## INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)
Four Year B.Tech III Semester End Examinations (Supplementary) - July, 2018
Regulation: IARE - R16
MATHEMATICAL TRANSFORM TECHNIQUES
Time: 3 Hours
(COMMON TO AE \| ECE)
Max Marks:
70
Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

## UNIT - I

1. (a) Obtain the Fourier series of $\sqrt{1-\cos x}$ in $(0,2 \pi)$ and hence deduce that $\frac{1}{2}=\sum \frac{1}{4 x^{2}-1}$.
(b) Find the Fourier series of $f(x)=\left\{\begin{array}{l}4-x, 3<x<4 \\ x-4, \\ 4<x<5\end{array}\right.$
2. (a) Find the Fourier series of $f(t)= \begin{cases}0 \text { if } & -\pi<t<-\frac{\pi}{2} \\ 5 \text { if } & -\frac{\pi}{2}<t<\frac{\pi}{2} \\ 0 \text { if } & \frac{\pi}{2}<t<\pi\end{cases}$
(b) Find the Fourier series of $f(t)=\left\{\begin{array}{cc}0 \text { if } & -\pi<t<0 \\ t \text { if } & 0<t<\pi\end{array}\right.$

UNIT - II
3. (a) Find the Fourier transform of $e^{-\frac{|t|}{T}}$.
(b) Find the Fourier sine transform of $e^{-|x|}$ and hence evalute $\int_{0}^{\infty} \frac{x \sin (m x)}{1+x^{2}}$.
4. (a) Find the Fourier sine transform of $\frac{e^{-a x}}{x}$.
(b) Find $\mathrm{f}(\mathrm{x})$ if its Fourier sine transform is $w /\left(w^{2}+1\right)$.

## UNIT - III

5. (a) Show that $L\{\sin \sqrt{t}\}=\frac{1}{s} e^{-s / w} \cdot \sqrt{\frac{\pi}{s}}$.
[7M]
(b) A periodic function of period $(2 \pi / w)$ defined by $f(t)=\left\{\begin{array}{c}E \sin w t, \quad 0 \leq t<\pi / w \\ 0, \quad \pi / w \leq t<2 \pi / w\end{array}\right.$

Where E and W are constants show that $L\{f(t)\}=E W \mid\left(s^{2}+w^{2}\right)\left(1-e^{-\pi s / w}\right)$.
[7M]
6. (a) Find i) $L^{-1}\left\{\frac{5 s+3}{(s-1)\left(s^{2}+2 s+5\right.}\right\}$ ii) $L\left\{\frac{\cos 6 t-\cos 4 t}{t}\right\}$
[7M]
(b) The current i and q in a series circuit containing an inductance L , a capacitance C , e.m.f. E satisfying the D.E. Express $L \frac{d i}{d t}+\frac{q}{c}=E$, i and q in terms of t given that L, C, E are constants and $\mathrm{i}, \mathrm{q}$ both are initially zero using Laplace transforms.

## UNIT - IV

7. (a) Find the Z-transform of $(n+p) C_{p}$.
(b) By resolving into partial fractions find $Z_{T}^{-1}\left[\frac{4 z^{2}-2 z}{z^{3}-5 z^{2}+8 z-4}\right]$.
8. (a) By using convolution theorem, find inverse Z-transform of $\frac{z}{(z-a)^{3}}$ and hence deduce for $\left(\frac{z}{z-1}\right)^{3}$.
(b) Solve the difference equation $u_{n+2}-2 u_{n+1}-3 u_{n}=3^{n}+2 n, u_{0}=0, u_{1}=0$.

## UNIT - V

9. (a) Find the temperature $u(x, t)$ in a homogeneous bar of heat conducting material of length $L$ in cm with its ends kept at zero temperature and initial temperature given by $d x(L-x) / L^{3}$.
(b) Solve $(y+z x) p-(x-y z) q=x^{2}-y^{2}$.
10. (a) Solve $\left(x^{2}-2 y z-y^{2}\right) d x+(x y+x z) d y=(x y-x z)$.
[7M]
(b) Solve $u_{x x}=u_{y}+2 u$ by separation of variables under the condition that $u=0, u_{x}=e^{-y}$ when $\mathrm{x}=0$ and for all y .
[7M]
