Hall Ticket No | | | | | | | | | | | Question Paper Code: AEC003

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech III Semester End Examinations (Regular) - December, 2017

Regulation: IARE – R16

PROBABILITY THEORY AND STOCHASTIC PROCESS

(Electronics and Communication Engineering)

Time: 3 Hours **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) A missile can be accidentally launched if two relays A and B both have failed. The Probabilities of A and B failing are known to be 0.01 and 0.03 respectively. It is also known that B is more likely to fail (probability 0.06) if A has failed. **[7M]**
   i. What is the probability that A will fail if B has failed?
   ii. Are the events A fails and B fails statistically independent?

   (b) The diameter of an electric cable X is a continuous random variable with PDF, **[7M]**

$$f(x) = \begin{cases} Kx(1-x), 0 \le x < 1 \\ 0, elsewhere \end{cases}$$ Find

   i. K
   ii. CDF of X P(X less than or equal to 1/2)

2. (a) State and expalin Baye's theorem. **[7M]**

   (b) An Urn contains 5 balls. Two balls are drawn at random and found to be white. What is the probability of all the balls being white? **[7M]**

UNIT – II

3. (a) A random variable X has the density function $e^{(-x)}$),x>0 Show that Chebyshev's inequality gives P[|X-1|>2]<1/4 and show that actual probability is e^{-3}. **[7M]**

   (b) If X is a random variable uniformly distributed in (0,1), Find the PDF of Y=SinX. Also find the mean and variance of Y. **[7M]**

4. (a) Define conditional distribution function and write its properties **[7M]**

   (b) If the probability density function of a random variable is given by **[7M]**

$$f_X(x) = \begin{cases} 0 & 3 > x \ge 13 \\ (x-3)/25 & 3 \le x < 8 \\ (8.2-x)/25 & 8 \le x < 13 \end{cases}$$

   Compute the probability that has values greater than 4.5 but not greater than 6.7.

UNIT – III

5. (a) Given the function $g(x,y) = \begin{cases} b\,e^{-x}\cos y, & 0 \le x \le 2 \text{ and } 0 \le y \le \frac{\pi}{2} \\ 0, & elsewhere \end{cases}$ Find the value of the constant so that is a valid probability density function. **[7M]**

   (b) A joint probability density function is **[7M]**

$$f_{x,y}(x,y) = \begin{cases} \frac{1}{ab}, & 0<x<a \text{ and } 0<y<b \\ 0 & elsewhere \end{cases}$$

   Find the value of the constant so that is a valid probability density function

6. (a) A random process has the power spectrum density $S_{xx}(w) = {}^{6w^2}/_{1+w^4}$ Find the average power of the process **[7M]**

   (b) The joint distribution function for the two random variables and is $F_{X,Y}(x,y) = u(x)u(y)[1 - e^{-0.5x} - e^{-0.5y} + e^{-0.5(x+y)}]$ where is the unit step function. Find **[7M]**

   i. $P[X \le 1, Y \le 2]$
   ii. $P[0.5 < X < 1.5]$
   iii. $P[-1.5 < X < 2, 1 < Y \le 3]$

UNIT – IV

7. (a) Explain cross correlation of two random processes with properties. **[7M]**

   (b) Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is wide-sense stationary if it is assumed that A and ω_0 are constants and θ is a uniformly distributed random variable on $(0,\ 2\pi)$. **[7M]**

8. (a) Define random process and classify the random processes. **[7M]**

   (b) A random process is defined by $Y(t) = X(t)\cos(\omega_0 t + \theta)$ where $X(t)$ is wide sense stationary random process that amplitude-modulates a carrier of constant angular frequency ω_0 with a random phase θ independent of $X(t)$ and uniformly distributed on $(-\pi,\ \pi)$. Determine $E[Y(t)]$ and autocorrelation of $Y(t)$. **[7M]**

UNIT – V

9. (a) Explain Power Density Spectrum with properties. **[7M]**

   (b) Determine the average power of the random process $X(t) = A_0\cos(\omega_0 t + \theta)$ where A_0 and ω_0 are constants and θ is a uniformly distributed random variable on $\left(0,\ \frac{\pi}{2}\right)$ **[7M]**

10. (a) Determine the rms bandwidth of power density spectrum **[7M]**

$$S_{XX}(\omega) = \begin{cases} P & |\omega| < W \\ 0 & |\omega| > W \end{cases}$$

   (b) Show that the power density spectrum of a random process $X(t)$ is an even function. **[7M]**

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