Hall Ticket No | | | | | | | | | | | Question Paper Code: AEC003

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech III Semester End Examinations (Supplementary) - January/February, 2018

Regulation: IARE – R16

PROBABILITY THEORY AND STOCHASTIC PROCESSES

(Electronics and Communication Engineering)

Time: 3 Hours **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) In a bolt factory, machines manufacture 20%, 30% and 50% of the total of their output and respectively 6%, 3% and 2% are defective bolts. A bolt is drawn at random and is found to be defective. Determine the probabilities that it was manufactured by the machine. **[7M]**

   (b) If A,B and C are mutually independent events, prove that $A \cup B$ and C are independent. **[7M]**

2. (a) If A,B and C are events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$, find **[7M]**

      i. $P\left(\frac{B}{A}\right)$

      ii. $P\left(\frac{B}{A^C}\right)$

   (b) State and prove total probability theorem. **[7M]**

UNIT – II

3. (a) Define distribution function and write its properties for a single random variable X. **[7M]**

   (b) A random variable is known to have a distribution function $F_X(x) = u(x)\left[1 - e^{\frac{-x^2}{b}}\right]$, where $b > 0$ is a constant. Find its density function. **[7M]**

4. (a) A random variable X can have values -4,-1,2,3 and 4 each with a probability 1/5. Find mean and variance of the random variable y=$3x^3$ **[7M]**

   (b) If X has the probability density function $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$, show that the characteristic function of X is given by $\varphi_x(x) = \frac{1}{1+t^2}$. Hence find the mean and variance of X. **[7M]**

UNIT – III

5. (a) Show that the density function of the sum of two statistically independent random variables is the convolution of their individual density functions. **[7M]**

   (b) Two random variable X and Y are defined by $\overline{X} = 0$, $\overline{X} = -1$, $\overline{X^2} = 2$, $\overline{Y^2} = 4$, $R_{XY} = -2$. Two new random variables W and U are W=2X+Y, U=-X-3Y. Find $\overline{W}$, $\overline{U}$, $\overline{W^2}$, $\overline{U^2}$, σ_{X^2}. **[7M]**

6. (a) State the properties of Gaussian random variables. **[7M]**

   (b) The joint density function of random variables X and Y is

$$f_{X,Y}(X,Y) = \begin{cases} \frac{25}{23ab}\left(\frac{y}{b}\right)\left[1-(x/b)^4(y/a)^3\right] & ,-\infty < x < b, o < y < a \text{ a, b>0.} \\ 0 & ,elsewhere \end{cases}$$

   Find the marginal densities of X and Y. **[7M]**

UNIT – IV

7. (a) Given a random process $X(t) = ACos(w_0t) + BSin(w_0t)$ where w_0 is a constant and A and B are uncorrelated non-zero random variables having different density functions but the same variances. Show that X(t) is WSS. **[7M]**

   (b) A Gaussian random process has an auto correlation function $R_{XY}(\tau) = 6exp\left[-\frac{|\tau|}{2}\right]$. Determine a covariance matrix for the random variables X(t), X(t+1) and X(t+2) and X(t+3). **[7M]**

8. (a) Define a random process by X(t)=Acos(t), where A is a Gaussian Random Variable with zero mean variance σ_A^2 **[7M]**

      i. Find the density function of X(0) and X(1)
      ii. Is X(t) stationary in any sense

   (b) If X(t) is a stationary random process having a mean value E[X(t)]=3 and auto correlation function $R_{XY}(\tau) = 9 + 2e^{-|\tau|}$ find **[7M]**

      i. The mean value and
      ii. The variance of the random variable $Y = \int\limits_0^2 X(t)\,dt$.

UNIT – V

9. (a) Consider a random process $X(t) = ACos(w_0t + \theta)$, where A and w_0 are real constants. Find average power of X. **[7M]**

   (b) The auto correlation function of a random process X(t) is $R_{XX}(\tau) = 3 + e^{-4\tau^2}$, find the power spectrum of X(t). **[7M]**

10. (a) Find the autocorrelation function corresponding to the power spectrum **[7M]**

   $S_{xx}(w) = {}^{8}\!\big/_{(9+w^2)^2}$

   (b) State properties of cross power density spectrum **[7M]**

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