INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech III Semester End Examinations (Regular) - December, 2017

Regulation: IARE – R16

MATHEMATICAL TRANSFORM AND TECHNIQUES

(Common to AE | ECE)

Time: 3 Hours   **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Find the Fourier series of $f(x) = x^2$ in (0,4) hence the value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$ **[7M]**

   (b) Find the Fourier series of $f(x) = |x|$ on $(-\pi, \pi)$. **[7M]**

2. (a) Obtain the Fourier series of **[7M]**

   $$f(x) = \begin{cases} 1 + (2x/\pi), & -\pi \le x \le 0 \\ 1 - (2x/\pi), & 0 \le x \le \pi \end{cases}$$

   (b) Find half range Fourier cosine series of $f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l-x), & l/2 \le x \le l \end{cases}$ and hence deduce

   that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + ...$ **[7M]**

UNIT – II

3. (a) Find the Fourier transform of $e^{-\alpha x^2}$ **[7M]**

   (b) Find the Fourier sine transform of $e^{-|x|}$ and hence evaluate $\int\limits_0^\infty \frac{x \sin(mx)}{1+x^2}$. **[7M]**

4. (a) Find the Fourier cosine and sine transform of $f(x) = \begin{cases} x \ \text{for} 0 < x < 1 \\ 2 - x \ \text{for} 1 < x < 2 \\ 0 \ \text{for} x > 2 \end{cases}$ **[7M]**

   (b) Find the finite Fourier cosine and sine transform of $f(x) = \begin{cases} 1 \ \text{for} 0 < x < \frac{\pi}{2} \\ -1 \ \text{for} \frac{\pi}{2} < x < \pi \end{cases}$ **[7M]**

UNIT – III

5. (a) Find $L\left\{\left(1 + te^{-t}\right)^3\right\}$. **[7M]**

   (b) Find the value of f(t), given that $f(t) - \int_0^t (t - \tau) f(\tau) d\tau = -t$. **[7M]**

6. (a) Obtain the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ where $f(t + 2a) = f(t)$

    and hence shown that $L\{f(t)\} = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$. **[7M]**

    (b) Solve using Laplace transforms, $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 3te^{-t}$, $x = 4, \frac{dx}{dt} = 2$ when t=0. **[7M]**

UNIT − IV

7. (a) Find Z-transform of **[7M]**
       i. $a^{-n}cosnq$.
       ii. $Z\left(\frac{1}{n!}\right)$ $and\, Z\left(\frac{1}{(n+2)}\right)$

    (b) Applying resolving into partial fractions, find inverse Z-transform of $\frac{z(2z+3)}{(z+2)(z-4)}$. **[7M]**

8. (a) Using the Z-transform, solve $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$ with $u_0=0$, $u_1=0$. **[7M]**

    (b) Using the inverse integral method, find the inverse Z-transform of $\frac{z}{(z-1)(z-3)}$. **[7M]**

UNIT − V

9. An insulated rod of length has its ends at A and B maintained at $0°C$ and $100°C$ respectively

   under steady conditions are reached. If B is then suddenly reduced to $0°C$ and maintained

   at $0°C$. Find the temperature at a distance x from A at time 't'. **[14M]**

10. (a) Form a partial differential equation by eliminating arbitrary function from **[7M]**
        $z = xf(ax + by) + g(ax + by)$.

    (b) Solve $z_{xx} - 2z_x + z_y = 0$ by separation of variables. **[7M]**

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