu_1}}\sin\theta_1 = \boldsymbol{H_{1t}} = \boldsymbol{H_{2t}} = \frac{\boldsymbol{B_2}}{\boldsymbol{\mu_2}}\sin\theta_2 \qquad \text{------------(11)}$$

Dividing eq. (11) by eq. (10) gives

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\boldsymbol{\mu_1}}{\boldsymbol{\mu_2}} \qquad \text{-----------(12)}$$

which is similar to the law of refraction for magnetic flux lines at a boundary with no surface current.

INDUCTORS AND INDUCTANCES:

A circuit (or closed conducting path) carrying current I produces a magnetic field B which causes a flux $\Psi = \int B \bullet dS$ to pass through each turn of the circuit as shown in below Figure.

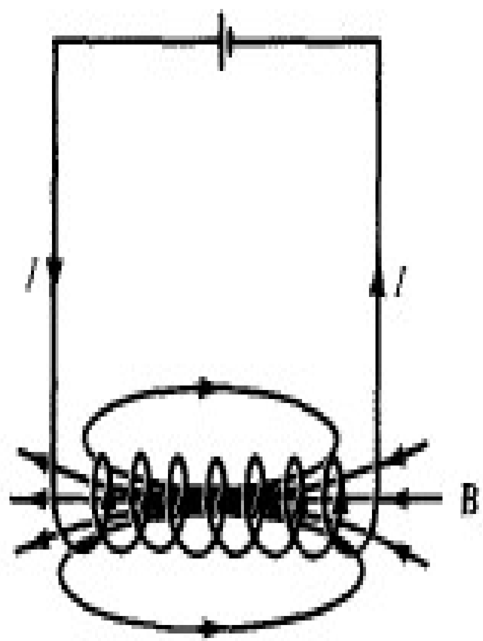


Figure 8.19 Magnetic field B produced by a circuit.

If the circuit has N identical turns, we define the flux linkage λ as

$$\lambda = N\,\Psi \qquad \text{-------------(1)}$$

Also, if the medium surrounding the circuit is linear, the flux linkage λ is proportional to the current I producing it; that is,

$$\lambda \propto I \qquad \text{or} \qquad \lambda = LI \quad \text{------------(2)}$$

where L is a constant of proportionality called the inductance of the circuit.

The inductance L is a property of the physical arrangement of the circuit. A circuit or part of a circuit that has inductance is called an inductor.

From eqs. (1) and (2), we may define inductance L of an inductor as the ratio of the magnetic flux linkage X to the current / through the inductor; that is,

$$L = \frac{\lambda}{I} = \frac{N\,\Psi}{I} \qquad \text{---------------(3)}$$

The magnetic energy (in joules) stored in an inductor is expressed in circuit theory as:

$$W_m = \tfrac{1}{2} L I^2 \qquad \text{---------------(4)}$$

Or $$L = \frac{2\,W_m}{I^2} \qquad \text{------------(5)}$$

Thus the self-inductance of a circuit may be defined or calculated from energy considerations.

If instead of having a single circuit we have two circuits carrying current I_1 and as I_2 shown in below Figure a magnetic interaction exists between the circuits. Four component fluxes Ψ_{11} , Ψ_{12} ,Ψ_{21} & Ψ_{22}. are produced.

Four component fluxes are Ψ_{11} , Ψ_{12} ,Ψ_{21} & Ψ_{22}.

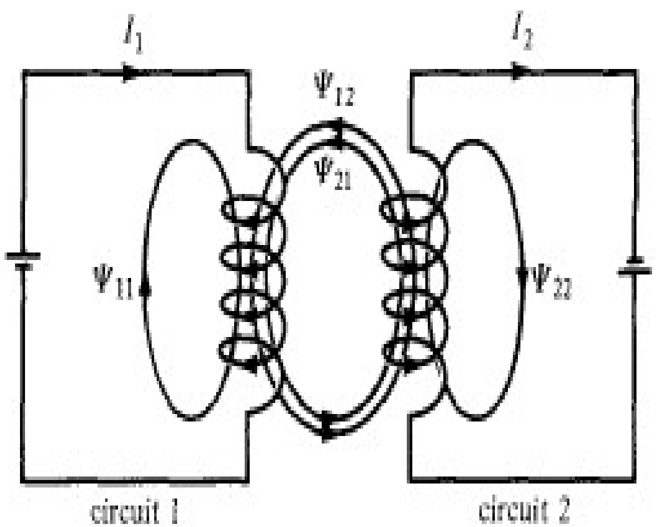


Figure 8.20 Magnetic interaction between two circuits.

The flux Ψ_{12}, for example, is the flux passing through circuit 1 due to current I_2 in circuit 2. If B_2 in the field due to I_2 and S_1 is the area of circuit 1, then

$$\Psi_{12} = \int_{S_1} B_2 \,.\mathrm{Ds} \qquad \text{-------------(6)}$$

We define the mutual inductance M_{12} as the ratio of the flux linkage

$\lambda_{12} = N_1 \Psi_{12}$ on circuit 1 to current I_2

that is,

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \Psi_{12}}{I_2} \qquad \text{-----------(7)}$$

Similarly, the mutual inductance M_{21} is defined as the flux linkages of circuit 2 per unit

current I_1; that is,

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \Psi_{21}}{I_1} \qquad \text{--------------(8)}$$

It can be shown by using energy concepts that if the medium surrounding the circuits is linear (i.e., in the absence of ferromagnetic material),

$$M_{12} = M_{21} \qquad \text{------------(9)}$$

The mutual inductance M_{12} or M_{21} is expressed in henrys and should not be confused with the magnetization vector M expressed in amperes/meter.

We define the self-inductance of circuits 1 and 2, respectively, as

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1\Psi_1}{I_1} \qquad \text{-------------(10)}$$

And $$L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2\Psi_2}{I_2} \qquad \text{------------(11)}$$

Where $\Psi_1 = \Psi_{11} + \Psi_{12}$ and $\Psi_2 = \Psi_{21} + \Psi_{22}$

The total energy in the magnetic field is the sum of the energies due to L_1 & L_2, and M_{12} (or M_{21}); that is,

$$W_m = W_1 + W_2 + W_{12}$$

$$W_m = ½\, L_1 I_1^{\,2} + ½\, L_2 I_2^{\,2} + M_{12} I_1 I_2 \quad \text{------------}(12)$$

- The positive sign is taken if currents I_1 & I_2 flow such that the magnetic fields of the two circuits strengthen each other. If the currents flow such that their magnetic fields oppose each other, the negative sign is taken.
- An inductor is a conductor arranged in a shape appropriate to store magnetic energy.

For a given inductor, we find the self-inductance L by taking these steps:

1. Choose a suitable coordinate system.

2. Let the inductor carry current I.

3. Determine B from Biot-Savart's law (or from Ampere's law if symmetry exists) and calculate Ψ from

$$\Psi = \int B \bullet dS.$$

4. Finally find L from

$$L = \frac{\lambda}{I} = \frac{N\Psi}{I}$$

In an inductor such as a coaxial or a parallel-wire transmission line, the inductance produced by the flux internal to the conductor is called the internal inductance L_{in} while that produced by the flux external to it is called external inductance L_{ext}.

The total inductance L is

$$L = L_{in} + L_{ext} \qquad \text{-----------(13)}$$

We know that for capacitors,

$$RC = \frac{\varepsilon}{\sigma} \qquad \text{--------------(14)}$$

$$L_{ext}C = \mu\varepsilon \qquad \text{------------(15)}$$

MAGNETIC ENERGY:

The potential energy in an electrostatic field as,

$$W_E = ½ \int D.E \text{ dv} = ½ \int \varepsilon E^2 \text{ dv} \text{ -----------(1)}$$

A similar expression for the energy in a magneto static field. A simple approach is using the magnetic energy in the field of an inductor.

$W_m = ½ \text{ L}I^2$, The energy is stored in the magnetic field B of the inductor.

Consider a differential volume in a magnetic field as shown in below Figure.

- Let the volume be covered with conducting sheets at the top and bottom surfaces with current ΔI.

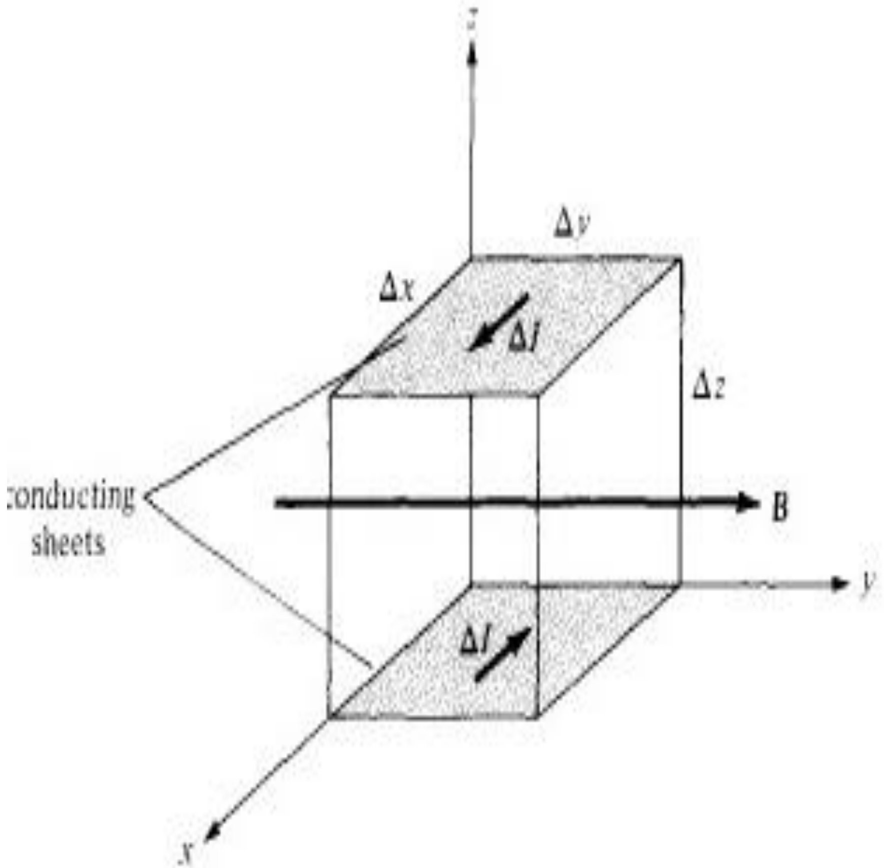


Figure 8.21 A differential volume in a magnetic field.

Each volume has an inductance

$$\Delta L = \frac{\Delta\Psi}{\Delta I} = \frac{\mu H \Delta x \Delta z}{\Delta I} \qquad \text{----------(2)}$$

Where ΔI = H Δy. Substituting eq. (2) into energy density equation,

$$\Delta W_m = ½\ \Delta L\ \Delta I^2 = ½\ \mu\ H^2 \Delta x \Delta y \Delta z \qquad \text{----------(3)}$$

$$\Delta W_m = ½\ \mu H^2 \Delta v$$

The magneto static energy density W_m (in J/ m^3) is defined as

$$W_m = \lim_{\Delta v \to 0} \frac{\Delta W_m}{\Delta v} = ½\, \mu H^2$$

Hence, $W_m = ½\, \mu H^2 = ½$ B.H $= \frac{B^2}{2\mu}$ -----------(4)

Thus the energy in a magneto static field in a linear medium is

$$\boldsymbol{W_m = \int W_m\, dv \quad or \quad W_m = ½ \int B.H\, dv = ½ \int \mu H^2 dv}$$ -------(5)

MAXWELL’S EQUATIONS (TIME VARYING FIELDS)

OBJECTIVES

FARADAY’S LAW

TRANSFORMER EMF

INCONSISTENCY OF AMPERE’S LAW

DISPLACEMENT CURRENT DENSITY

MAXWELL’S EQUATIONS IN FINAL FORMS

BOUNDARY CONDITIONS

INTRODUCTION:

- Electrostatic fields denoted by E(x, y, z) and these are usually produced by static electric charges.
- Magneto static fields denoted by H(x, y, z) and these are due to motion of electric charges with uniform velocity or static magnetic charges .
- Time-varying fields or waves are usually due to accelerated charges or time-varying currents

- Stationary charges → electrostatic fields
- Steady currents → magnetosiatic fields
- Time-varying currents → electromagnetic fields (or waves)

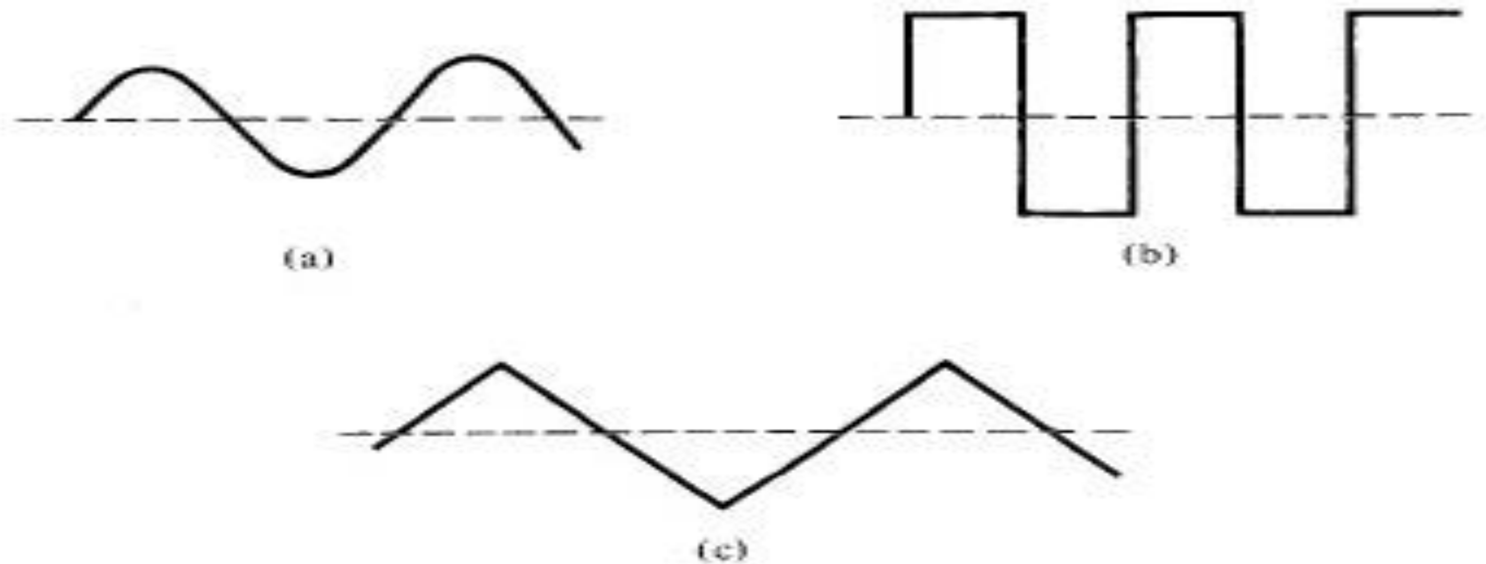


Figure 9.1 Various types of time-varying current: **(a)** sinusoidal, **(b)** rectangular, **(c)** triangular.

FARADAY'S LAW:

A static magnetic field produces no current flow, but a time-varying field produces an induced voltage (called electromotive force or emf) in a closed circuit, which causes a flow of current.

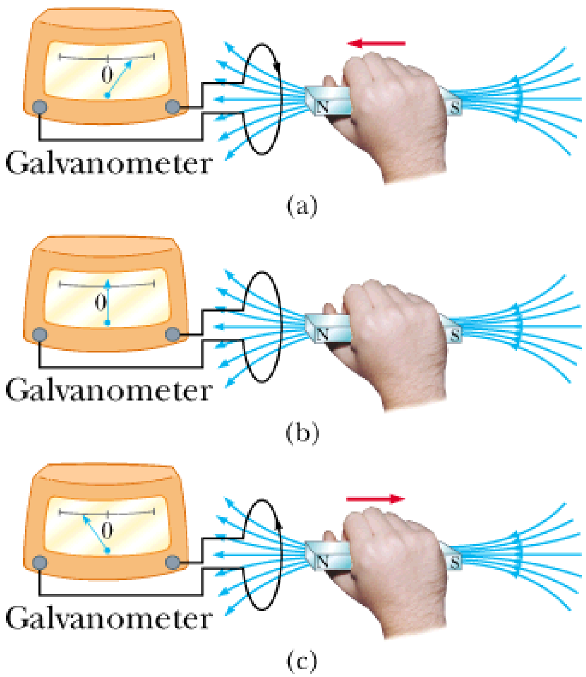
Faraday discovered that the induced emf V_{emf} in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit. This is called Faraday's law.

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt} (\because \lambda = N\Psi)$$

Where 'n ' is no. of turns in the circuit

'Ψ' is the flux through each turn

Induced EMF:



A current flows through the loop when a magnet is moved near it, without any batteries.

LENZ'S LAW:

The direction of current flow in the circuit is such that the induced magnetic field produced by the induced current will oppose the original magnetic field.

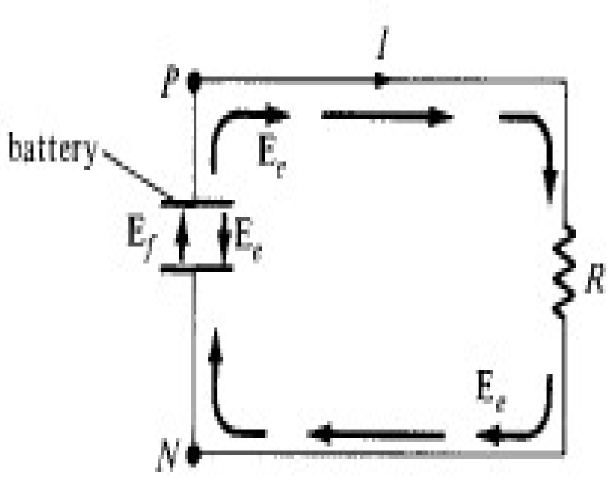


Figure 9.2 A circuit showing emf-producing field $\mathbf{E}_f$ and electrostatic field $\mathbf{E}_e$.

The total electric field at any point is,

$$E = E_f + E_e$$

Note that E_f is zero outside the battery, E_f and E_e have opposite directions in the battery, and the direction of E_e inside the battery is opposite to that outside it.

$\oint \mathrm{E}.\,\mathrm{dl} = \oint \mathrm{E_f}.\,\mathrm{dl} + 0 = \int_N^P \mathrm{E_f}.\,\mathrm{dl}$ $\quad (\because \oint \mathrm{E_e}.\,\mathrm{dl} = 0)$

The emf of the battery is the line integral of the emf-produced field

$\mathrm{V_{emf}} = \int_N^P \mathrm{E_f}.\,\mathrm{dl} = -\int_N^P \mathrm{E_e}.\,\mathrm{dl} = \mathrm{IR}$

Note:

An electrostatic field E_e cannot maintain a steady current in a closed circuit since $\oint E_e.\,dl = 0$

An emf produced field E_f is nonconservative.

TRANSFORMER emf:

For a single turn(N=1) , Faraday's law is

$$V_{emf} = -\frac{d\Psi}{dt}$$

In terms of E and B is,

$$V_{emf} = \oint E.dl = -\frac{d}{dt}\int B.ds \qquad (\because \Psi = \int B.ds)$$

The variation of flux with time may be caused in three ways:

- By having a stationary loop in a time-varying B field
- By having a time-varying loop area in a static B field
- By having a time-varying loop area in a time-varying B field.

A STATIONARYLOOPIN A TIME VARYING B FIELD (TRANSFORMER emf):

This emf induced by the time-varying current (producing the time-varying B field) in a stationary loop is often referred to as transformer emf in power analysis since it is due to transformer action.

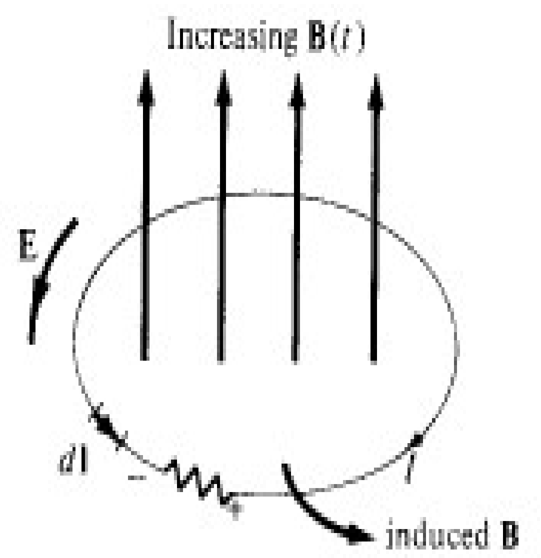


Figure 9.3 Induced emf due to a stationary loop in a time-varying **B** field.

$$V_{emf} = \oint E.dl = -\frac{d}{dt}\int B.ds \qquad (\because \Psi = \int B.ds)$$

Due to the transformer action,

$$\int(\nabla \times E).ds = -\int \frac{\partial B}{\partial t}.ds \qquad \text{(stokes's theorem)}$$

For the two integrals to be equal, their integrands must be equal,

$\nabla \times E = -\frac{\partial B}{\partial t}$ Maxwell's equation for time varying fields.

It shows that the time varying E field is not conservative

i.e. $(\nabla \times E \neq 0)$

Moving loop in a static B field(Motional emf):

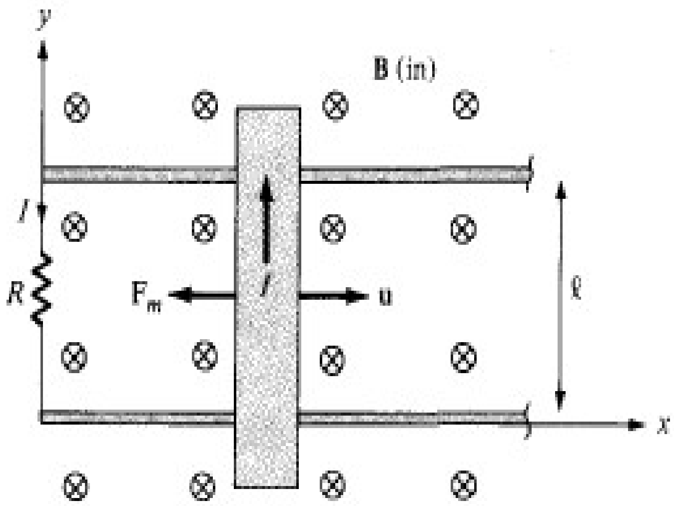


Figure 9.5 Induced emf due to a moving loop in a static **B** field.

When a conducting loop is moving in a static B field, an emf is induced in the loop.

The force on a charge moving with uniform velocity u in a magnetic field B is

$F_m = Q_u$ X B

E_m = F_m/Q = u X B

V_{emf} = $\oint E_m . dl$ = $\oint$ u X B . dl

∇XE_m = ∇ Xu X B

Moving loop in a time-varying field:

A moving conducting loop is in time varying field constitutes both the transformer emf and motional emf.

V_{emf} = $\oint E_m . dl$ = -$\int \frac{\partial B}{\partial t}$ .ds + $\oint$ (u X B) . dl

∇XE =-$\frac{\partial B}{\partial t}$ + ∇ Xu X B

DISPLACEMENT CURRENT:

For static EM fields, $\nabla \times H = J$

The divergence of the curl of any vector field is identically zero,

$\nabla.(\nabla \times H) = 0 = \nabla.J$

The continuity of current equation, $\nabla.J = -\frac{\partial \rho_v}{\partial t} \neq 0$

$\nabla \times H = J + J_d$, Where $J_d = \frac{\partial D}{\partial t}$

$\nabla \times H = J + \frac{\partial D}{\partial t}$

The term $J_d = \frac{\partial D}{\partial t}$ is known as displacement current density and

J is the conduction current density ($J = \sigma E$).

INCONSISTANCY OF AMPERE'S LAW:

- Without the term J_d, the propagation of electromagnetic waves would be impossible.
- At low frequencies, 'J_d' is usually neglected compared with 'J'.

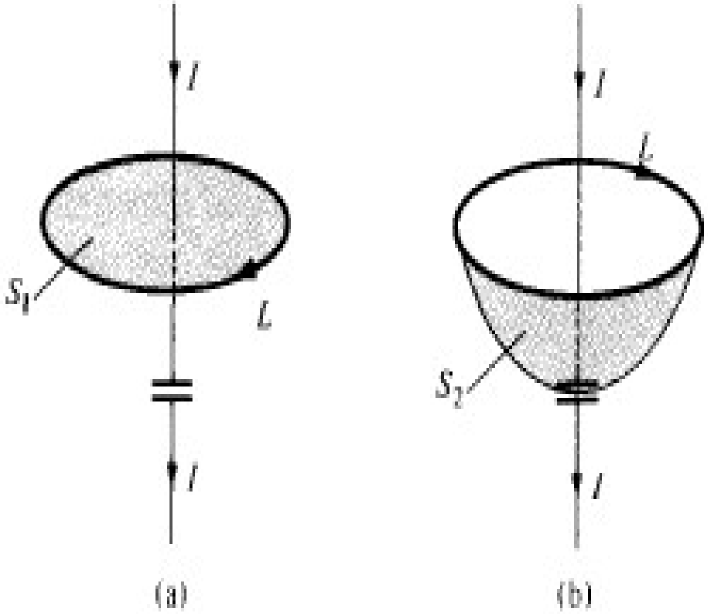


Figure 9.10 Two surfaces of integration showing the need for $\mathbf{J}_d$ in Ampere's circuit law.

Based on the displacement current density, we define the displacement current is given by,

$I_d = \int J_d.dS = \int \frac{\partial D}{\partial t}.dS$ ----------(1)

Applying an unmodified form of Ampere's circuit law to a closed path L,

$\oint H.dl = \int J.dS = I_{enq} = I$ -------(2)

$\oint H.dl = \int J.dS = I_{enq} = 0$ -------(3) (∵ no conduction current (J = 0) flows through S2)

To resolve the conflict, we need to include the displacement current in Ampere's circuit law.

$$\boldsymbol{\oint H.dl = \int J_d.dS = \frac{d}{dt}\int D.dS = \frac{dQ}{dt} = I}$$

MAXWELL'S EQUATIONS IN FINAL FORMS

Differential form	Integral form	Remarks
$\nabla . D = \rho_V$	$\oint D.dS = \int \rho_V$ dv	Gauss's law
$\nabla . B = 0$	$\oint B.dS = 0$	Nonexistence of isolated magnetic charge
$\nabla \times$ E =- $\frac{\partial B}{\partial t}$	$\oint E.dl$ = -$\frac{d}{dt} \int B.ds$	Faraday's law
∇x H = J + $\frac{\partial D}{\partial t}$	$\oint \boldsymbol{H}.\boldsymbol{dl} = \int (J + \frac{\partial D}{\partial t})$.dS	Ampere's circuit law

CONDITIONS AT A BOUNDARY SURFACES:

- The concepts of linearity, isotropy, and homogeneity of a material medium still apply for time-varying fields;
- In a linear, homogeneous, and isotropic medium characterized by σ, ε, μ.

$$\left.\begin{aligned} &D = \varepsilon E = \varepsilon_0 E + P \\ &B = \mu H = \mu_0 (H + M) \\ &J = \sigma E + \rho_V u \end{aligned}\right\} \quad \text{constitutive relations}$$

For time-varying fields, The boundary conditions ,

$E_{1t} = E_{2t}$ or $(E_1 - E_2) \times a_{n12} = 0$

$H_{1t} - H_{2t} = K$ or $(H_1 - H_2) \times a_{n12} = K$

$D_{1n} - D_{2n} = \rho_s$ or $(D_1 - D_2).a_{n12} = \rho_s$

$B_{1n} - B_{2n} = 0$ or $(B_2 - B_1).a_{n12} = 0$

For a perfect conductor ($\sigma \cong \infty$)in a time varying field,

E = 0, H = 0, J = 0

$B_n = 0$, $E_t = 0$

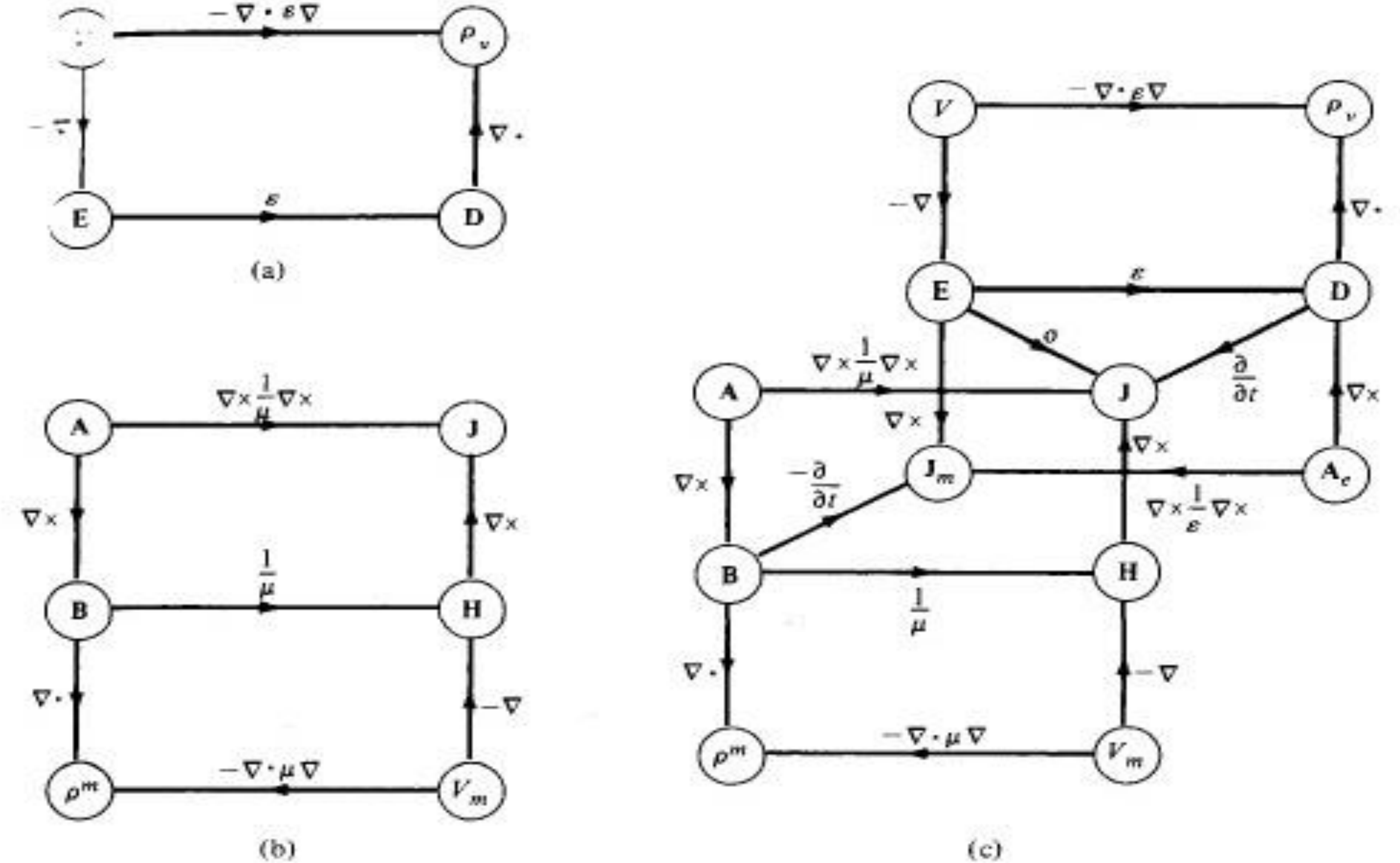


Figure 9.11 Electromagnetic flow diagram showing the relationship between the potentials and vector fields: **(a)** electrostatic system, **(b)** magnetostatic system, **(c)** electromagnetic system. [Adapted with permission from IEE Publishing Dept.]

For a perfect dielectric (σ = 0),

i) **Compatibility equations:**

$$\mathrm{V} \bullet \mathrm{B} = \rho^{m} , \qquad \nabla \mathrm{X} E = -\frac{\partial B}{\partial t} = \mathrm{J}$$

ii) **Constitutive equations:**

$$\mathrm{B} = \mu \mathrm{H} , \qquad \mathrm{D} = \varepsilon \mathrm{E}$$

iii) **Equilibrium equations:**

$$\nabla . \mathrm{D} = \rho_{\mathrm{V}} , \nabla \mathrm{x}\ \mathrm{H} = \mathrm{J} + \frac{\partial D}{\partial t}$$

TIME-HARMONIC MAXWELL'S EQUATIONS

POINT FORM

$\nabla . D_s = \rho_{vs}$

$\nabla . B_s = 0$

$\nabla \times E_s$ = -jωB_s

$\nabla \times H_s$ =J_s +jωD_s

INTEGRAL FORM

$\oint D_s . ds = \int \rho_{vs}$ dv

$\oint B_s . ds = 0$

$\oint E_s . dl = -j\omega \int B_s . ds$

$\oint H_s . dl = \int (J_s + \text{j}\omega D_s)$ ds

EM WAVE CHARACTERISTICS-I

OBJECTIVES

WAVE EQUATIONS FOR MEDIA

UNIFORM PLANE WAVES

RELATIONS BETWEEN E&H

WAVE PROPAGATION IN LOSSLESS MEDIA

WAVE PROPAGATION IN GOOD CONDUCTORS

WAVE PROPAGATION IN GOOD DIELECTRICS

POLARIZATION

WHAT ARE WAVES?

Definition: A disturbance that transfers energy from place to place.

What carries waves? A medium, a medium is the material through which a wave travels.

A medium can be a gas, liquid, or solid.

Not all waves require a medium to travel.

Light from the sun travels through empty space.

- Waves are means of transporting energy or information.
- A wave is a function of both space and time.

Ex: Radio waves, TV signals, radar beams, and light rays.

EM wave motion in the following media:

1. Free space $(\sigma = 0, \varepsilon = \varepsilon_0, \mu = \mu_0)$
2. Lossless dielectrics $(\sigma = 0, \varepsilon = \varepsilon_0\varepsilon_r, \mu = \mu_0\mu_r$, or $\sigma \ll \omega\varepsilon)$
3. Lossy dielectrics $(\sigma \neq 0, \varepsilon = \varepsilon_0\varepsilon_r, \mu = \mu_0\mu_r)$
4. Good conductors $(\sigma \cong \infty, \varepsilon = \varepsilon_0, \mu = \mu_0\mu_r$, or $\sigma \gg \omega\varepsilon)$

TABLE 10.1 Electromagnetic Spectrum

EM Phenomena	Examples of Uses	Approximate Frequency Range
Cosmic rays	Physics, astronomy	10^{14} GHz and above
Gamma rays	Cancer therapy	10^{10}–10^{13} GHz
X-rays	X-ray examination	10^{8}–10^{9} GHz
Ultraviolet radiation	Sterilization	10^{6}–10^{8} GHz
Visible light	Human vision	10^{5}–10^{6} GHz
Infrared radiation	Photography	10^{3}–10^{4} GHz
Microwave waves	Radar, microwave relays, satellite communication	3–300 GHz
Radio waves	UHF television	470–806 MHz
	VHF television, FM radio	54–216 MHz
	Short-wave radio	3–26 MHz
	AM radio	535–1605 kHz

UNIFORM PLANE WAVES:

Definition:*The wave that will have variation only in the direction of propagation and its characteristics remain constant across the planes normal to the direction of propagation.*

UNIFORM PLANE WAVES IN FREE SPACE:

Assume an EM wave travelling in free space. Consider an electric field is in x- direction and a magnetic field is in y- direction.

$$\nabla \text{x H} = \text{J} + \frac{\partial D}{\partial t}$$

$$\nabla \text{x H} = \text{J} + \frac{\partial D}{\partial t} \quad (\text{if} \quad \text{J} = 0)$$

$$\nabla \text{x H} = \frac{\partial}{\partial t}(D_x a_x + D_y a_y + D_z a_z)$$

$$\nabla \text{x H} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

As H is in Y- direction, H_x = H_z =0,

$$\therefore -\frac{\partial H_y}{\partial z} a_x + \frac{\partial H_y}{\partial x} a_z = \frac{\partial}{\partial t}(D_x a_x + D_y a_y + D_z a_z)$$

H_y is not changing with x, and it is uniform in x-y plane,

$$-\frac{\partial H_y}{\partial x} = 0,$$

$$\therefore -\frac{\partial H_y}{\partial z} a_x = \frac{\partial}{\partial t}(D_x a_x + D_y a_y + D_z a_z)$$

$$-\frac{\partial H_y}{\partial z} = \frac{\partial D_x}{\partial t} \quad \text{or} \quad \boldsymbol{\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_x}{\partial t}} \text{---------(1) (since D=}\varepsilon\text{E)}$$

Form faraday's law ,

∇x E = - $\frac{\partial B}{\partial t}$

$$\nabla \text{x E} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

As E is in X- direction, E_y = E_z =0,

$$\therefore - \frac{\partial E_x}{\partial z} a_y + \frac{\partial E_x}{\partial y} a_z = -\frac{\partial}{\partial t}(B_x a_x + B_y a_y + B_z a_z)$$

E_x is not changing with y, and it is uniform in x-y plane,

$$\frac{\partial E_x}{\partial y} = 0, \therefore \frac{\partial E_x}{\partial z} a_y = -\frac{\partial}{\partial t}(B_x a_x + B_y a_y + B_z a_z)$$

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\boldsymbol{\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\frac{\partial E_x}{\partial z}}$$ ----------(2) (since $B = \mu H$)

Differentiating eq(1) with respect to 't'

$$\frac{\partial}{\partial t}\left(\frac{\partial H_y}{\partial z}\right) = -\varepsilon \frac{\boldsymbol{\partial}^2 \mathrm{E_X}}{\boldsymbol{\partial \mathrm{t}^2}}$$ ----------(3)

Differentiating eq(2) with respect to 'z'

$$\frac{\partial}{\partial z}\left(\frac{\partial H_y}{\partial t}\right) = -\frac{1}{\mu}\frac{\boldsymbol{\partial}^2 E_X}{\boldsymbol{\partial Z^2}}$$ ------------(4)

$$\frac{\partial^2 E_X}{\partial \mathrm{t}^2} = \frac{1}{\mu\varepsilon}\frac{\boldsymbol{\partial}^2 E_X}{\boldsymbol{\partial Z^2}}$$ ----------(5)

$$\frac{\partial^2 E_X}{\partial \mathrm{t}^2} = v^2\frac{\boldsymbol{\partial}^2 E_X}{\boldsymbol{\partial Z^2}}$$,

$$\mathbf{v} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{\mu\varepsilon}}}$$ **---------(6)**

Attenuation constant (α):

When any wave propagates in the medium, it gets attenuated. The amplitude of the signal reduces. It is represented by an attenuation constant (α)

α is measured in neper per meter (Np/m)

1 Np = 8.686 Db

Phase constant(β):

When any wave propagates in the medium, phase change also takes place. Such a phase change is expressed by a phase constant(β)

β is measured in radians per meter (rad/m)

Propagation constant(γ):

Attenuation constant (α) and phase constant(β) together constitutes a propagation constant (γ)

$$\gamma = \alpha + j\beta$$

Intrinsic Impedance(η):

The ratio of amplitudes of E to H of the waves in either direction is called intrinsic impedance of the material in which wave is travelling.

$$\eta = \frac{E_{m+}}{H_{m+}} = -\frac{E_{m-}}{H_{m-}} = \frac{\omega\mu}{\beta} = v\mu = \sqrt{\frac{\mu}{\varepsilon}}$$

WAVE EQUATIONS IN PHASOR FORM:

From the faraday's law,

$$\nabla \text{x E} = - \frac{\partial B}{\partial t} = - \mu \frac{\partial H}{\partial t} \quad \text{--------(1)}$$

$$\nabla \text{x} \nabla \text{x E} = - \mu[\nabla \text{x} \frac{\partial H}{\partial t}]$$

$$\nabla \text{x} \nabla \text{x E} = - \mu[\frac{\partial}{\partial t} (\nabla \text{xH})] \quad \text{--------(2)}$$

Using vector identity,

$$\nabla (\nabla .)\text{E-}\nabla^2 \text{E} = - \mu[\frac{\partial}{\partial t} (\nabla \text{xH})] \quad \text{---------(3)}$$

$$\nabla \text{x H} = \text{J} + \frac{\partial D}{\partial t}$$

$$\nabla (\nabla .\text{E})\text{-}\nabla^2 \text{E} = - \mu[\frac{\partial}{\partial t} (\text{J} + \frac{\partial D}{\partial t})] \quad \text{-------(4)}$$

$-\nabla^2 E = -\mu[\frac{\partial}{\partial t}(J + \frac{\partial D}{\partial t})]$ (since $\nabla.E = 0$) or

$\nabla^2 E = \mu[\frac{\partial}{\partial t}(J + \frac{\partial D}{\partial t})]$ ---------(5)

When any field varies with respect to time, its partial derivative taken with respect to time can be replaced by **$j\omega$**

$\nabla^2 E = \mu[j\omega(J + j\omega D)]$

$\nabla^2 E = [j\omega\mu(\sigma + j\omega\varepsilon)]E$ --------(6)

$\nabla^2 H = [j\omega\mu(\sigma + j\omega\varepsilon)]H$ ---------(7)

$$\nabla^2 \mathrm{E} = \gamma^2 \mathrm{E} \quad \& \nabla^2 \mathrm{H} = \gamma^2 \mathrm{H},$$

$$\gamma = \alpha + \mathrm{j}\beta = \sqrt{\mathrm{j}\omega\mu(\sigma + \mathrm{j}\omega\varepsilon)} \text{ ------(8)}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\sqrt{1 + (\frac{\sigma}{\omega\varepsilon})^2 - 1}} \text{-------(9)}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\sqrt{1 + (\frac{\sigma}{\omega\varepsilon})^2 + 1}} \text{-------(10)}$$

$$\eta = \sqrt{\frac{\mathrm{j}\omega\mu}{\sigma + \mathrm{j}\omega\varepsilon}} \quad \text{-------(11)}$$

$$|\eta| = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1 + (\frac{\sigma}{\omega\varepsilon})^2}}$$

$$\tan 2\theta = \frac{\sigma}{\omega\varepsilon}, \quad 0 < \theta < 45$$

UNIFORM PLANE WAVE IN LOSSY DIELECTRICS:

- A lossy dielectric is a medium in which an EM wave loses power as it propagates

  due to poor conduction.

- A lossy dielectric is a partially conducting medium (imperfect dielectric or imperfect conductor) with $\sigma \neq 0$

$\gamma = \alpha + j\beta = \pm\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ -------(1)

By rearranging the terms,

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}} \qquad \text{-------(2)}$$

$\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \leq \theta_n\Omega$ or -----(3)

$\eta = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}} \Omega$ --------(4)

$\theta_n = \frac{1}{2} [\frac{\pi}{2}$ - $\tan^{-1}(\frac{\omega\varepsilon}{\sigma})$ rad ------(5) , $0 < \theta_n < \frac{\pi}{4}$

- This angle depends on the properties of the lossy dielectric medium as well as the frequency of a signal.
- For low frequency signal, ω becomes very small, $\theta_n = \frac{\pi}{4}$
- For every high frequency signal, $\theta_n = 0$

UNIFORM PLANE WAVES IN PRACTICAL DIELECTRICS:

For a perfect dielectric, σ = 0,But for practical dielectric, $\sigma \neq 0$ (i. e. $\sigma \ll \omega\varepsilon$)

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}$$

Consider the radical term, Mathematically using binomial theorem,

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \text{--------}, \quad \text{where } |x|<1,$$

If $|x|<1$ and n= ½ , then neglecting the higher order terms,

$$(1 + \frac{\sigma}{j\omega\varepsilon})^{\frac{1}{2}} = 1 + \frac{1}{2}\frac{\sigma}{j\omega\varepsilon}$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\,[1 + \frac{\sigma}{2j\omega\varepsilon}]$$

$$\gamma = \alpha + j\beta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon} \quad \text{-------}(1)$$

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \quad \text{--------}(2)$$

$$\beta = \omega\sqrt{\mu\varepsilon} \quad \text{--------}(3)$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon(1+\frac{\sigma}{j\omega\varepsilon})}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}(1 + \frac{\sigma}{j\omega\varepsilon})^{-\frac{1}{2}},$$

Using binomial theorem,

$$(1 + x)^{-n} = 1\text{-nx}+\frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \text{--------}$$

As $x = \frac{\sigma}{j\omega\varepsilon}$ is very small as compared to 1, neglecting the higher order terms,

$(1+x)^{-n}$= 1-nx

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left[1 - \frac{\sigma}{2j\omega\varepsilon}\right] \quad \text{or} \quad \sqrt{\frac{\mu}{\varepsilon}}\left[1 + j\frac{\sigma}{2\omega\varepsilon}\right]\,\Omega$$

Maxwell's equation,

$$\nabla \text{x } H = J + \frac{\partial D}{\partial t}$$

$$= \sigma E + \varepsilon \frac{\partial E}{\partial t}$$

$$= \sigma E + j\omega\varepsilon E = E(\sigma + j\omega\varepsilon)$$

$$\nabla \text{x } H = J_c + J_D$$

$$\frac{J_c}{J_D} = \frac{\sigma E}{j\omega\varepsilon E} = \frac{\sigma}{j\omega\varepsilon}$$

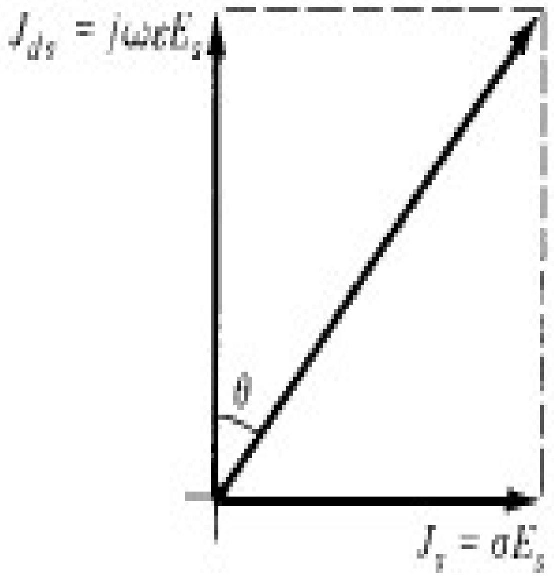


Figure 10.6 Loss angle of a lossy medium.

Loss angle , $\theta = \tan^{-1}\frac{\sigma}{\omega\varepsilon}$

Loss tangent of dielectric, $\tan\theta = \frac{\sigma}{\omega\varepsilon}$

If $\sigma \ll \omega\varepsilon$, the loss tangent is small, and medium is said to be good dielectric.

If $\sigma \gg \omega\varepsilon$, the loss tangent is high, and medium is said to be good conductor.

UNIFORM PLANE WAVES IN GOOD CONDUCTORS:

A perfect, or good conductor, or $\sigma \gg \omega\varepsilon$,

$\sigma \cong \infty,\ \varepsilon = \varepsilon_0,\ \mu = \mu_0\mu_r$

$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ --------(1)

$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}\sqrt{j}$

but $j = 1\angle 90$

$\gamma = \sqrt{\omega\mu\sigma}\sqrt{1\angle 90} = \sqrt{\omega\mu\sigma}\angle 45$

$\gamma = \sqrt{\omega\mu\sigma}\ (\cos 45 + j \sin 45)$

$\gamma = \sqrt{2\pi f\mu\sigma}\ [\frac{1}{\sqrt{2}}(1+j)]$

$\gamma = \alpha + j\beta = \sqrt{\pi f \mu \sigma} + j\sqrt{\pi f \mu \sigma}$ ------(2),

$\alpha = \sqrt{\pi f \mu \sigma}$ Np/m , $\beta = \sqrt{\pi f \mu \sigma}$ rad/m

For a good conductor, α and β are equal and both are directly proportional to the square root of frequency and conductivity(σ)

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ ------(3)

$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\sqrt{j}$ (since $\sigma \gg \omega\varepsilon$)

$\eta = \sqrt{\frac{\pi f \mu}{\sigma}}(1+j)$ --------(4) , T**he angle of intrinsic impedance is 45^0**

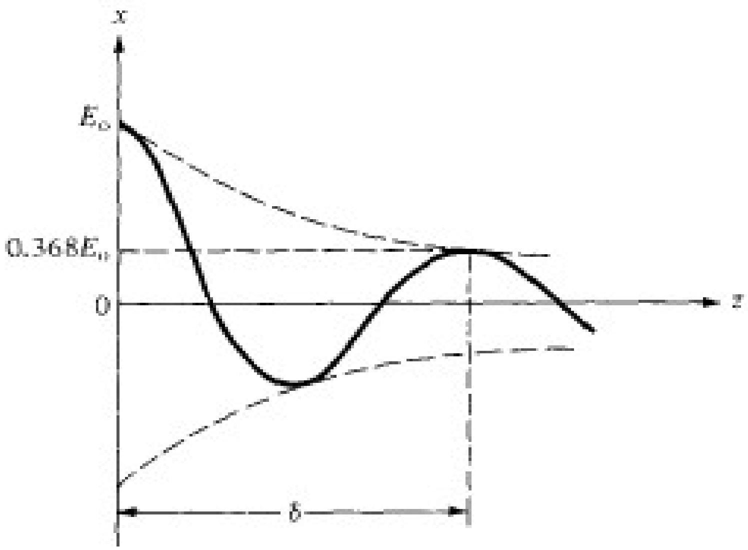


Figure 10.8 Illustration of skin depth.

The component of the electric field E_x is travelling in positive z- direction.

$E_x = E_m + e^{-\alpha z} = E_m + e^{-j\beta z}$

- At z = 0, amplitude of the component E_x is E_0,
- At z = δ, amplitude is $E_0 e^{-\alpha d}$.

In distance z = δ, the amplitude is gets reduced by a factor $e^{-\alpha d}$.

If we select $\delta = 1/\alpha$, then the factor becomes $e^{-1} = 0.368$

The skin depth is a measure of the depth to which an EM wave can penetrate the medium.

E (or H) wave travels in a conducting medium, its amplitude is attenuated by the factor $e^{-\alpha z}$. The distance δ, through which the wave amplitude decreases by a factor e^{-1} (about 37%) is called skin depth or penetration depth of the medium.

$$E_0 e^{-\delta\alpha} = E_0 e^{-1}$$

$$\boldsymbol{\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

UNIFORM PLANE WAVES IN LOSSLESS DIELECTRICS:

In a lossless dielectric, $\sigma \ll \omega\varepsilon$,

$$\sigma = 0,\ \varepsilon = \varepsilon_0\varepsilon_r\ ,\ \mu = \mu_0\mu_r$$

$\alpha = 0$, $\beta = \omega\sqrt{\mu\varepsilon}$

$\gamma = \omega\sqrt{\mu\varepsilon}$

$$v = \omega/\beta = \frac{1}{\sqrt{\mu\varepsilon}},$$

$$\lambda = \frac{2\pi}{\beta}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon\angle 0^o}}$$

E and H are in time phase with each other.

UNIFORM PLANE WAVES IN FREE SPACE:

In a free space medium, $\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$

$\alpha = 0$, $\beta = \omega\sqrt{\mu_0\varepsilon_0} = \omega/c$

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c,$$

$\lambda = \frac{2\pi}{\beta}$ where $c = 3 \text{ X } 10^8$ m/s

$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega$ is called the intrinsic impedance of free space

EM WAVE CHARACTERISTICS-II

OBJECTIVES

REFLECTION AND REFRACTION OF PLANE WAVES

NORMAL AND OBLIQUE INCIDENCES

BREWSTER ANGLE

CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

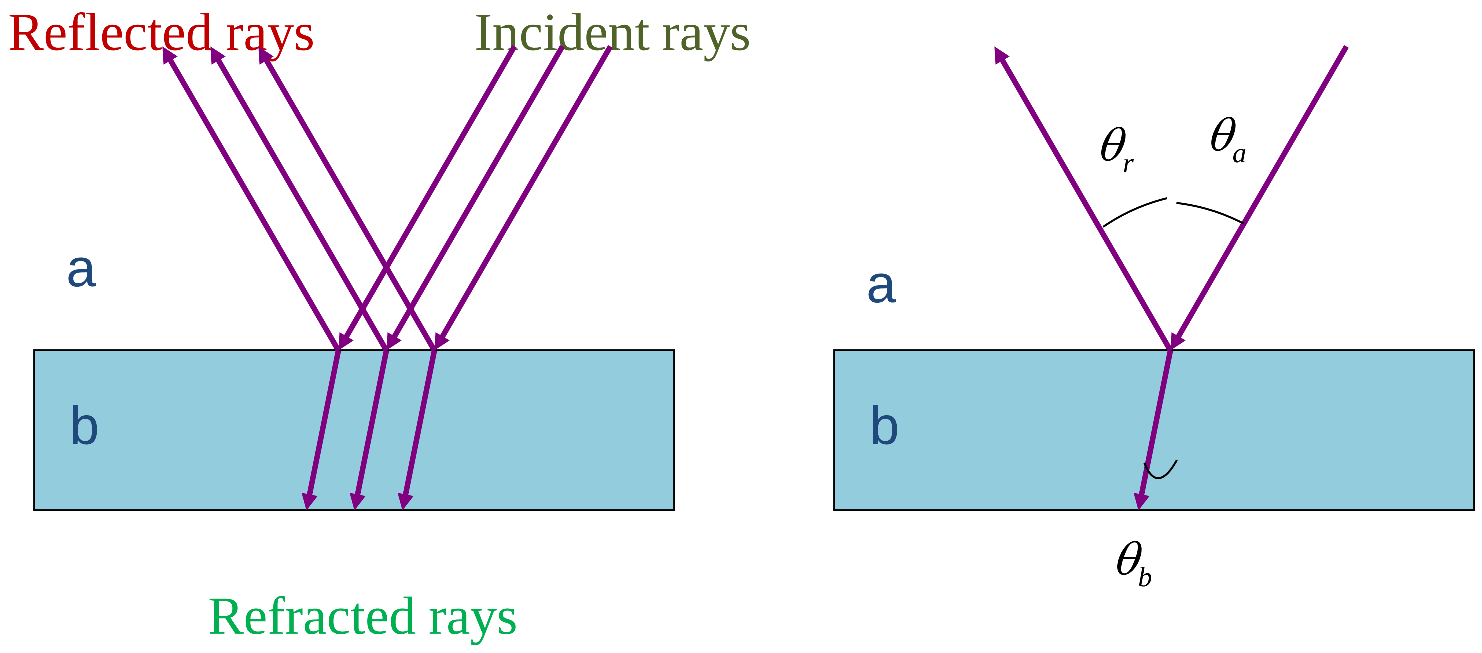
SURFACE IMPEDANCE

POTNTING VECTOR AND POYNTING THEOREM-APPLICATIONS

POWER LOSS IN A PLANE CONDUCTOR

Reflection and refraction:

When a light wave strikes a smooth interface of two transparent media (such as air, glass, water etc.), the wave is in general partly reflected and partly refracted (transmitted).



- If a transmission line having a characteristic impedance $'Z_0'$ and that line is terminated in load impedance $'Z_L'$.
- If $Z_L \neq Z_0$, then there is no mismatch between two impedances and the line is not properly terminated, at this case reflection occurs.
- If $Z_L = Z_0$, the line is properly terminated

Types of incidences:

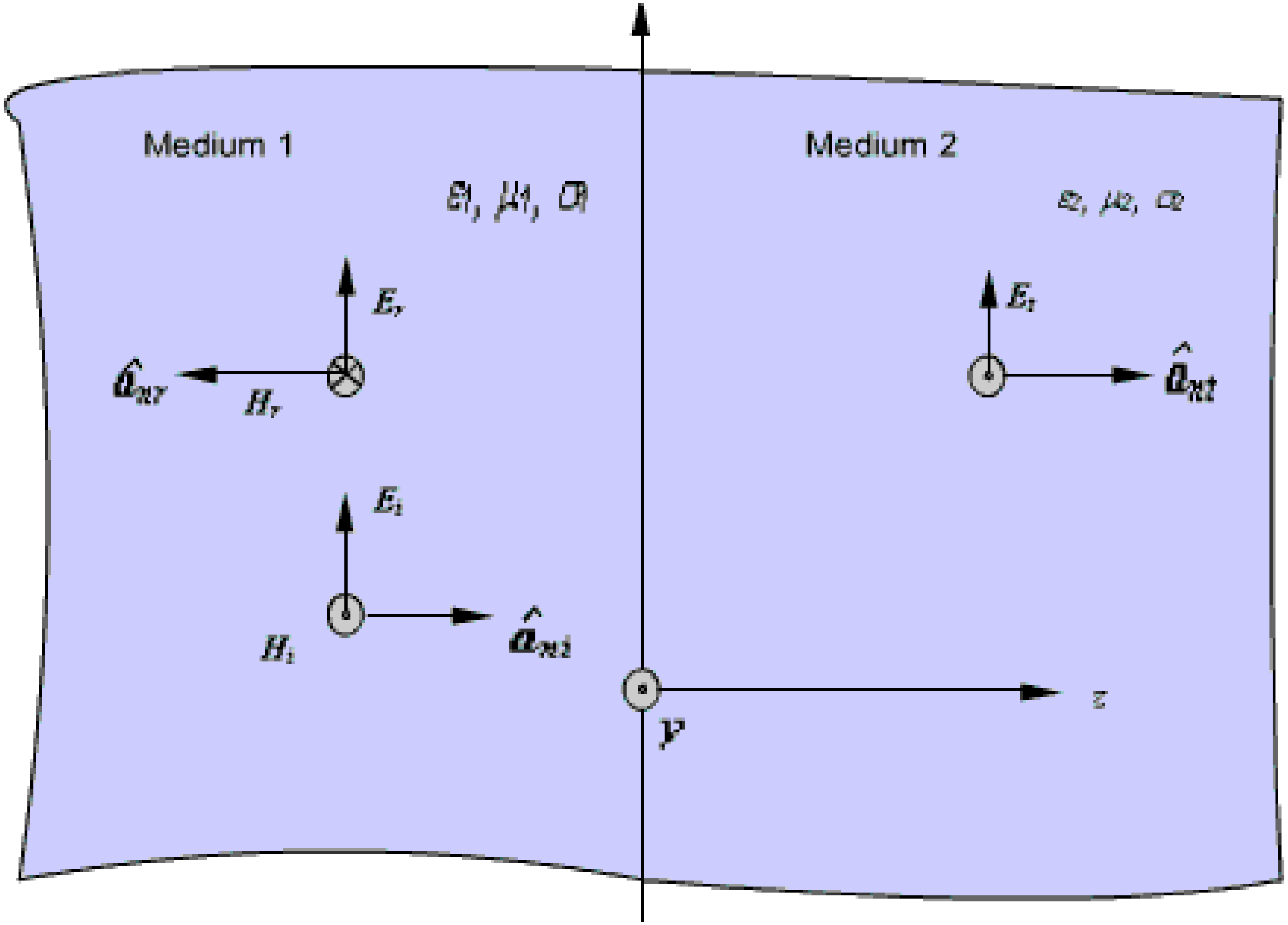
1. **Normal Incidence:**

When a uniform plane wave is incidences normally to the boundary between the media then it is known as normal incidence.

1. **Oblique incidence:**

When a uniform plane wave is incidences normally to the boundary between the media then it is known as normal incidence.

Normal Incidence at plane dielectric Boundary:



- A uniform plane wave striking the interface between the two dielectrics at right angles.
- A uniform plane wave travels along +z- direction and incidence at right angles at z=0.

- Below z = 0, let the properties of medium 1 be $\varepsilon_1, \mu_1, \sigma_1, \eta_1$ and above z=0, the properties of medium 2 be $\varepsilon_2, \mu_2, \sigma_2, \eta_2$.
- Let E_i, H_i be the field strengths of the incident wave striking the boundary.
- Let E_t, H_t be the field strengths of the transmitted wave in the medium 2.
- Let E_r, H_r be the field strengths of the reflected wave in the medium 1returning back from the interface.

For medium 1,

$\mathrm{E_1} = \mathrm{E_i} + \mathrm{E_r}$ & $\mathrm{H_1} = \mathrm{H_i} + \mathrm{H_r}$

For medium 2,

$\mathrm{E_2} = \mathrm{E_t}$ & $\mathrm{H_2} = \mathrm{H_t}$

According to the boundary conditions, the tangential components of E and H must be continuous at the interface, z = 0.

The waves are transverse in nature, at the boundary the fields E and H both tangential to the interface.

$E_{1tan} = E_{2tan} \& H_{1tan} = H_{2tan}$

At the interface, z = 0

$E_i = \eta_1 H_i$, $E_r = -\eta_1 H_r \& E_t = \eta_2 H_t$

$E_t = E_i + E_r$ -------(1)

$H_t = H_i + H_r$ -------(2)

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2} \Rightarrow \frac{E_i - E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$E_i - E_r = \frac{\eta_1}{\eta_2} E_t$ --------(3)

Add eq(1) &eq(2),

$$E_i + E_r + E_i \cancel{-} E_r = E_t \cancel{+} \frac{\eta_1}{\eta_2} E_t$$

$$2E_i = \left(1+\frac{\eta_1}{\eta_2}\right)E_t \Rightarrow E_t = \frac{2\eta_2}{\eta_1+\eta_2} E_i \text{ -------(4)}$$

The transmission coefficient (τ),

$$\boldsymbol{\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1+\eta_2}} \text{-------------(5)}$$

Eliminating the E_t from eq(1) &(3),

$$E_i + E_r = \frac{\eta_2}{\eta_1}(E_i - E_r) \quad \Rightarrow \eta_1(E_i + E_r) = \eta_2(E_i - E_r)$$

$$E_r = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} E_i \quad \text{----------(6)}$$

The reflection coefficient (Γ),

$$\boldsymbol{\Gamma = \frac{E_r}{E_i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}} \text{ -----------(7)}$$

From eq(5) & (7),

a) $1 + \Gamma = \tau$ b) $0 \leq |\Gamma| \leq 1$

c) Both the coefficients, Γ, τ are dimensionless and may be complex in nature.

According to the poynting theorem,

$$\boldsymbol{P_{avg} = \frac{1}{2}\frac{E_m{}^2}{\eta}} \quad \mathbf{W/\,m^2}$$

The average power incident in medium1 ,

$$P_{iavg} = \frac{1}{2}\frac{E_i{}^2}{\eta_1} \quad W/\,m^2$$

The average power reflected in medium1 ,

$$P_{ravg} = \frac{1}{2}\frac{E_r{}^2}{\eta_1} \quad W/\,m^2$$

The average power transmitted in medium2 ,

$$P_{tavg} = \frac{1}{2}\frac{E_t{}^2}{\eta_2} \quad W/\,m^2$$

The ratio of power transmitted to power incident,

$$\frac{P_{tavg}}{P_{iavg}} = \frac{\frac{1}{2}\frac{E_t^{\,2}}{\eta_2}}{\frac{1}{2}\frac{E_i^{\,2}}{\eta_1}} = \frac{\eta_1}{\eta_2}\left(\frac{\cancel{E_t}}{\cancel{E_i}}\right)^2$$

$$\boldsymbol{\frac{P_{tavg}}{P_{iavg}} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2}} \text{-------(8)} \quad \left(\text{since, } \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1+\eta_2}\right)$$

The ratio of power reflected to power incident,

$$\frac{P_{ravg}}{P_{iavg}} = \frac{\frac{1}{2}\frac{E_r^{\,2}}{\eta_1}}{\frac{1}{2}\frac{E_i^{\,2}}{\eta_1}} = \left(\frac{\cancel{E_r}}{\cancel{E_i}}\right)^2$$

$$\boldsymbol{\frac{P_{ravg}}{P_{iavg}} = \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}} \text{--------(9)} \quad \left(\text{since, } \frac{E_r}{E_i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right)$$

Add eq(8) &(9),

$$\frac{P_{tavg}}{P_{iavg}} + \frac{P_{ravg}}{P_{iavg}} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} + \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}$$

$$\boldsymbol{P_{tavg} + P_{ravg} = P_{ravg}} \quad \textbf{--------(10)}$$

Normal Incidence at Plane Conducting Boundary:

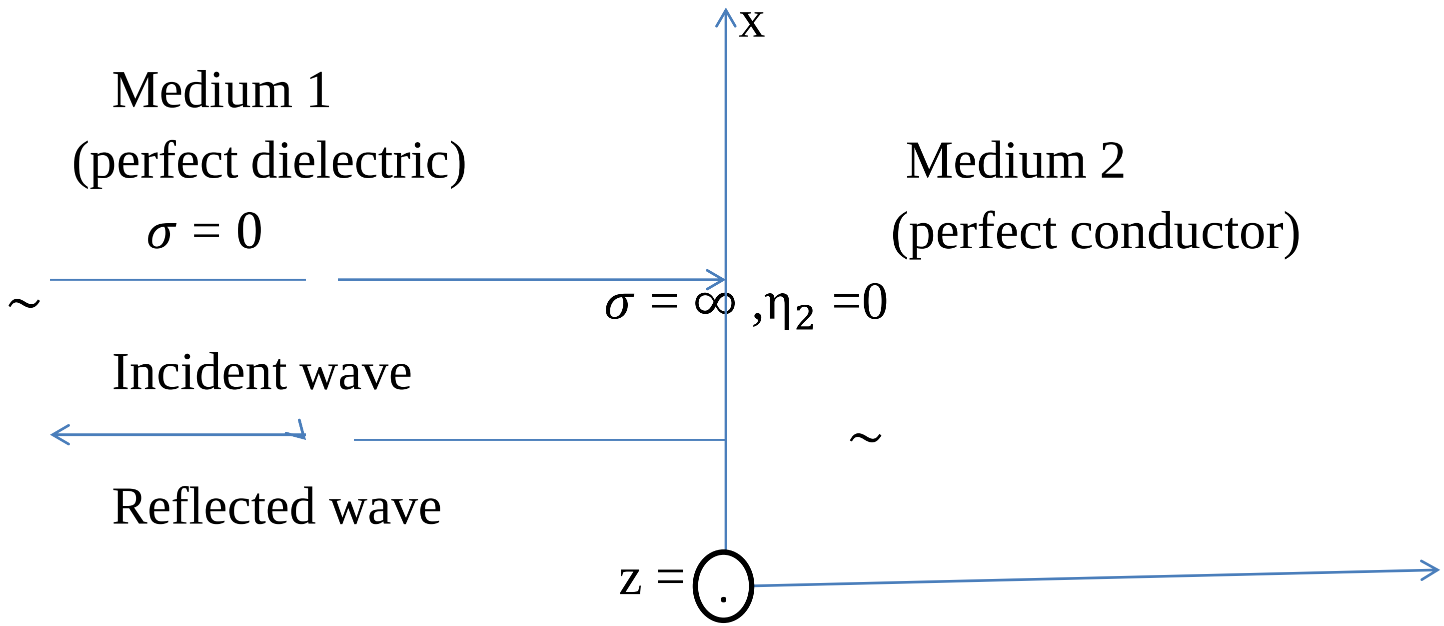


Fig: Normal incident at plane conducting boundary

- A uniform plane wave striking the interface between the two media.
- Where medium1 is perfect dielectric ($\sigma = 0$, lossless) and medium2 is perfect conductor ($\sigma = \infty$)

For medium2 ,η_2 =0, being a perfect conductor.

STANDING WAVES:

- Standing waves are nothing but it consists of two travelling waves, one is incident and the other is reflected and are does not travel.
- Both the waves have same amplitudes but the directions in which they propagate are different.

Let the standing wave in medium be denoted by E_{1s},

E_{1s}= E_i+ E_r = $(E_i e^{-\gamma_1 z} + E_r e^{\gamma_1 z})a_x$ ------(1)

But $\Gamma = \frac{E_r}{E_i}$ = -1

For medium 1, $\sigma = 0$,

$\gamma_1 = \alpha + j\beta_1$ where $\alpha = 0$ for $\sigma = 0$,

$\gamma_1 = j\beta_1$

$E_{1s} = (E_i e^{-j\beta_1 z} - E_i e^{j\beta_1 z}) a_x$ (since $\frac{E_r}{E_i} = -1$)

$E_{1s} = -2j\, E_i \frac{e^{j\beta_1 z} - e^{-j\beta_1 z})}{2j} a_x$

$E_{1s} = -2j\, E_i \sin\beta_1 z\, a_x$ ------------(2)

The field in the medium1,

$E_1 = \mathrm{Re}\,(E_{1s} e^{j\omega t})$

$E_1 = 2\, E_i \sin\beta_1 z \sin\omega t\; a_x$ -----------(3)

$H_1 = \frac{2E_i}{\eta_1} \cos\beta_1 z \cos\omega t\, a_y$ -----------(4)

If $\beta_1 z = n\pi$, $n = 0, \pm 1, \pm 2$ ----------

$z = \frac{n\pi}{\beta_1}$, but $\beta_1 = \frac{2\pi}{\lambda_1}$,

$z = \frac{n\pi}{\frac{2\pi}{\lambda_1}} \Rightarrow z = n\frac{\lambda_1}{2}$

Representation of instantaneous values of the total electric field at $t=\lambda_1/2$ in medium1.

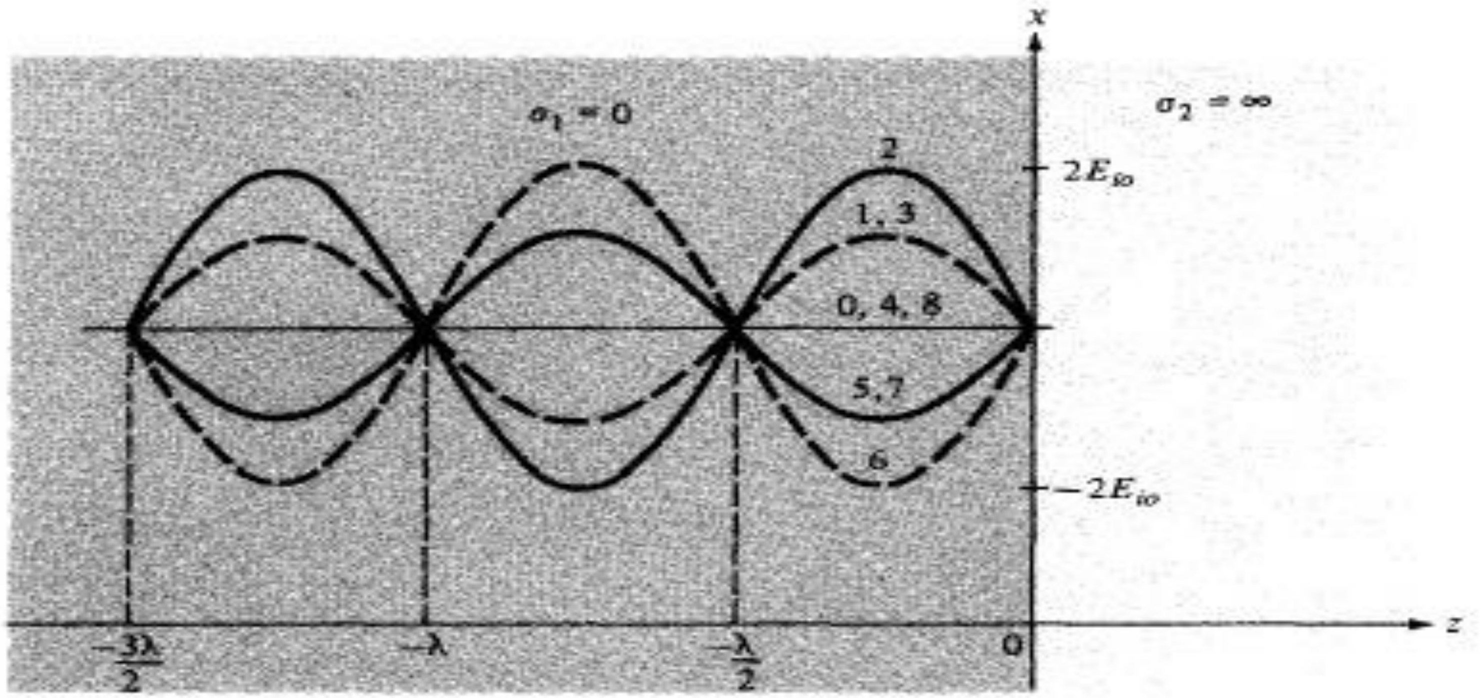


Figure 10.12 Standing waves $E = 2E_{io} \sin \beta_1 z \sin \omega t \, \mathbf{a}_x$; curves 0, 1, 2, 3, 4, . . . are, respectively, at times $t = 0, T/8, T/4, 3T/8, T/2, \ldots$; $\lambda = 2\pi/\beta_1$.

The magnitude of the magnetic field is maximum at the positions where the electric field is zero.

STANDING WAVE RATIO(SWR):

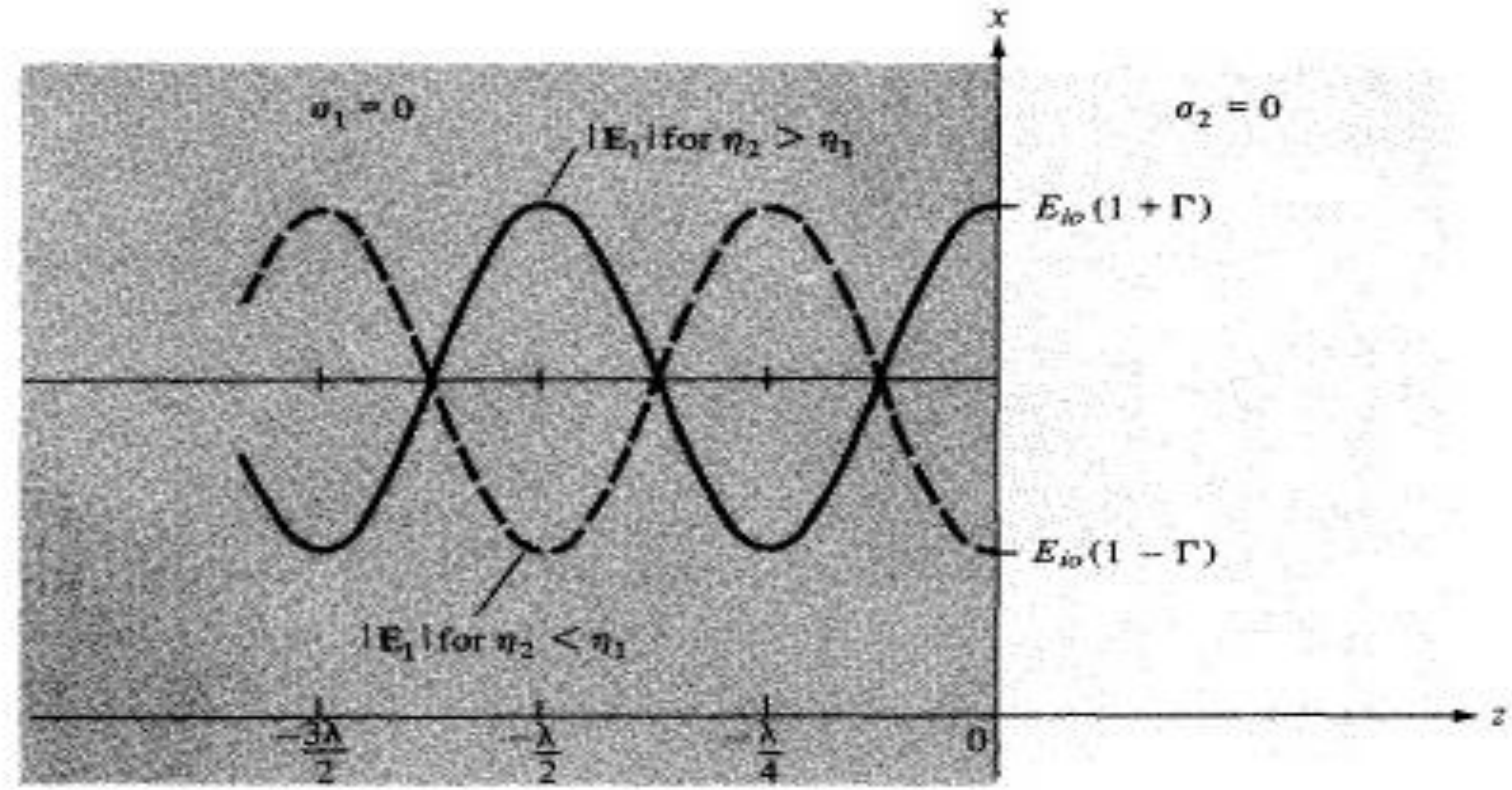


Figure 10.13 Standing waves due to reflection at an interface between two lossless media; $\lambda = 2\pi/\beta_1$.

A uniform plane wave travelling in a lossless medium , it gets reflected back by the perfect conductor, results in which a standing wave is generated.

The total field in the medium1 along the standing wave,

$E_{1s} = E_i + E_r = (E_i e^{-\gamma_1 z} + E_r e^{\gamma_1 z})$ ------------(1)

The reflection coefficient,

$\Gamma = \frac{E_r}{E_i}$ or $E_r = \Gamma E_i$

$E_{1s} = E_i e^{-j\beta_1 z} + \Gamma E_i e^{j\beta_1 z}$ -----------(2)

The reflection coefficient in phasor form,

$\Gamma = |\Gamma| e^{j\phi}$

$E_{1s} = (e^{-j\beta_1 z} + |\Gamma| e^{j(\beta_1 z + \phi)})E_i$ ----------(3)

$E_{1smax} = (1 + |\Gamma|)E_i$ -----------(4)

$E_{1smin} = (1 - |\Gamma|)E_i$ -----------(5)

⇒*Standing wave ratio is defined as the ratio of maximum to minimum amplitudes of voltage.*

$$\mathbf{S = \frac{E_{1smax}}{E_{1smin}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$ (or)

$$|\Gamma| = \frac{S-1}{S+1}$$

OBLIQUE INCIDENCE AT A PLANE DIELECTRIC BOUNDARY:

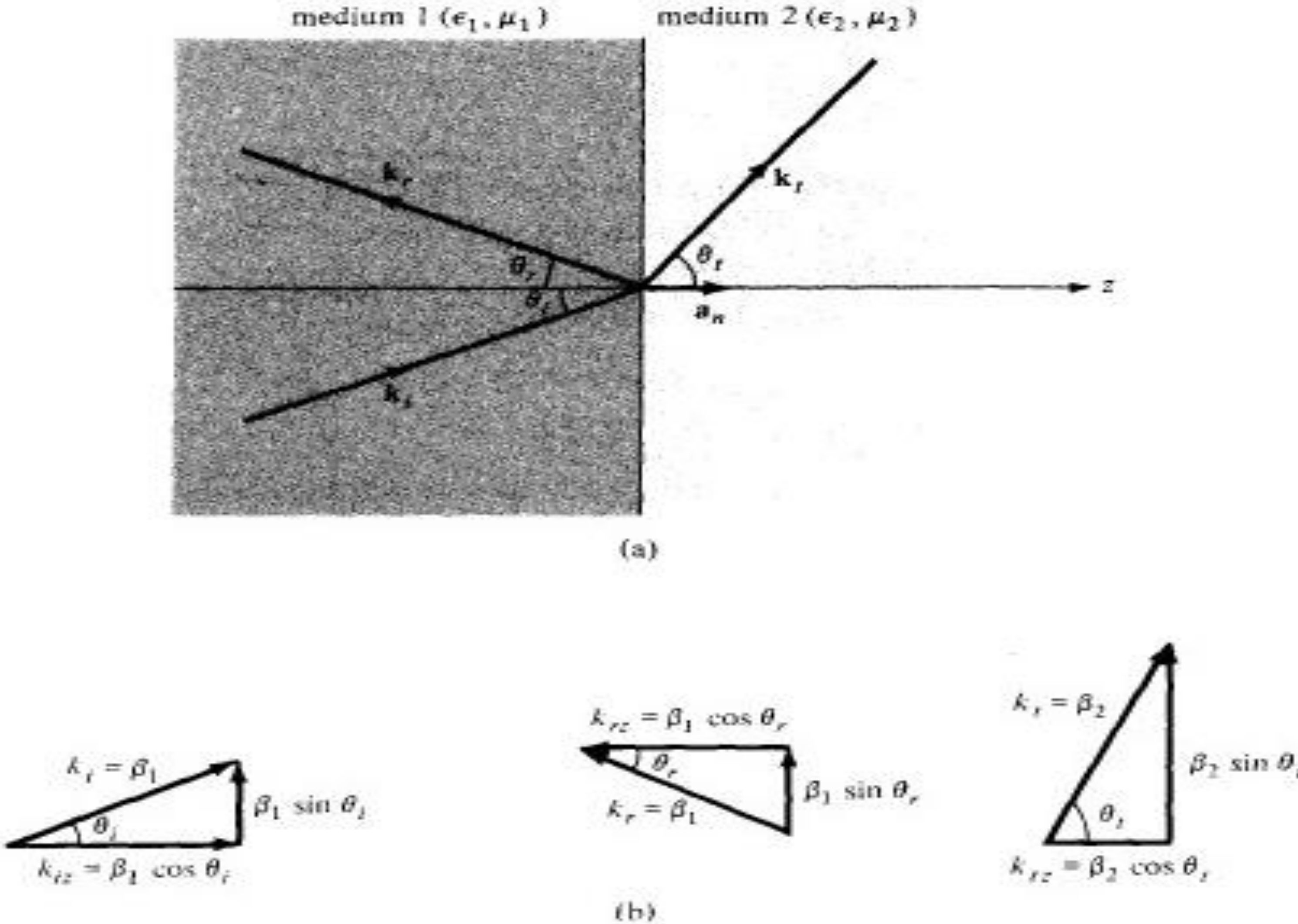


Figure 10.15 Oblique incidence of a plane wave: **(a)** illustration of θ_i, θ_r, and θ_t; **(b)** illustration of the normal and tangential components of **k**.

- The incident and reflected waves travel in medium1, while transmitted wave in medium2.
- The velocity for the waves in medium 1 is same and the distances travel by the waves are same.

$\boldsymbol{\theta_i = \theta_r}$-------(1)

Snell's law of reflection:

The angle of incidence and angle of reflection waves are equal.

$$\frac{\sin \boldsymbol{\theta_t}}{\sin \boldsymbol{\theta_i}} = \frac{v_2}{v_1} = \frac{\beta_1}{\beta_2} \text{------------(2)} \quad (\because\ v = \omega/\beta)$$

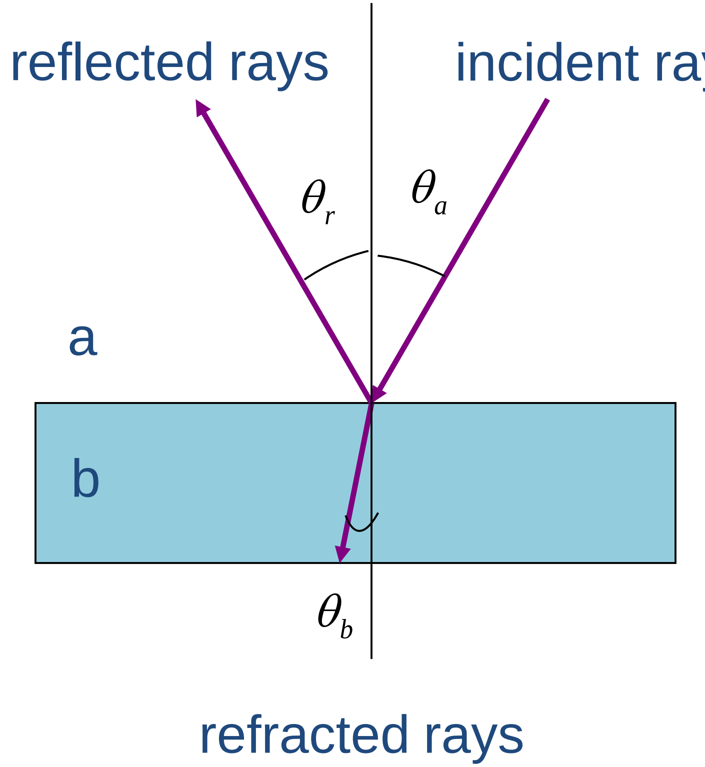
$$\boldsymbol{\frac{\sin \theta_t}{\sin \theta_i} = \frac{n_1}{n_2}} \text{-----------(3)} \quad (\because\ n = c/v)$$

which is snell's law of refraction

$$\frac{\sin \boldsymbol{\theta_t}}{\sin \boldsymbol{\theta_i}} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \quad \text{------}(4) \quad (\because \ \mathrm{v} = \frac{1}{\sqrt{\mu\varepsilon}})$$

$$\frac{\sin \boldsymbol{\theta_t}}{\sin \boldsymbol{\theta_i}} = \frac{\eta_2}{\eta_1} \quad \text{--------}(5) \quad (\because \eta = \sqrt{\frac{\mu}{\varepsilon}})$$

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{v_2}{v_1} = \frac{\beta_1}{\beta_2} = \frac{\mathrm{n}_1}{\mathrm{n}_2} = \frac{\eta_2}{\eta_1} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$



CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION:

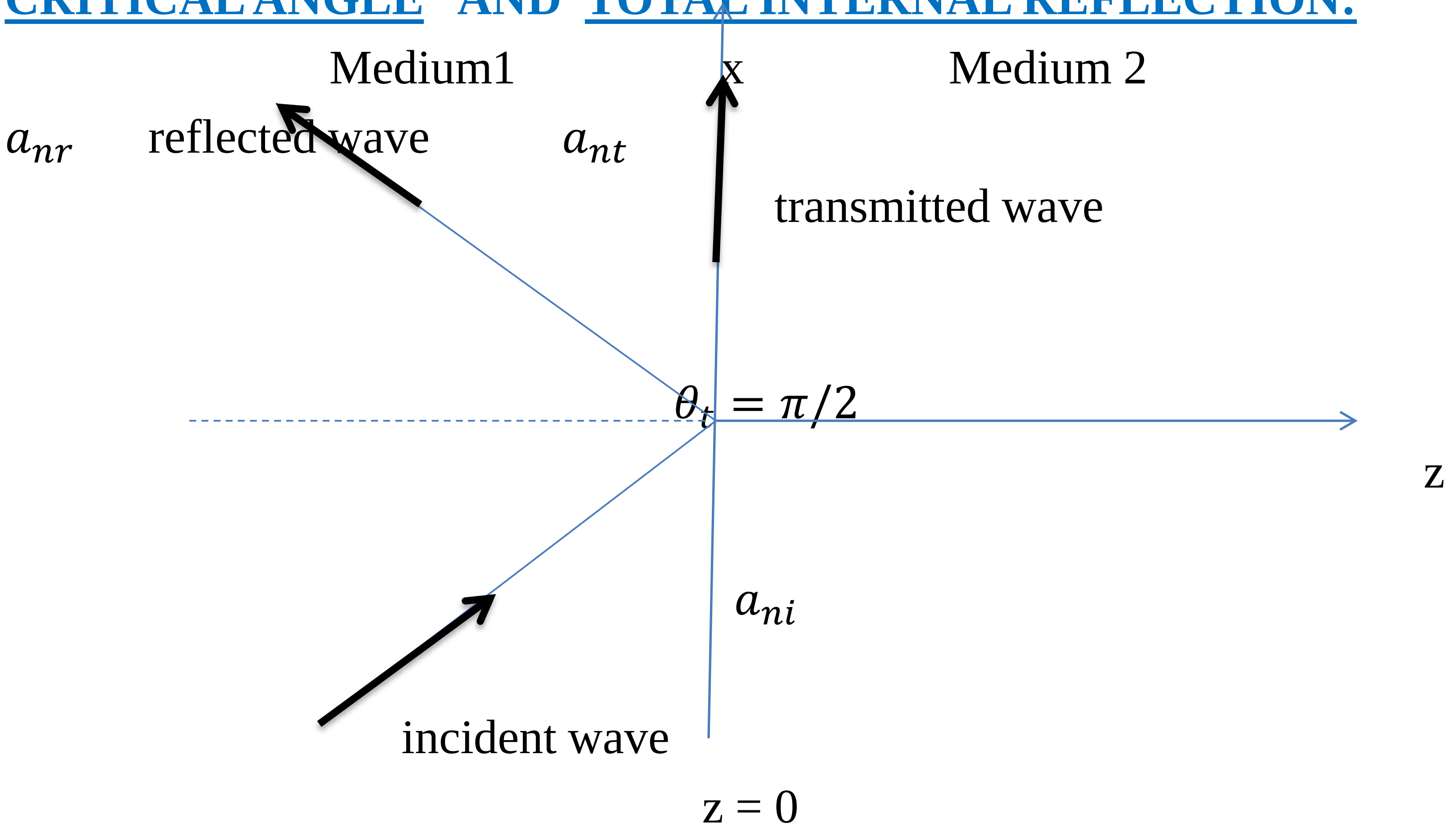


Fig: total reflection at $\theta_t = \pi/2$

- The medium1 is denser than the medium2 i.e. $\varepsilon_1 > \varepsilon_2$
- The angle of transmission θ_t becomes greater than the angle of incidence.

CRITICAL ANGLE: (θ_c)

The angle of incidence at which the total reflection takes place is known as critical angle.

At $\theta_t = \pi/2$, $\theta_i = \theta_c$,

$$\frac{\sin \boldsymbol{\theta_t}}{\sin \boldsymbol{\theta_i}} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

$$\Rightarrow \qquad \frac{\sin\frac{\pi}{2}}{\sin \boldsymbol{\theta_c}} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$$

$$\sin \boldsymbol{\theta_c} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\boldsymbol{\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}} \qquad \textbf{(or)}$$

$$\boldsymbol{\theta_c = sin^{-1}\frac{n_2}{n_1}}$$

POLARIZATION:

- Horizontal or perpendicular polarization.
- Vertical or parallel polarization.

HORIZONTAL OR PERPENDICULAR POLARIZATION:

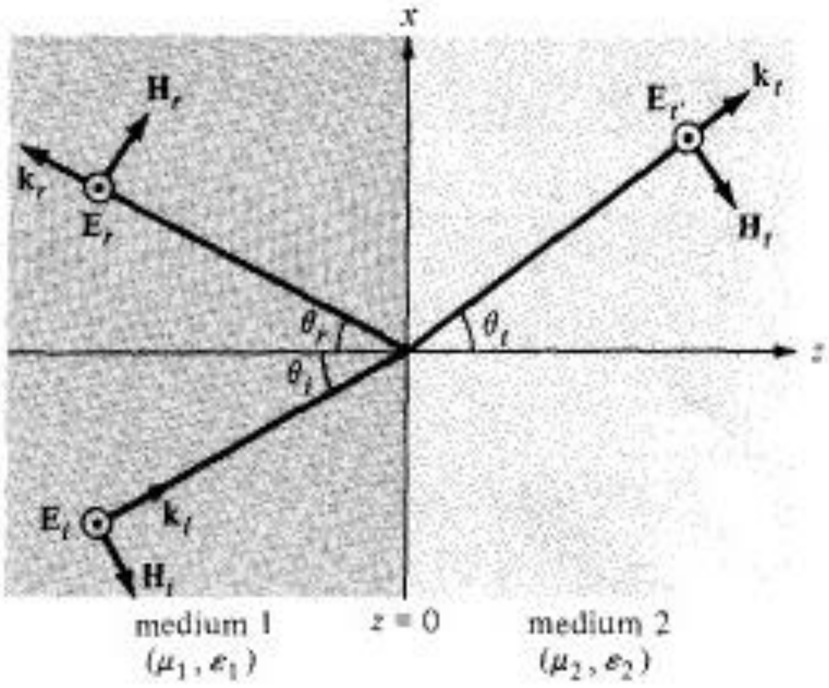


Figure 10.17 Oblique incidence with **E** perpendicular to the plane of incidence.

The incident electric field intensity vector in medium1,

$E_i = E_i e^{-j\beta_1(x \sin\theta_i + z\cos\theta_i)} a_y$ ------(1)

The incident magnetic field intensity vector in medium1,

$H_i = \frac{E_i}{\eta_1}(-\cos\theta_i a_X + \sin\theta_i\, a_Z)\; e^{-j\beta_1(x \sin\theta_i + z\cos\theta_i)}$------(2)

The reflected electric and magnetic fields,

$E_r = E_r e^{-j\beta_1(x \sin\theta_r - z\cos\theta_r)} a_y$ ------(3)

$H_r = \frac{E_r}{\eta_1}(\cos\theta_r a_X + \sin\theta_r\, a_Z)\; e^{-j\beta_1(x \sin\theta_r - z\cos\theta_r)}$ ----------(4)

The transmitted electric and magnetic fields,

$E_t = E_t e^{-j\beta_2(x \sin\theta_t + z\cos\theta_t)} a_y$------(5)

$H_t = \frac{E_t}{\eta_2}(-\cos\theta_t a_X + \sin\theta_t\, a_Z)\; e^{-j\beta_2(x \sin\theta_t + z\cos\theta_t)}$ ---------(6)

From the boundary conditions, the tangential components of E and H must be continuous at the interface, z=0. i.e

$E_1 = E_2 \Rightarrow E_i + E_r = E_t$

$E_i e^{-j\beta_1(x \sin\theta_i)} + E_r e^{-j\beta_1(x \sin\theta_r)} = E_t e^{-j\beta_2(x \sin\theta_t)}$ -------(7)

$H_1 = H_2 \Rightarrow H_i + H_r = H_t$

$\therefore \frac{1}{\eta_1}(- E_i \cos\theta_i \, e^{-j\beta_1(x \sin\theta_i)} + E_r \cos\theta_r \, e^{-j\beta_1(x \sin\theta_r)})$

$= - \frac{1}{\eta_2} E_t(\cos\theta_t \, e^{-j\beta_2(x \sin\theta_t)}$ ------------(8)

$\beta_1 x \sin\theta_i = \beta_1 x \sin\theta_r = \beta_2 x \sin\theta_t$ ----------(9)

We know that, Snell's law of reflection ($\theta_i = \theta_r$)

Snell's law of reflection ($\frac{\sin \boldsymbol{\theta_t}}{\sin \boldsymbol{\theta_i}} = \frac{\beta_1}{\beta_2}$)

POLARIZATION OF UNIFORM PLANE WAVES:

The polarization of a plane wave can be defined as the orientation of the electric field vector as a function of time at a fixed point in space.

For an electromagnetic wave, the specification of the orientation of the electric field is sufficient as the magnetic field components are related to electric field vector by the Maxwell's equations.

Types:

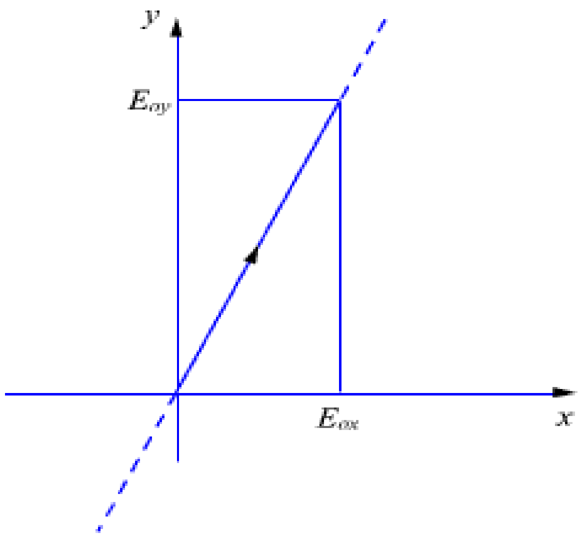
1. Linear polarization
2. Elliptical polarization
3. Circular polarization

LINEAR POLARIZATION

The electric field E has only x component and y component of E is 0. Then the wave is said to be linearly polarized in x- direction.

(or)

The resultant vector E is oriented in a direction which is constant with time, and the wave is said to be linearly polarized.

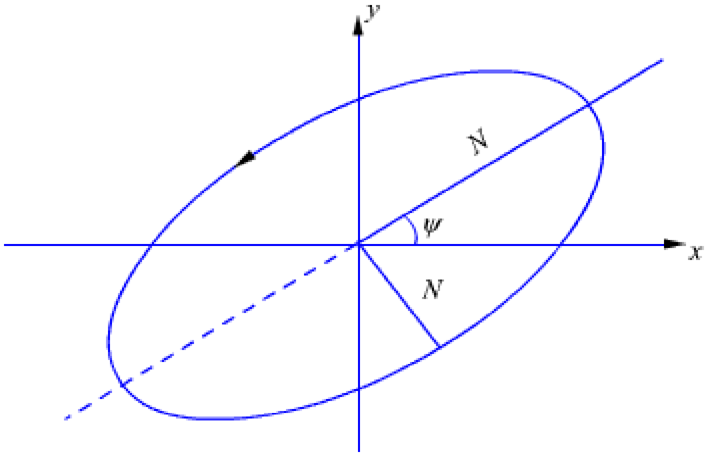


Ex and Ey components are in phase with either equal or unequal amplitudes , for a uniform plane wave travelling in z- direction, the polarization is linear.

ELLIPTICAL POLARIZATION

The electric field E has both the components which are not having same amplitudes and are not in phase.

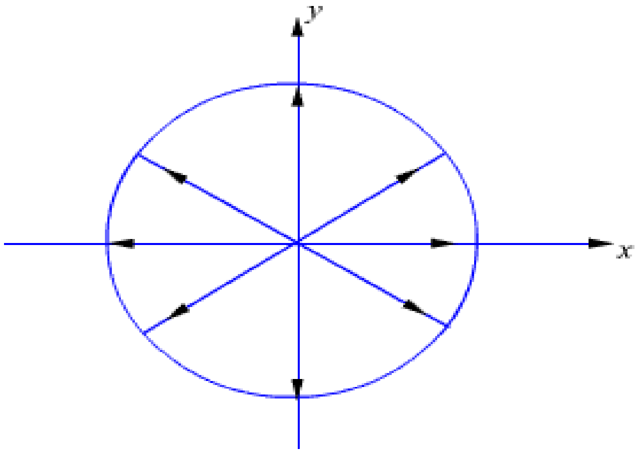
The amplitudes of Ex and Ey are different and the phase difference between the two is other than 90, then the axis of the ellipse are inclined at an angle θ with the coordinate axis.



The components Ex and Ey of unequal amplitudes have a constant ,non zero phase difference between two, for a uniform plane wave travelling in z-direction, the polarization is elliptical.

CIRCULAR POLARIZATION

- The amplitudes of Ex and Ey are same and the phase difference between the two is exactly 90.
- In one wavelength span, the resultant vector E completes one cycle of rotation such a wave is said to be circularly polarized.
-



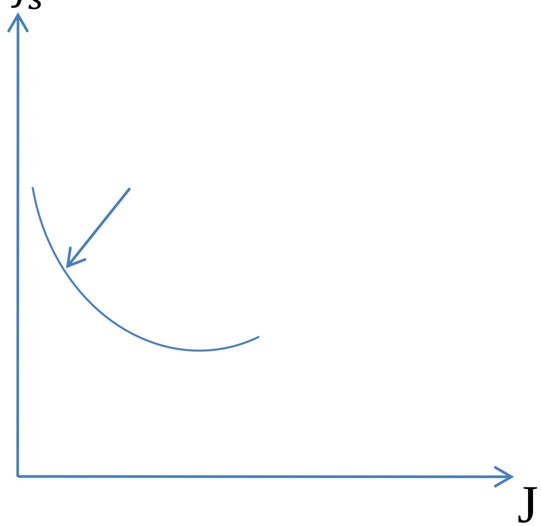
circular polarization

SURFACE IMPEDANCE

DEFINITION:

The ratio of the tangential component of the electric field to the surface current density at the conductor surface.

$$Z_s = \frac{E_{tan}}{J_s} \quad \text{--------(1)}$$



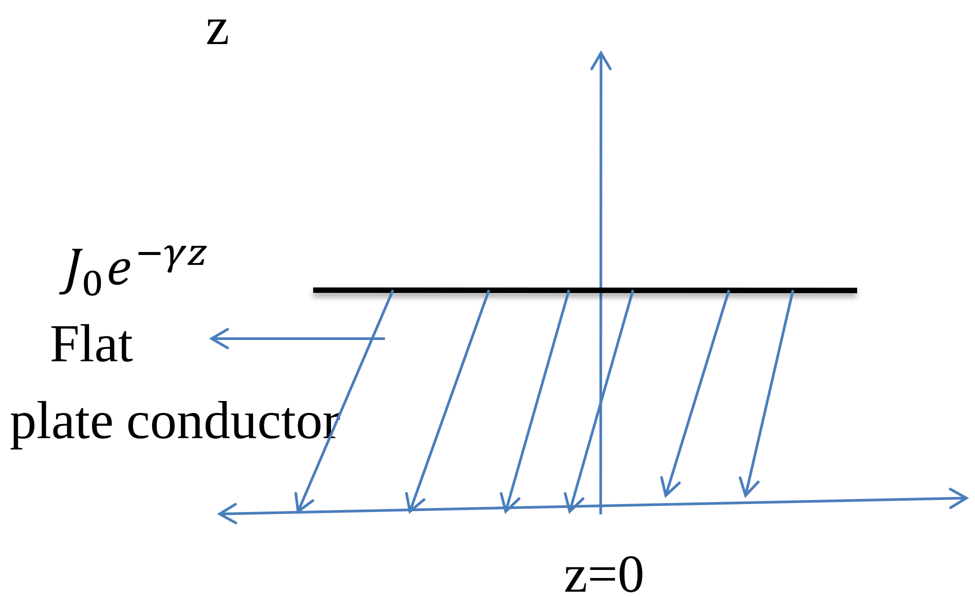


Fig: Current distribution in flat plate conductor

With the surface at z=0, plane, then the current distribution in z-direction is given by,

$$J = J_0 e^{-\gamma z} \quad \text{-------------(2)}$$

Linear current density is given by,

$$J_s = \int_0^{\infty} J_0 e^{-\gamma z} dz$$

$$J_s = \frac{J_0}{\gamma}$$

But we know that, $J_0 = \sigma E_{tan}$

$$J_s = \frac{\sigma E_{tan}}{\gamma}$$

$$Z_s = \frac{\gamma}{\sigma}$$

The propagation constant ‘ γ’ is given by,

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

For conducting medium, $\sigma >> \omega\varepsilon$

$$\gamma = \sqrt{j\omega\mu\sigma}$$

$$Z_s = \frac{\sqrt{j\omega\mu\sigma}}{\sigma}$$

$$Z_s = \sqrt{\frac{j\omega\mu}{\sigma}} = \eta \qquad (\because \eta = \sqrt{\frac{j\omega\mu}{\sigma}})$$

For a good conductor, The surface impedance of a plane conductor with thickness greater than the skin depth of a conductor is equal to the characteristic impedance of the conductor.

$$\boldsymbol{Z_s = \eta}$$

POYNTING VECTOR & POYNTING THEOREM:

The energy stored in an electric field and magnetic field is transmitted at a certain rate of energy flow which can be calculated with the help of poynting theorem.

The power density is given by,

$\bar{P} = \bar{E} \times \bar{H}$

Where $\bar{P}$ is called poynting vector.

- Poynting theorem is based on law of conservation of energy in electromagnetism.
- The direction of $\bar{P}$ indicates instantaneous power flow at that point.

DEFINITION:

"The net power flowing out of a given volume 'v' is equal to the time rate of decrease in the energy stored within volume 'v' minus the ohmic power dissipated".

$\overline{E} = E_x \overline{a_x}$ & $\overline{H} = H_y \overline{a_y}$

Then $\overline{P} = (E_x \overline{a_x}) \times (H_y \overline{a_y})$

$\overline{P} = E_x H_y \overline{a_z}$ $(\because \overline{a_x} \times \overline{a_y} = \overline{a_z})$

$\mathbf{\overline{P} = P_Z \overline{a_z}}$

Where $\bar{P}$, $\bar{E}$, $\bar{H}$ are mutually perpenicular to each other.

The electric field propagates in free space given by,

$\overline{\mathrm{E}} = \mathrm{E_m} \cos(\omega t - \beta z)\,\overline{\mathrm{a_x}}$

In the medium , the ratio of magnitudes of $\overline{\mathrm{E}}$ and $\overline{\mathrm{H}}$ depends on its intrinsic impedance (η),

$\overline{\mathrm{H}} = \mathrm{H_m} \cos(\omega t - \beta z)\,\overline{\mathrm{a_y}}$

or $\overline{\mathrm{H}} = \frac{\mathrm{E_m}}{\eta_0} \cos(\omega t - \beta z)\,\overline{\mathrm{a_y}}$

According to the poynting theorem, $\bar{P} = \bar{E} \times \bar{H}$

$\bar{P} = (\mathrm{E_m} \cos(\omega t - \beta z)(\overline{\mathrm{a_x}}) \times (\frac{\mathrm{E_m}}{\eta_0} \cos(\omega t - \beta z)\,\overline{\mathrm{a_y}})$

$\bar{P} = \frac{\mathrm{E_m}^2}{\eta_0} cos^2(\omega t - \beta z)\overline{\mathrm{a_z}}$ W/m^2

The power passing particular area is given by,

power = power density × area

AVERAGE POWER DENSITY: (P_{avg}):

To find the average power density , let us integrate power density in z-direction over one cycle and divide by the period T of one cycle.

$$P_{avg} = \frac{1}{T}\int_0^T \frac{\mathrm{E_m}^2}{\eta} \cos^2(\omega t - \beta z)\,\mathrm{dt}$$

$$= \frac{\mathrm{E_m}^2}{\eta T}\int_0^T \frac{1+\cos 2(\omega t - \beta z)}{2}\,\mathrm{dt}$$

$$= \frac{\mathrm{E_m}^2}{\eta T}\left[\frac{t}{2} + \frac{\sin 2(\omega t - \beta z)}{4\omega}\right]_0^T$$

$$P_{avg} = \frac{\mathrm{E_m}^2}{\eta T}\left[\frac{T}{2} + \frac{\sin(2\omega T - \beta z)}{4\omega} + \frac{\sin(2\beta z)}{4\omega}\right]$$

Let $\omega T = 2\pi$,

$$P_{avg} = \frac{\mathrm{E_m}^2}{\eta T}\left[\frac{T}{2} + \frac{\sin(4\pi - \beta z)}{4\omega} + \frac{\sin(2\beta z)}{4\omega}\right]$$

$$P_{avg} = \frac{\mathrm{E_m}^2}{\eta T}\left[\frac{T}{2} - \cancel{\frac{\sin(2\beta z)}{4\omega} + \frac{\sin(2\beta z)}{4\omega}}\right]$$

$$P_{avg} = \frac{\mathrm{E_m}^2}{\eta\ T}\left[\frac{T}{2}\right]$$

$$\boldsymbol{P_{avg} = \frac{1}{2}\frac{E_m{}^2}{\eta}}\mathrm{W}/m^2$$

TRANSMISSION LINES-I

OBJECTIVES

TRANSMISSION LINES TYPES

TRANSMISSION LINE EQUATIONS

CHARACTERISTIC IMPEDANCE

PROPAGATION CONSTANT

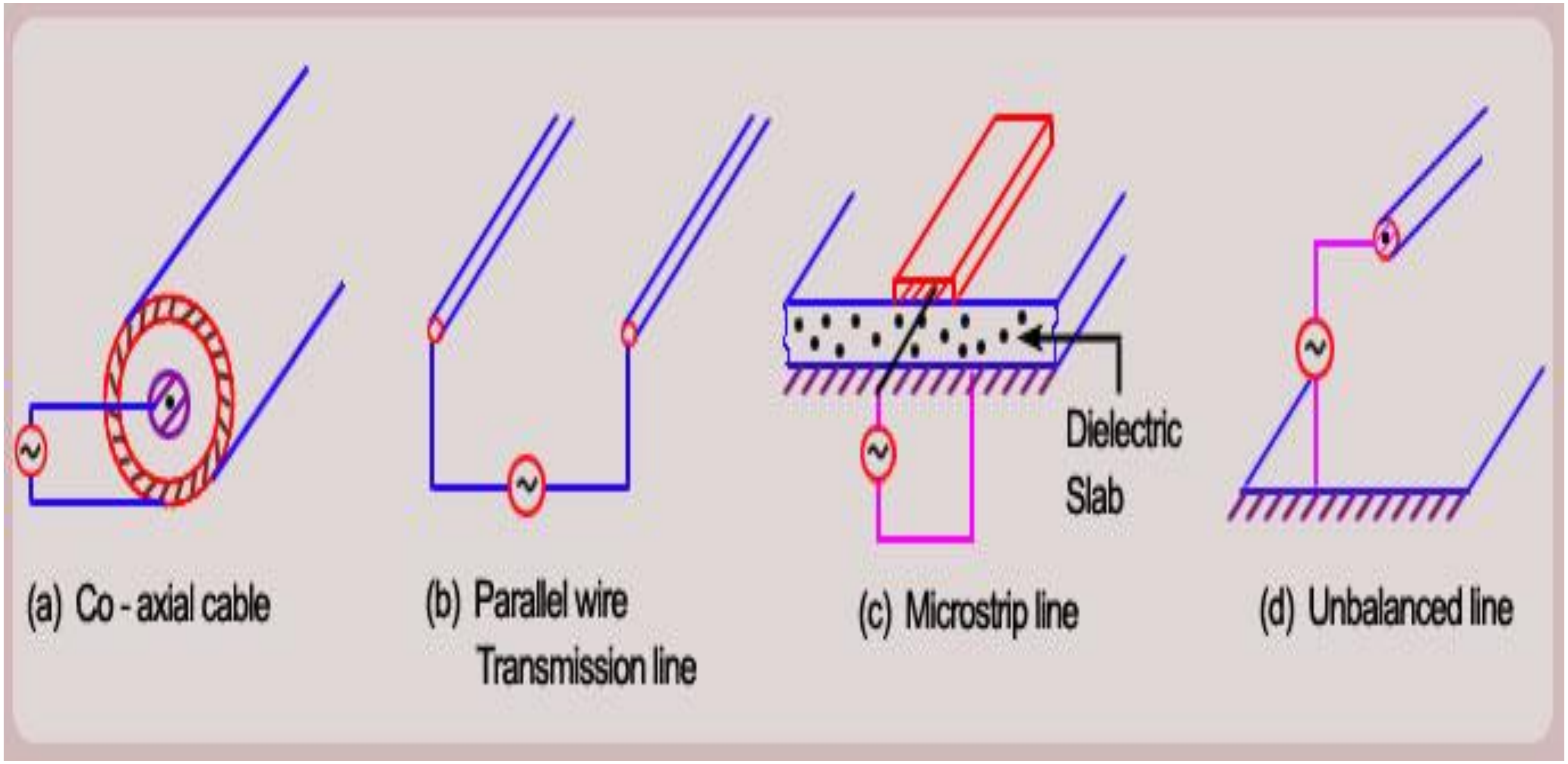
LOSSLESS TRANSMISSION LINE

DISTORTIONLESS TRANSMISSION LINE

LOADING- TYPES

DEFINITION:

The transmission line is a structure which can transport electrical energy from one point to another.



•At low frequencies, a transmission line consists of two linear conductors separated by a distance.

•When an electrical source is applied between the two conductors, the line gets energized and the electrical energy flows along the length of the conductors.

Co-axial cable:

Consists of a solid conducting rod surrounded by the two conductors. This line has good isolation of the electrical energy and has low Electromagnetic Interference (EMI).

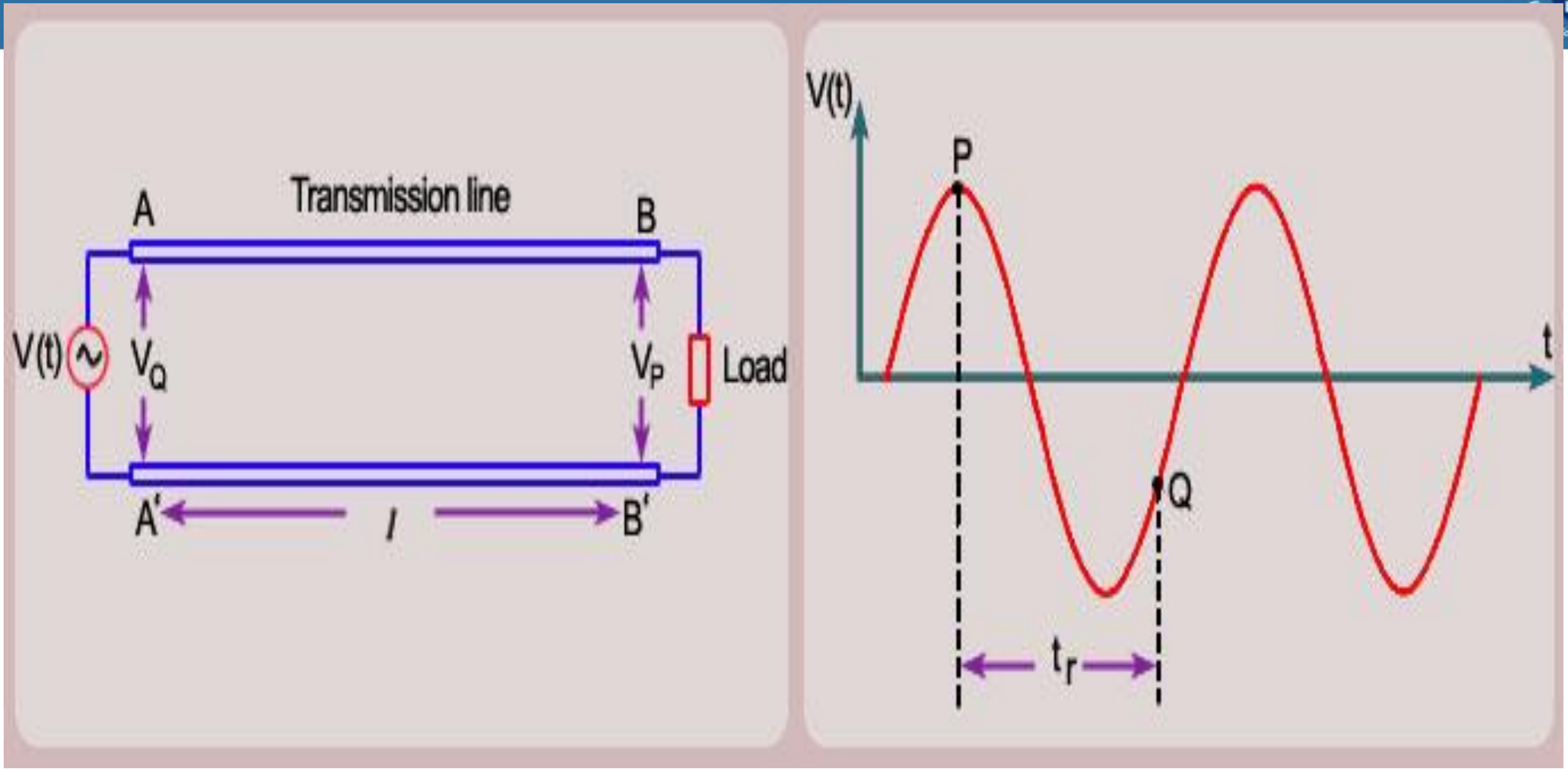
Parallel wire transmission line:

- Consists of two parallel conducting rods. In this case the electrical energy is distributed between and around the rods.
- Theoretically the electric and magnetic fields extend over infinite distance though their strength reduces as the distance from the line. Obviously this line has higher EMI.

- **Micro strip line:**
- Consists of a dielectric substrate having ground plane on one side and a thin metallic strip on the other side.
- The majority of the fields are confined in the dielectric substrate between the strip and the ground plane.
- Some fringing field exist above the substrate which decay rapidly as a function of height.
- This line is usually found in printed circuit boards at high frequencies.

Balanced and Un-balanced line:

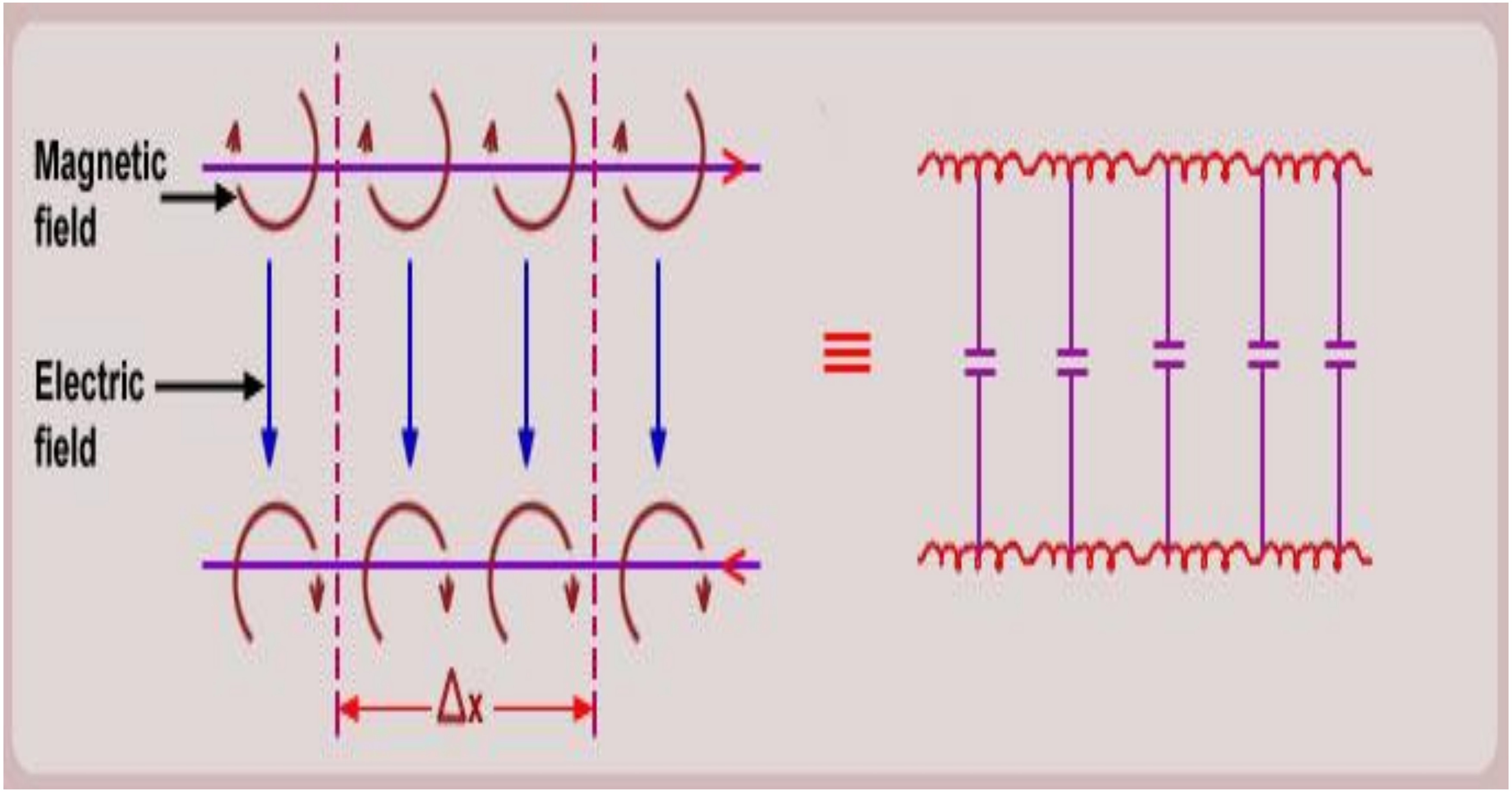
- If the two conductors are symmetric around the ground, then the line is called the balanced line, otherwise the line is an un-balanced line.
- Transmission lines (a), (c) and (d) are un-balanced line, whereas the line (b) is a balanced line.



•No Signal can travel with infinite velocity. That is to say that if a voltage or current changes at some location, its effect cannot be felt instantaneously at some other location.

•There is a finite delay between the 'cause' and the effect. This is called the ' *Transit Time'* effect.

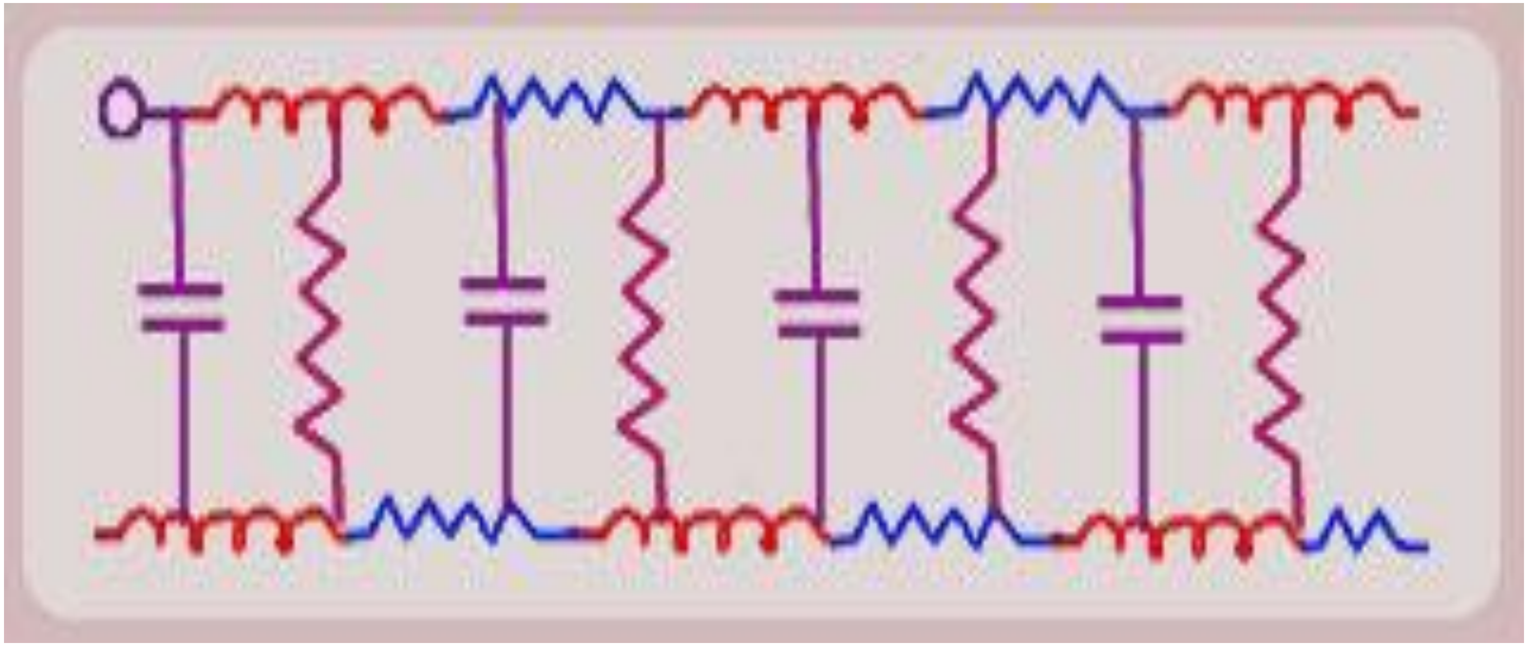
DISTRIBUTED CIRCUIT ELEMENTS:



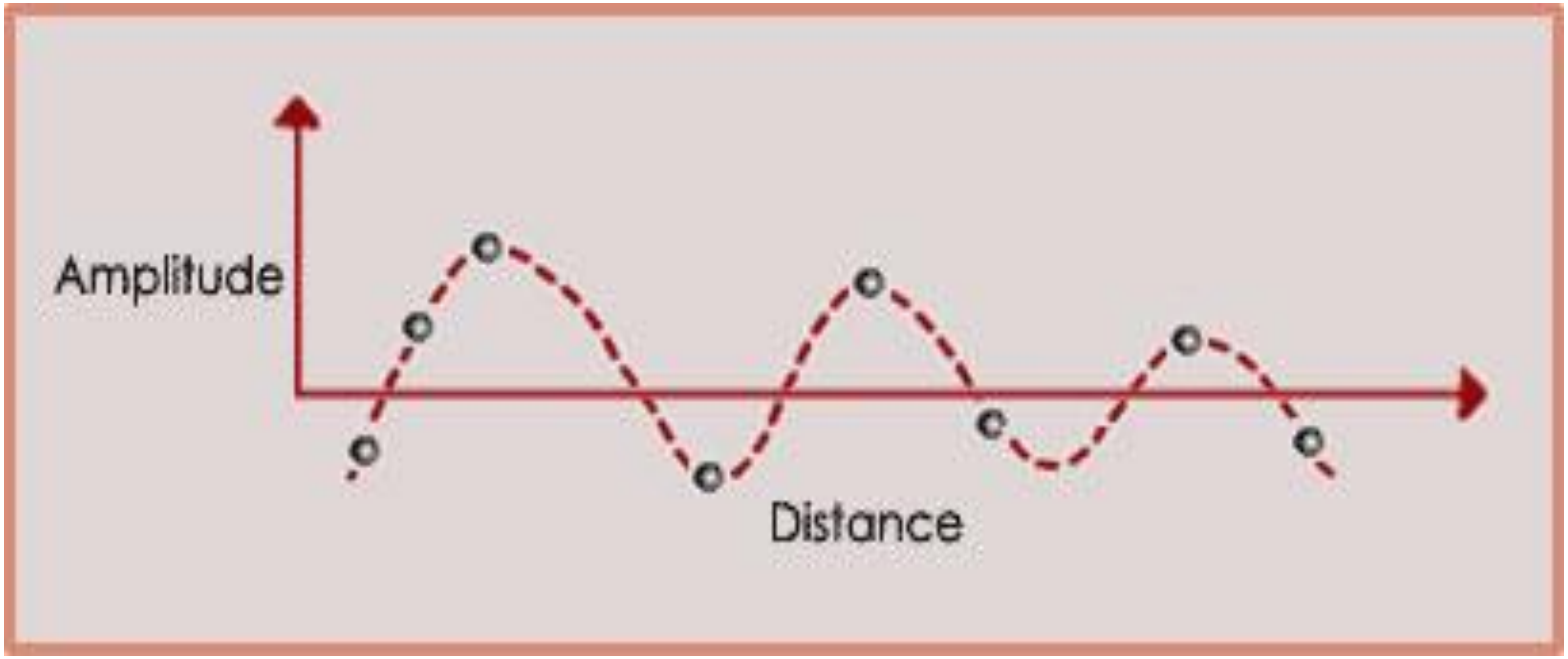
A conductor carrying a current has magnetic field and consequently has flux linkage. The conductor therefore has inductance.

- Due to transit time effect the whole line inductance or capacitance cannot be assumed to be located at a particular point in space.

- The inductance and capacitance are distributed throughout the length of the line. These are therefore called the ' Distributed Parameters' of the line.



PROPAGATION CONSTANT:(β)



- The propagation constant in general is complex.
- The wave amplitude varies as $e^{-\alpha x}$.That is $'\alpha'$ denotes the exponential decay of the wave along its direction of propagation. Therefore is called the 'Attenuation Constant' of the line.

The wave phase has two components:

- Time phase ωt
- Space phase $\pm\beta x$

The parameter β gives the phase change per unit length and hence called the *'Phase Constant'* of the line. Its units are Radian/m.

For a wave the distance over which the phase changes by 2π is called the *wavelength (λ)*.

$$\boldsymbol{\beta = \frac{2\pi}{\lambda}}$$

CHARACTERISTIC IMPEDANCE OF TRANSMISSION LINE:

$$\frac{V^+}{I^+} = \frac{R+J\omega L}{\gamma} = \sqrt{\frac{R+J\omega L}{G+J\omega C}}$$

$$\frac{V^-}{I^-} = - \frac{R+J\omega L}{\gamma} = -\sqrt{\frac{R+J\omega L}{G+J\omega C}}$$

$$Z_0 = \sqrt{\frac{R+J\omega L}{G+J\omega C}}$$

The ratio of Forward Voltage and Current waves is always Z_0,and the ratio of the Backward Voltage and Current waves is always -Z_0

LOSS LESS TRANSMISSION LINE:

In any electrical circuit the power loss is due to ohmic elements. A loss less transmission line therefore implies R= 0, G = 0.

For a loss less transmission line,Propagation constant is,

$\gamma = \sqrt{J\omega L J\omega C} = j\omega\sqrt{LC}$ (purely imaginary)

$$\alpha = 0, \qquad \beta = \omega\sqrt{LC}$$

The characteristic impedance is,

$Z_0 = \sqrt{\frac{J\omega L}{J\omega C}} = \sqrt{\frac{L}{C}}$ (purely real)

The reflection coefficient at any point on the line is,

$$\Gamma(l) = \Gamma_L e^{j2\beta l} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{j2\beta l}$$

DISTORTIONLESS LINE:

A line in which there is no phase or frequency distortion and also it is correctly terminated , is known as distortion less line.

To derive the condition for distortion less line,

$\gamma = \sqrt{(R + J\omega L)(G + J\omega C)}$

$\gamma^2 = (R + J\omega L)(G + J\omega C)$

$\gamma^2 = (RG - \omega^2 LC) + j\omega C(RC + LG)$

We know that for minimum attenuation L= CR/G or LG = CR

$\gamma^2 = (RG - \omega^2 LC) + j2\omega RC$

but, RC = LG= $\sqrt{RCLG}$

$\gamma^2 = RG - \omega^2 LC + 2j\omega\sqrt{RCLG}$

$\gamma^2 = \sqrt{RG} + j\omega\sqrt{LC}$

But $\gamma = \alpha + j\beta$

Then, $\alpha = \sqrt{RG}$ & $\beta = \omega\sqrt{LC}$

α does not vary with frequency which eliminates the frequency distortion.

$$\mathbf{v} = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{LC}}$$

For the condition RC = LG , the velocity becomes independent of frequency.

This eliminates the phase distortion.

All the distortions are eliminated for a condition,

RC = LG i.e. $\frac{R}{G} = \frac{L}{C}$

LOADING:

Introduction of inductance in series with the line is called loading and such lines are called loaded lines.

EFFECT OF LOADING:

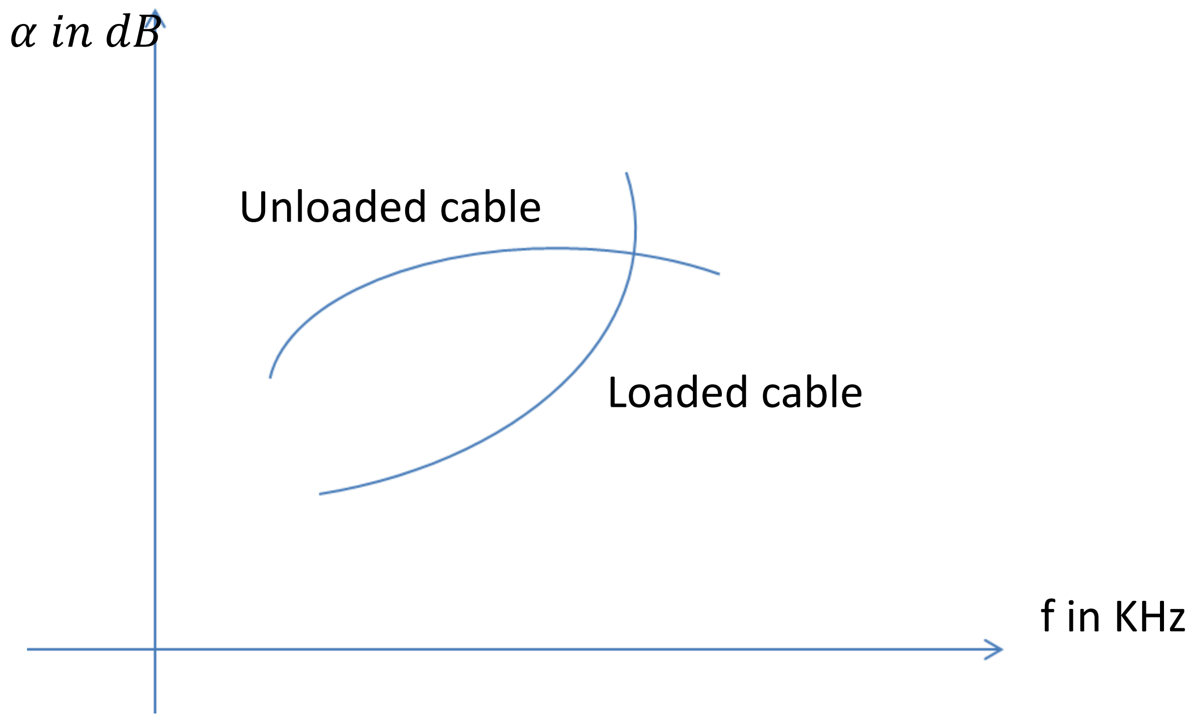


Fig: Effect of loading on the cable

TYPES:

1.CONTINUOUS LOADING:

Here loading is done by winding a type of iron around the conductor.

This increases inductance but it is expensive.

2.PATCH LOADING:

This type of loading uses sections of continuously loaded cable separated by sections of unloaded cable. Hence cost is reduced.

3.LUMPED LOADING:

Here loading is introduced at uniform intervals. It may be noted that hysteresis and eddy current losses are introduced by loading and hence, design should be optimal.

TRANSMISSION LINES-II

OBJECTIVES

INPUT IMPEDANCE RELATIONS

REFLECTION COEFFICIENT

VSWR

IMPEDANCE TRANSFORMATIONS

SMITH CHART AND APPLICATIONS,

SINGLE AND DOUBLE STUB MATCHING

VOLTAGE STANDING WAVE RATIO:(VSWR)

The maximum and minimum peak voltages measured on the line are,

$|V|_{max} = |V^+|(1+|\Gamma_L|)$

$|V|_{min} = |V^+|(1-|\Gamma_L|)$

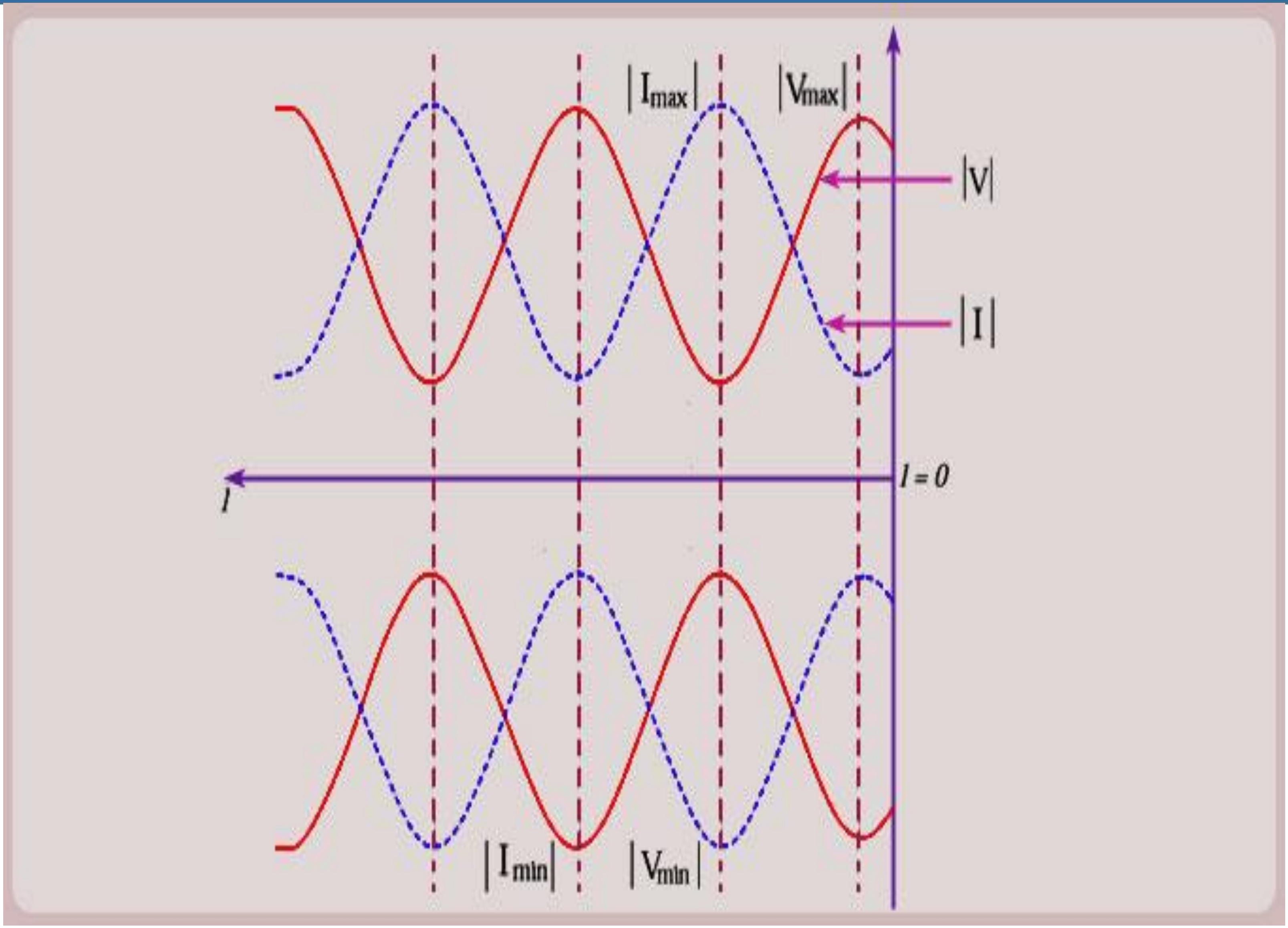
$$\rho = \frac{|V|_{max}}{|V|_{min}}$$

$$\rho = \frac{|V^+|(1+|\Gamma_L|)}{|V^+|(1-|\Gamma_L|)}$$

$$\boldsymbol{\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \text{ (or) } |\Gamma_L| = \frac{\rho-1}{\rho+1}}$$

Higher the value of VSWR, higher is $|\Gamma_L|$ i.e., higher is the reflection and is lesser the power transfer to the load.

$$0 \leq |\Gamma_L| \leq 1 \quad \text{and} \quad 1 \leq \rho \leq \infty$$


|Imax|
|Vmax|
|V|
|I|
l
l = 0
|Imin|
|Vmin|

REFLECTION COEFFICIENT:

The ratio of the amplitudes of the reflected and incident voltage waves at the receiving end of the line is called the reflection coefficient.

$$\Gamma = \frac{\text{reflected voltage at the load}}{\text{incident voltage at the load}}$$

The reflected voltage at load is component E_2 at the receiving end with s=0.

$$E_2 | s = 0 = \frac{E_{R(Z_L - Z_0)}}{2Z_L}$$

The incident voltage at load is component E_1 at the receiving end with s=0.

$$E_1 | s = 0 = \frac{E_{R(Z_L + Z_0)}}{2Z_L}$$

$$\Gamma = \frac{\frac{E_{R(Z_L - Z_{0})}}{2Z_L}}{\frac{E_{R(Z_L + Z_{0})}}{2Z_L}}$$

$$\Gamma = \frac{\frac{E_R}{2Z_L}}{\frac{E_R}{2Z_L}} \frac{(Z_L - Z_{0})}{Z_L + Z_{0})}$$

$$\Gamma = \frac{(Z_L - Z_{0})}{Z_L + Z_{0})}$$

Observations:

1.When $Z_L = Z_{0,}$ Γ =0, there is no reflection.

2.When Z_L = 0, i.e. The line is short circuited.

Γ = -1 = $1\angle + 180^0$ The reflection is maximum.

3. When Z_L = ∞, i.e. The line is open circuited.

Γ = 1 = $1\angle 0^0$ The reflection is minimum.

4.Where K ranges from 0 to 1, and its phase angle ranges from 0 to 180.

INPUT IMPEDANCE RELATIONS:

- Input impedance of a resonant lossless line is either 0 or ∞.
- In practice, the lines have finite loss. This loss has to be included in the calculations while analyzing the resonant lines.
- The complex propagation constant has to be used in impedance calculations of a resonant line.
- The input impedance of a short or open circuited line having propagation constant γ can be written as ,

$Z_{sc} = Z_0 \tan h\gamma l$ for short circuit load

$Z_{oc} = Z_0 \cot h\gamma l$ for open circuit load

$$Z_{sc} = Z_0 \tan h(\alpha + j\beta)l = Z_0\left[\frac{\tan h\alpha l+\tan h(j\beta)l}{1+\tan h\alpha l \tan h(j\beta)l}\right]$$

For a low-loss line, taking $\alpha l \ll 1$ $and,$

tan hαl= αl tan $h(j\beta)l$ = jtan βl

$$Z_{sc} \approx Z_0 \left[\frac{\alpha l + j\tan\beta l}{1 + j\alpha l\tan\beta l}\right]$$

Similarly for an open circuited line we get

$$Z_{oc} \approx Z_0 \left[\frac{1 + j\alpha l\tan\beta l}{\alpha l + j\tan\beta l}\right]$$

If we take l even multiples of $\frac{\lambda}{4}$, tan βl =0,

$$Z_{sc} \approx Z_0\, \alpha l$$

$$Z_{oc} \approx \frac{Z_0}{\alpha l}$$

SMITH CHART:

Smith chart is polar plot of the reflection coefficient in terms of normalized impedance, r+jx.

(or)

It is a graphical plot of normalized resistance and reactance in the reflection coefficient plane.

CONSTRUCTION:ρ_i

It is constructed within a circle of unit radius $\rho \leq 1$

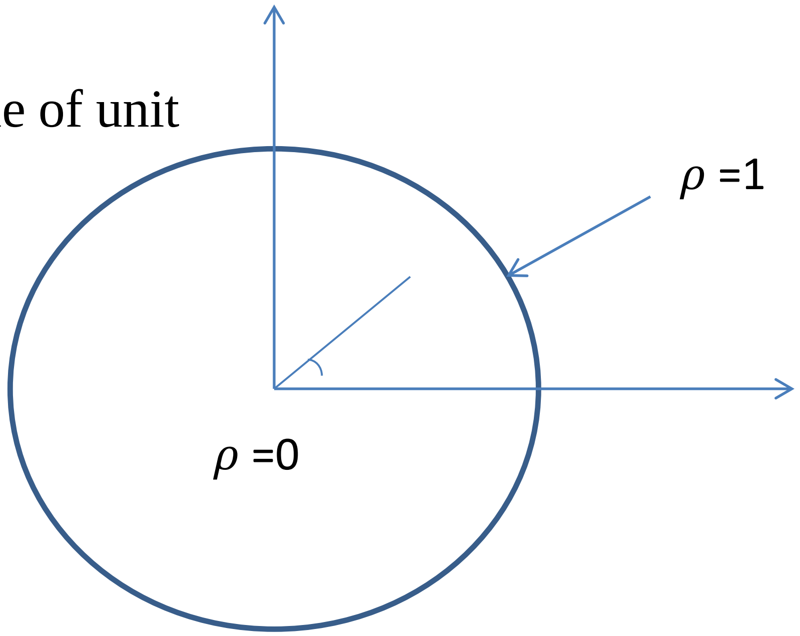


Fig: Construction of smith chart on unit circle

Smith chart provides the relation between reflection coefficient, ρ load, z_L and characteristic impedance, z_0.

$$\rho = \frac{z_L - z_0}{z_L + z_0}$$

$$\rho = |\rho|\angle\theta_\rho = \rho_r + \text{j } \rho_i$$

$$\rho = \rho_r + \text{j } \rho_i = \frac{z_n - 1}{z_n + 1} \qquad (\because z_n = \frac{z_L}{z_0} = \text{r+jx})$$

$$z_n = \frac{(1+\rho_r) + \text{j } \rho_i}{(1-\rho_r) - \text{j } \rho_i}$$

$$\text{r} = \frac{1-\rho_r{}^2-\rho_i{}^2}{(1-\rho_r)^2+\rho_i{}^2} \ \& \ \ \text{x} = \frac{2\rho_i}{(1-\rho_r)^2+\rho_i{}^2}$$

$$[\rho_i - \frac{r}{r+1}]^2 + \rho_i{}^2 = [\frac{1}{r+1}]^2 \quad \text{--------(1)}$$

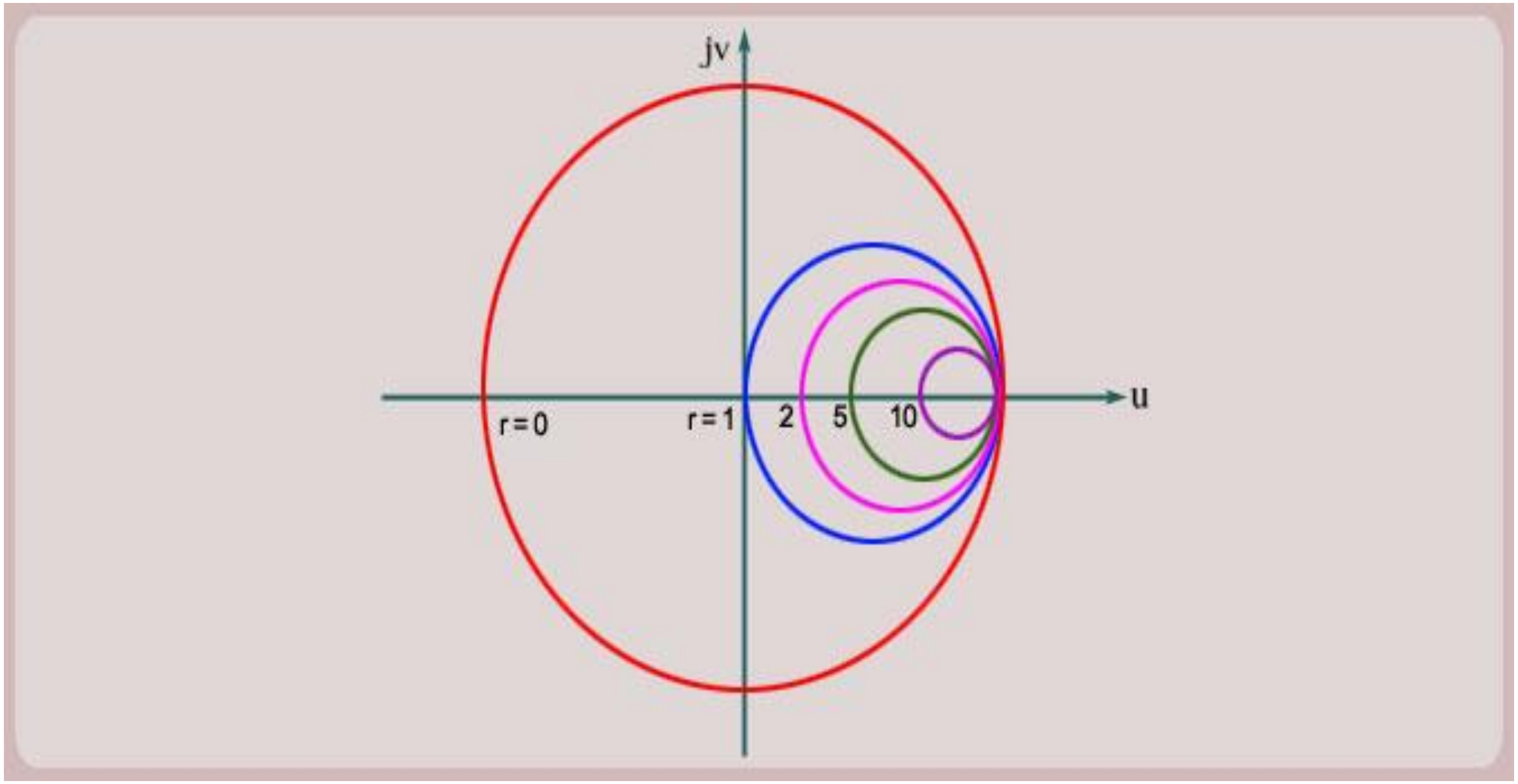
$$(\rho_i - 1)^2 + [\rho_i - \frac{1}{x}]^2 = [\frac{1}{x}]^2 \quad \text{----------(2)}$$

Eq(1) is known as r- circles and eq(2) is known as x- circles.

Resistance circles:

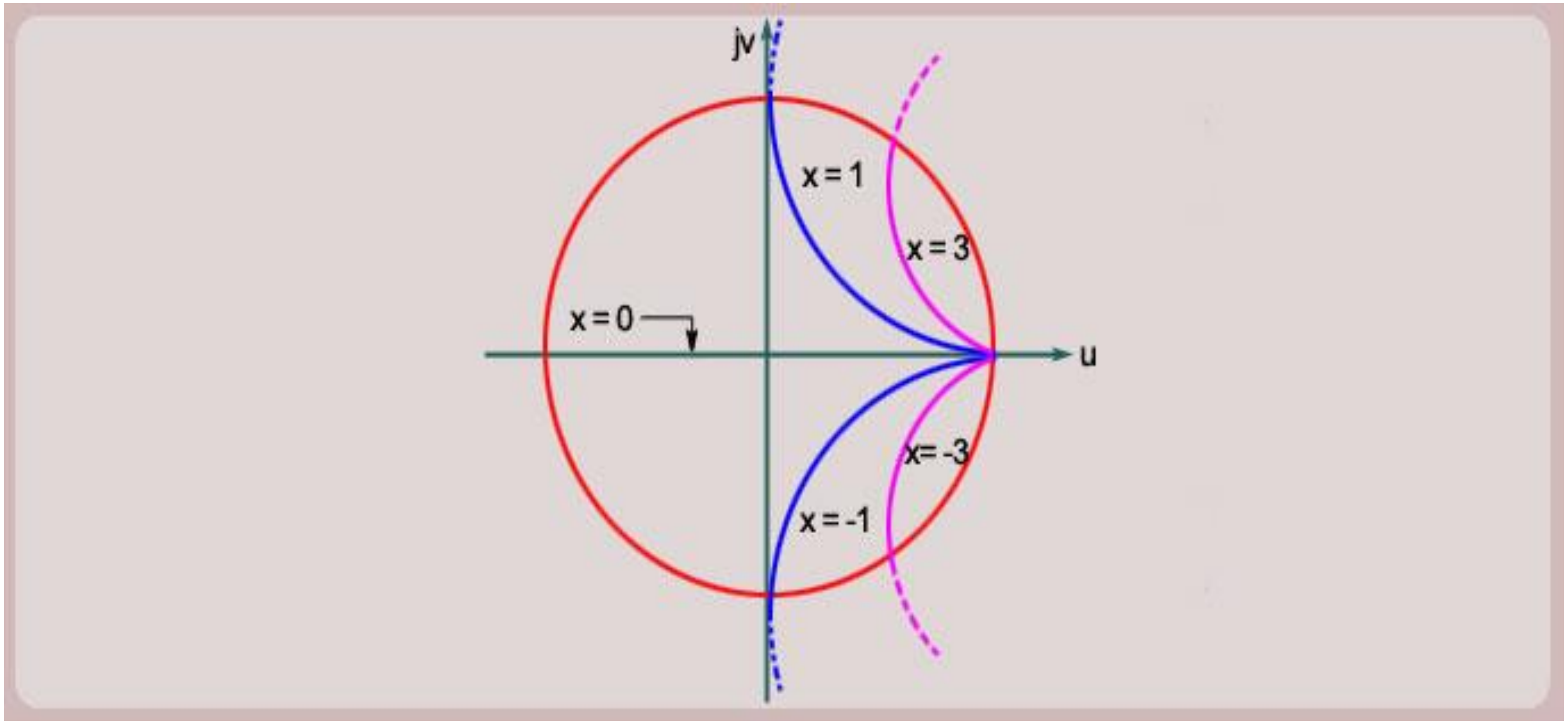
The constant resistance circles have their centers at $(\frac{r}{r+1}$, 0) and radii $(\frac{1}{r+1})$.

Figure below shows the constant resistance circles for different values of ranging between 0 and ∞ .

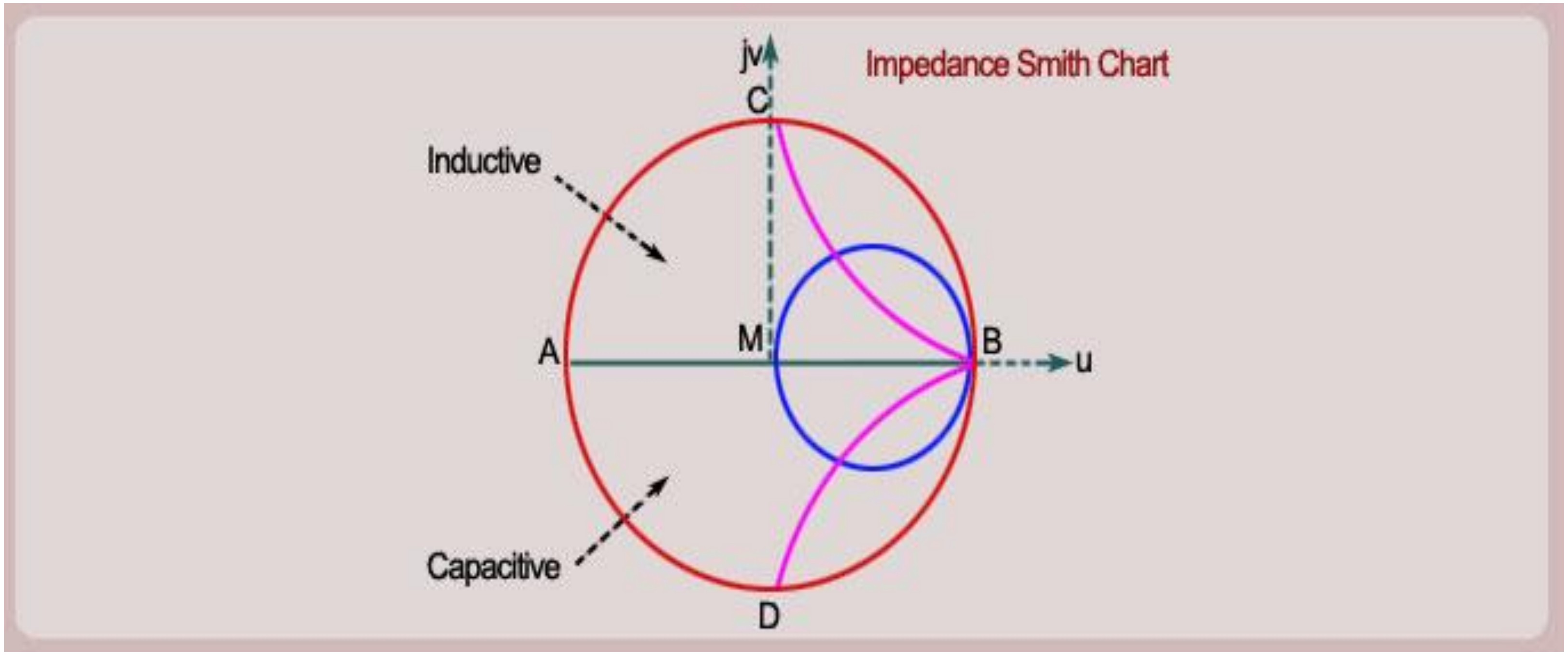


Reactance circles:

The constant reactance circles have their centers at (1 , 1/x) and radii (1/x). The centers for these circles lie on a vertical line passing through point (1,0) in the -plane.



The Smith chart is a graphical figure which is obtained by superposing the constant resistance and the constant reactance circles within the unity circle in the complex Γ -plane. Since we have mapped here the impedances to the - Γ plane, let us call this Smith chart the Impedance Smith chart.



Let us identify some special points on the Smith Chart.

a) The left most point A on the smith chart corresponds to $\gamma = 0, x = 0$ and therefore represents ideal short-circuit load.

b) The right most point B on the Smith chart corresponds to $\gamma = \infty, x = \infty$ and therefore represents ideal open circuit load.

c) The center of the Smith chart M , corresponds to $\gamma = 1, x = 0$ and hence represents the matched load.

d) Line AB represents pure resistive loads and the outermost circle passing through A and B represents pure reactive loads.

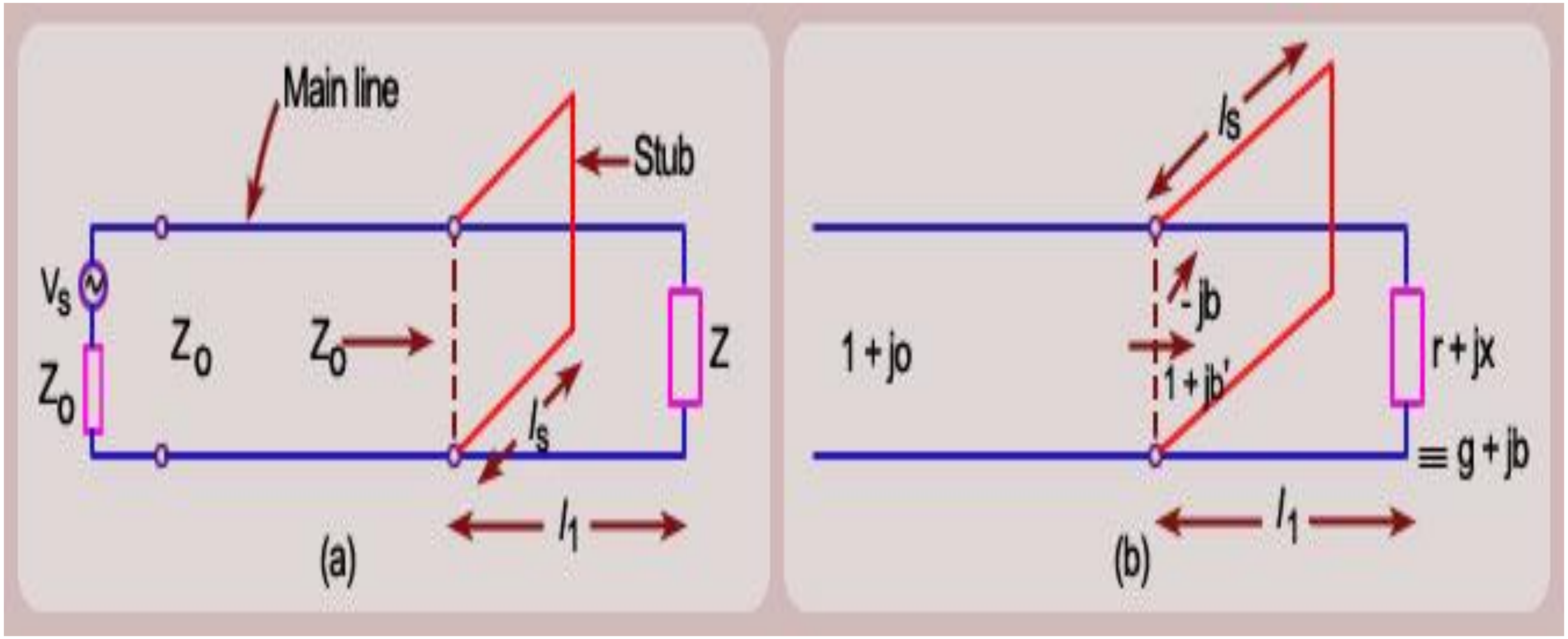
e) The upper most point C represents a pure inductive load of unity reactance and the lower most point D represents a pure capacitive load of unity reactance.

f) In general the upper half of the Impedance Smith Chart represents the complex inductive loads and the lower half represents the complex capacitive loads.

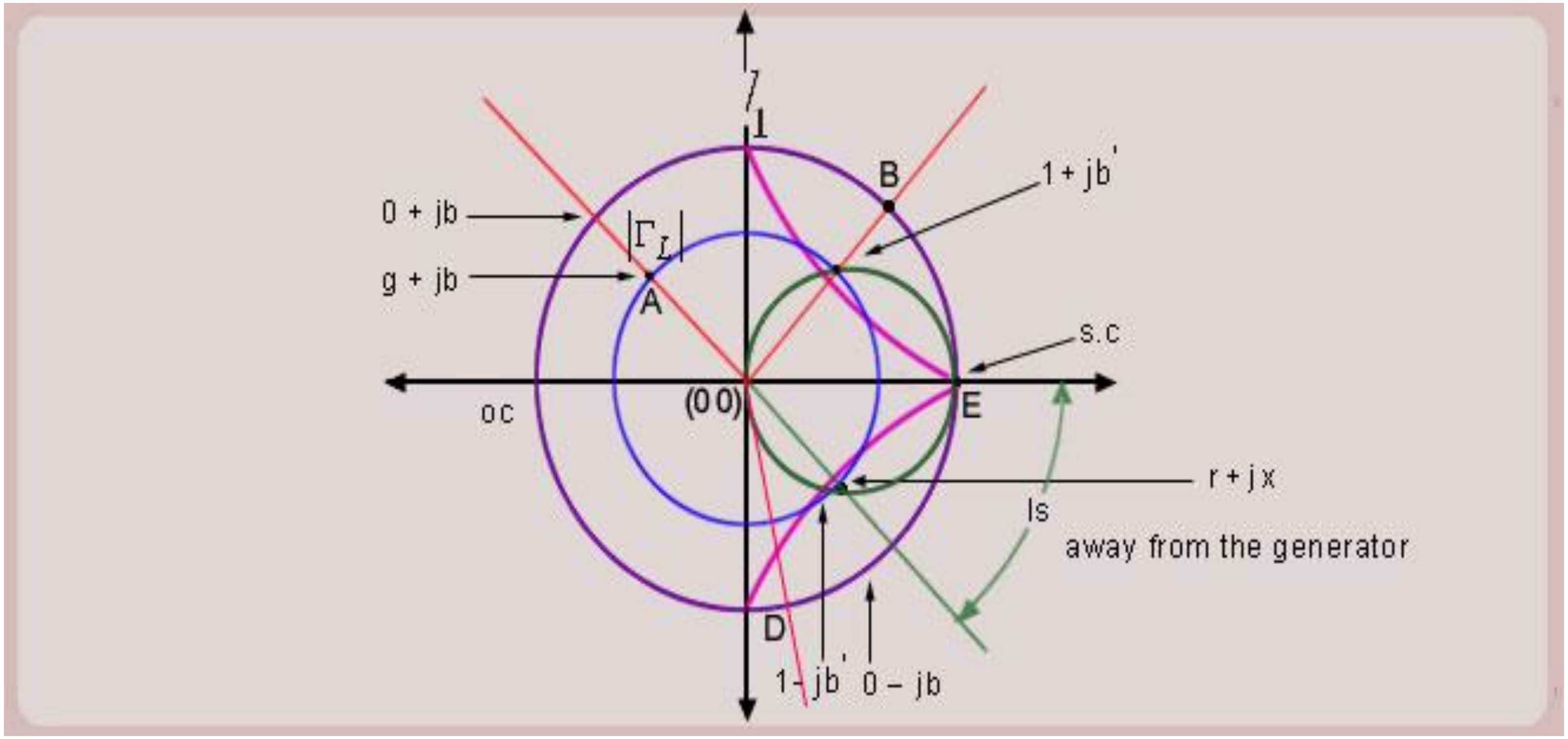
SINGLE STUB MATCHING TECHNIQUE:

- A stub is a short-circuited section of a transmission line connected in parallel to the main transmission line.
- A stub of appropriate length is placed at some distance from the load such that the impedance seen beyond the stub is equal to the characteristic impedance.
- Suppose we have a load impedance Z_L connected to a transmission line with characteristic impedance Z_O.The objective here is that no reflection should be seen by the generator.
- In other words, even if there are standing wavesin the vicinity of the load Z_L, the standing waves must vanish beyond certain distance from the load.

Conceptually this can be achieved by adding a stub to the main line such that the reflected wave from the short-circuit end of the stub and the reflected wave from the load on the main line completely cancel each other at point B to give no net reflected wave beyond point B towards the generator.



- We have a parallel connection of transmission lines, it is more convenient to solve the problem using admittances rather than impedances.
- To convert the impedance into admittance also we make use of the Smith chart and avoid any analytical calculation.
- Now onwards treat the Smith chart as the admittance chart.



Matching procedure:

- First mark the load admittance on the admittance smith chart (A).
- Plot the constant $|\Gamma|$ circle on the smith chart .Move on the constant $|\Gamma|$ circle till you intersect the constant g=1 circle this point of intersection corresponds to point 1+jb' (B). The distance traversed on the constant $|\Gamma|$ circle is l_1 . This is the location of placing the ***stub*** on the transmission line from the load end .
- Find constant susceptance jb' circle.
- Find mirror image of the circle to get –jb' circle.
- Mark 0-jb' on the outer most circle (D).
- From (D) move circular clockwise uptos.c point (E) to get the stub length l_s.

Advantages:

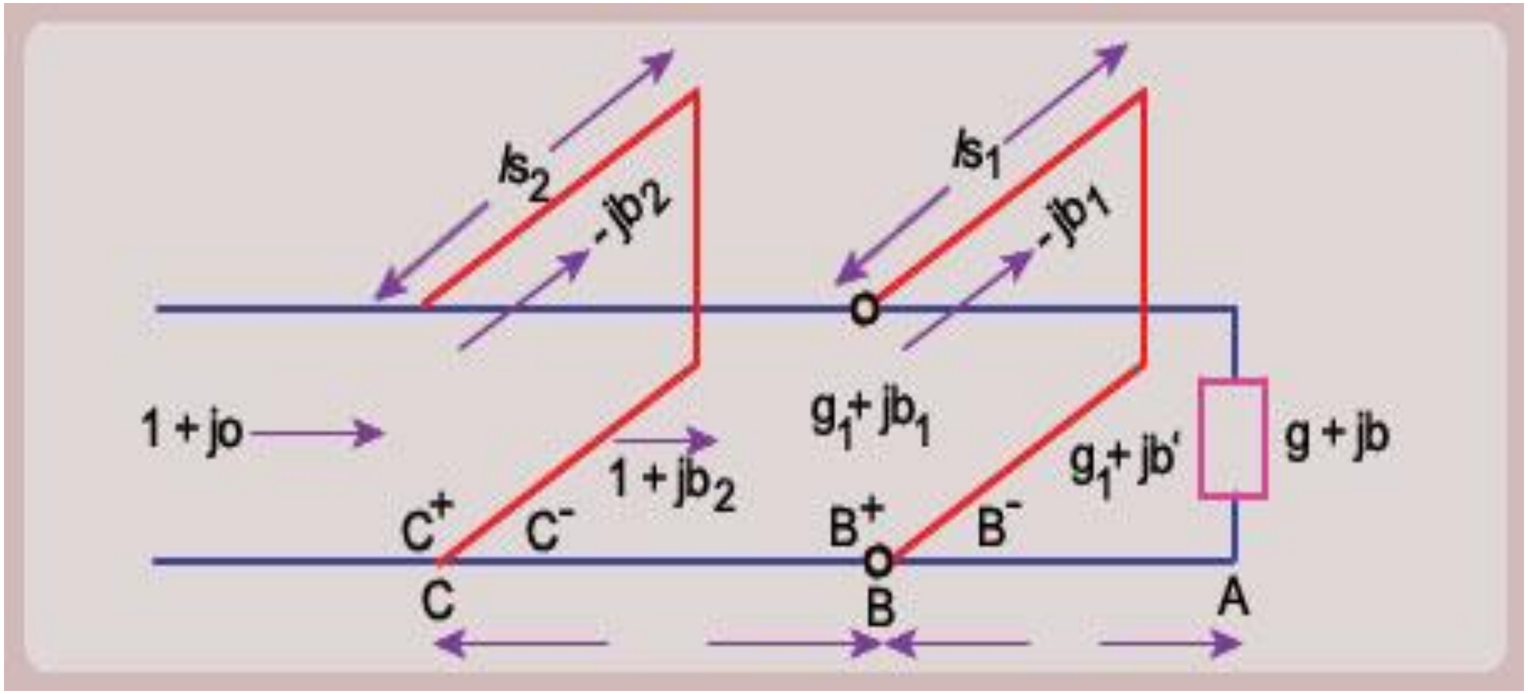
- The single-stub matching technique is superior to the quarter wavelength transformer as it makes use of only one type of transmission line for the main line as well as the stub.
- This technique also in principle is capable of matching any complex load to the characteristic impedance/admittance.
- The single stub matching technique is quite popular in matching fixed impedances at microwave frequencies.

Drawback:

- The single stub matching technique although has overcome the drawback of the quarter wavelength transformer technique, it still is not suitable for matching variable impedances.
- A change in load impedance results in a change in the length as well as the location of the stub.
- Even if changing length of a stub is a simpler task, changing the location of a stub may not be easy in certain transmission line configurations.
- For example, if the transmission line is a co-axial cable, the connection of a stub would need drilling of a hole in the outer conductor.

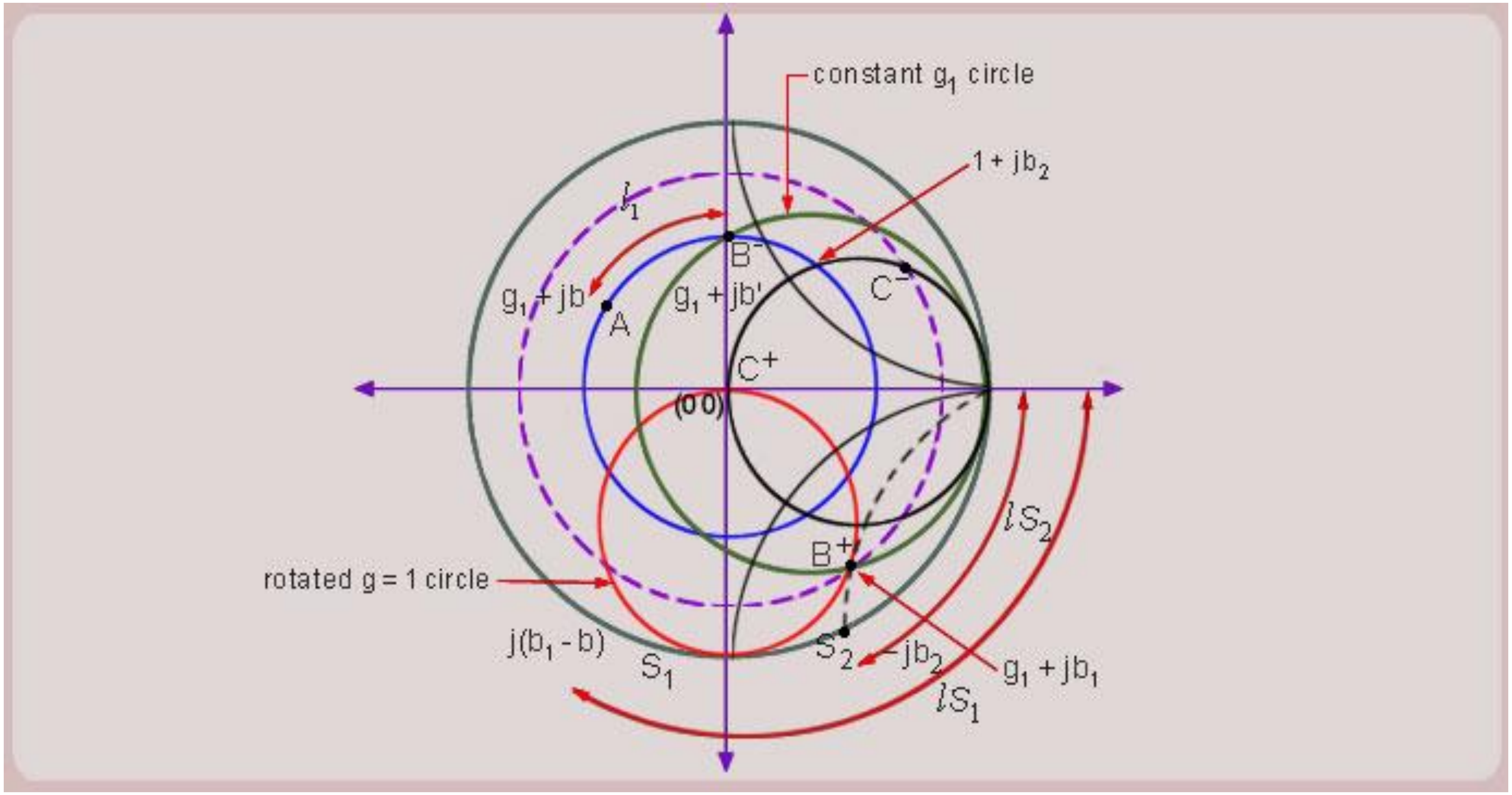
DOUBLE STUB MATCHING TECHNIQUE:

The technique uses two stubs with fixed locations. As the load changes only the lengths of the stubs are adjusted to achieve matching.



Let us assume that a normalized admittance g+jb is to be matched using the double stub matching technique.

The first stub is located at a convenient distance from the load say l_1 . The second stub is located at a distance of $3\lambda/8$ from the first stub.



Matching procedure:

- Mark the admittance g+jb on the Smith chart (Point A).
- Move on constant VSWR circle passing through A by a distance l_1 to reach B^-.
- Move along the constant-conductance (constant- g) circle to reach B^+ (a point on the rotated g=1 circle). Note that a stub at B will change only the reactive part and therefore we move on a circle which keeps the real part of g_1+jb' same while going from B^- to B^+ .
- Transform admittance g_1+j b_1 at B^+ to C^- by moving a distance of $3\lambda/8$ on a constant VSWR circle passing through B^+ . The point C^- must be lying to the g=1 circle. Let the transformed admittance at point C^- be 1+j b_2
- Add a stub to give susceptance -jb_2 at location C so as to move the point C^- to C^+ which is the matched point

- To calculate the length of the first stub l_{s1} we note that this stub must provide a susceptance which is the difference between the susceptances at B^+ and B^- . That is, the stub susceptance b_{s1} is equal to $b_1 - b'$. Mark the susceptance j(b-b') on the chart to get point S_1 .Distance from S_1 to S in the anticlockwise direction gives the length l_{s1} of the first stub.
- The second stub should have a susceptance of -jb_2 . To get the length l_{s2} of the second stub the procedure is same as that used in the single stub matching. That is, mark -jb_2 on the Smith chart to get point S_2 . Measure distance S_2S in anti-clockwise direction to give ls_2 .

Limitation:

- The whole matching process relies on the fact that by moving along a constant conductance circle one can go from point B^- to B^+ . (B^+ lies on the g = 1 rotated circle). If this step is not realizable then the whole matching process is unrealizable.

- If point B^- lies in the hatched region, moving along constant-g circle can never bring a point on rotated g=1 circle. Hence that admittance cannot be matched by the Double Stub method.